THE FULL CONTROLLED DEFORMATIONS OF ACTIVE COMPOSITE PLATES AND SHELLS WITH SHAPE MEMORY ALLOYS

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SUMMARY Active Composites with Shape Memory Alloys (SMA) filaments and layers are considered with the goal to elaborate methods of simulation of the SMA behavior in composite structures, to create the analytical method for solution of simplest non-connected problems and to set algorithms of the numerical procedure for connected problems, to derive the conditions for closed two way shape memory effects in such composites and to elaborate the optimal design of mechanical parameters of the active materials structures. The constitutive equations are formulated, refined theories of the SMA beams, plates and shells and methods of solutions are proposed. The variation of the Stress-strain State in composite structures with the phase transformations is discussed.

KEYWORDS: thermodynamic constitutive equations, composites reinforced by shape memory alloys, refined theories of composite structures, optimal design.

INTRODUCTION

Shape Memory Alloys show a considerable promise for new structural and advanced technology creation. Restriction to widespread use of SMA and SMA composites is connected with lack of complete set of the constitutive equations, models of deformations and methods of solutions that would describe the whole set of their unique mechanical properties of the SMA [1-6]. Well-known properties of SMA, like shape memory itself and high damping, exclusive of the phenomena of oriented transformation are used as a rule in practice [7-10]. The constitutive equations for phase, elastic and thermal deformations, equation for martensitic volume fraction must be solved in conjunction with equilibrium and compatibility equations. In this paper the thermodynamic constitutive model is derived and constitutive equations for SMA are formulated. The refined theories of the SMA beams, plate and shell are proposed and refined methods of stress investigations are discussed for SMA composites. The active composites reinforced by SMA are investigated based on the composite structure mechanics, and optimal parameters are established.

THERMODYNAMIC CONSTITUTIVE EQUATIONS

The thermodynamic constitutive model is used to derive constitutive equations for SMA[11]. Assume that the dissipation heat increment dW is proportional to increment of the martensitic volume fraction ξ:

$$dW = k\frac{d\xi}{dt}d\xi \tag{1}$$

Here k is scalar function of variables ξ, σ, ε, T. Assume also that the martensitic volume fraction ξ, temperature T, stress tensor σ, strain tensor ε are independent variables. Then we can prove that the following relation must satisfies

$$d\varepsilon = Ad\sigma + BdT + Cd\xi \tag{2}$$

where A, B–thermoelastic compliance tensor and thermal expansion coefficient tensor. We will use the generalized relations for A and B similar to thermomechanical equations in the theory of elasticity

$$A_{ij\alpha\beta} = \hat{\lambda}\delta_{ij}\delta_{\alpha\beta} + \hat{\mu}\left(\delta_{i\alpha}\delta_{j\beta} + \delta_{j\alpha}\delta_{i\beta}\right) \tag{3}$$

Here $\hat{\lambda}, \hat{\mu}$, α are thermoelasticity coefficients that depend on ξ, T;. The third member in Eqn 2 is phase deformation connected with phase martensite-austenit and austenit-martensite transformations. The scalar coefficient C and tensor coefficients B and A in Eqs.2, 3 are functions of the variables ξ, T, σ, ε.

Taking into account the thermodynamic equations the martensite volume fraction dependence on temperature T and stresses σ can be obtained (for forward transformation) Eqn 1

$$\xi = \psi^{-1}\left[s^*\left(T - M_s^0\right) + \sigma C + \frac{1}{2}\frac{\partial\hat{\lambda}}{\partial\xi}I_1^2(\sigma) + \frac{\partial\hat{\mu}}{\partial\xi}I_2(\sigma) - \Delta W^*\ \right] \tag{4}$$

Here $\frac{df}{d\xi} = \psi(\xi)$, $f(\xi)$ describes transformation induced strain hardening in the SMA material; s is specific entropy [13, 14]; ΔW^* is critical value of the thermodynamic force that corresponds to the phase deformation start and end; $I_{1,2}(\sigma)$ are the first and second invariant of stresses $f(\xi)$; M_s^0 is critical temperature of the phase transformation start.

The similar relations can be written for reversible transformations. As a result, we can write the constitutive equations for phase deformations

$$d\varepsilon = d_{\xi=const}\varepsilon_k + Cd\xi\,, \qquad C_{ij} = a\delta_{ij} + c\Lambda_{ij} \tag{5}$$

where a, c are scalar functions of ξ, T and invariant of σ and ε; Λ is tensor of the second rank that depends on ξ, T, σ, ε; ε_k is deformation tensor of classical thermoelasticity. Using the linear approximation for function C, Eqns 3, 4 and 5, we can write a general system of the constitutive equations in the following form:

$$\varepsilon = \varepsilon^e + \varepsilon^T + \varepsilon^{ph}, \qquad \frac{d\varepsilon_{ij}^{ph}}{d\xi} = a^{\pm}\delta_{ij} + b^{\pm}\sigma_{ij}' + \gamma^{\pm}\varepsilon_{ij}^{ph} \tag{6}$$

$$\sigma = E\varepsilon^e, \qquad \varepsilon^T = BT, \qquad E = E\left[\xi(\sigma,T)\right], \qquad B = B\left[\xi(\sigma,T)\right]$$

$$\xi = \varphi^+\left[s^+\left(T - M_s^0\right) + kI(\sigma,\varepsilon)\right],\ d\xi > 0,\ \xi = \varphi^-\left[s^-\left(T - A_s^0\right) + kI(\sigma,\varepsilon)\right],\ d\xi < 0$$

Where ε is the total strain tensor, ε^e, ε^T are elastic and temperature strain tensors, ε^{ph} is transformation strain tensor, σ'_{ij} is deviator stress tensors, I is some invariant of stress and strain, $a^{\pm}, b^{\pm}, \gamma^{\pm}, s^{\pm}, k$ are scalar factors that may depend on the ξ, T; M^0_s is martensitic start temperature without stresses ($\sigma_{ij}=0$), A^0_s is austenit start temperature, superscripts ″±″ refer to forward and reverse phase transformations respectively, E is modulus of the elasticity.

The differential equations for phase deformations of system (6) must be solved taking into account the initial conditions $\varepsilon^{ph}\big|_{\xi=\xi_0} = \varepsilon^{ph}{}_0$. The functions $\varphi^{\pm}$ describe ξ explicitly in terms of temperature and applied stresses. These functions have a specific form for exponential, polynomial and cosine models, respectively. It is observed that factor b is taken as zero in Eqn 6 for volume part of phase deformation ε_{ii}^{ph}. The above equations for phase deformations allow correctly describe the whole set of the known experimental data of SMA, including the oriented transformation phenomena. The simplified forms of constitutive equations can be used to describe almost all set of the forward and reversible transformations phenomena's in the SMA: *forward transformations (*$d\xi > 0$*)*

$$\frac{d\varepsilon_{ij}^{ph}}{d\xi} = b(\xi)\sigma_{ij}' + \gamma(\xi)\varepsilon_{ij}^{ph}, \quad b(\xi) = b_0(1-\xi), \quad \gamma = \gamma_0(1-\xi), \quad ,$$

$$\xi = (T - M^0{}_s - k|\sigma|)/(M^0{}_f - M^0{}_s)\,; \tag{7}$$

reversible process ($d\xi < 0$*)* $$\frac{d\varepsilon_{ij}^{ph}}{d\xi} = B_{ij} + \gamma(\xi)\varepsilon_{ij}^{ph}, \xi = (T - A^0{}_f - k|\sigma|)/(A^0{}_s - A^0{}_f) \tag{8}$$

Here b_0, γ_0 are constants of the model, $M^0{}_{s,f}$ are martensite start and finish temperatures without stresses, $A^0_{s,f}$ are austenite start and finish temperatures without stresses, k is scalar factor which points to connection of stress state and phase transformations problems. The value B must be found to satisfy initial conditions: $\varepsilon^{ph}\big|_{\xi=1} = \varepsilon^{ph}{}_0$ and $\varepsilon^{ph}\big|_{\xi=0} = 0$.

METHODS OF SOLUTIONS

The major problem consists in solution of forward transformation because phase deformations depend on the stresses during all the process. For reversible transformation the problem is solved as elastic one with initial temperature and phase deformations known from the previous stage.

Non-connected problems for forward and reversible transformations.

In-connected thermomechanical problem of forward transformation can be solved analytically in the same way as the linear viscoelasticity problem [11,12] with the aid of the Laplace transform. Assume that parameter k=0 in Eqns. 6,7 and modulus of elasticity and thermal expansion coefficient do not depend on parameter ξ. Then constitutive equations for phase deformation tensor may be written as Convolution form $\varepsilon^{ph} = \left[\tilde{N}(\xi,T) * \sigma(\xi)\right]$ and forward transformation problem (when volumetric reaction effect and thermal expansion can be neglected) is reduced to elastic problem with the aid of Laplace transform [12,11] with pseudomodulus $G(s),K(s)$. Here G is the shear modulus G, K is the bulk modulus of strain, γ, b are constant parameters in Eqn 7. pseudomodulus $G(s),K(s)$ may be written as

$$\begin{aligned} &G(s) = G(s-b)/(s-\gamma_0), \quad K(s) = K, \quad \gamma_0 = b - 2\gamma G, \\ &E(s) = (s-b)/(s-\gamma_1), \quad \gamma_1 = b - 2/3\gamma E. \end{aligned} \tag{9}$$

We can consider also phase transformation taking into account temperature expansion deformation and growth rate of the volume part of phase deformation (γ_*) the similar way. Using Laplace transform leads here to the equivalent quasi-elastic problem with initial volumetric strain that depend on $\xi_*\left[(s-b)s\right]^{-1}$ and Laplace transform of thermal strain. Thus, to receive solution of forward transformation it is necessary to find solution of the equivalent elastic problem and to do inverse Laplace transforms.

Now consider the reversible transformations. Using the Laplace integral transform we get the usual elastic boundary problem with distribution of the initial deformations for Laplace transform of the stain stress state. This distribution is defined in the following form $\psi(s) = B_{ij} / s(s-\gamma_0)$. Thus, we get usual elastic problem with addition distributions of the body forces $\psi(s)E$ and surface forces $\psi(s)(En_j)_{S_\sigma}$ for static part of the boundary conditions on the surface S_σ. The result solution can be obtained using the inverse Laplace transforms.

Coupled problems for forward and reversible transformations.

In the general case for connected problem the incremental relationships between stress and strain are to be used in the shape memory alloys (SMA). Using formulas (6),(7),(8) we have arrived at the of incremental equations for components of stresses and strains tensors:

$$\Delta\sigma = \tilde{A}\Delta\varepsilon + \tilde{B}\Delta T \tag{10}$$

Here $\tilde{A} = \tilde{A}(\sigma,\varepsilon^{ph},\varepsilon^{T},\varepsilon,T)$, $\tilde{B} = \tilde{B}(\sigma,\varepsilon^{ph},\varepsilon^{T},\varepsilon,T)$

In particular, for plane thermomechanical problem of the phase transformation we have the following relations for $\tilde{A}$ *and* $\tilde{B}$ in the incremental constitutive Eqn 10:

$$\tilde{A} = \frac{\overline{E}}{1+\dfrac{k}{\Delta M}(D\overline{\sigma} - C\overline{\varepsilon} - KT)I^{T}_{,\sigma}}, \quad \tilde{B} = \frac{1}{\Delta M}\frac{D\overline{\sigma} - C\overline{\varepsilon} - KT}{1+\dfrac{k}{\Delta M}(D\overline{\sigma} - C\overline{\varepsilon} - KT)I^{T}_{,\sigma}} \tag{11}$$

$$D = \Delta\overline{E}\overline{E}^{-1} - \overline{E}A + \overline{E}B\overline{E}^{-1}, \quad C = \overline{E}B, \quad K = \overline{E}B\Gamma - \overline{E}\Gamma,$$

where $\overline{E}(\xi), \Delta\overline{E}(\xi), \Gamma(\xi)$ are elastic modulus matrix, increment of the elastic modulus matrix with $\Delta\xi$ and thermal expansion coefficient vector:

$$\overline{E}(\xi) = \begin{pmatrix} E_{11}(\xi) & E_{12}(\xi) & 0 \\ E_{21}(\xi) & E_{22}(\xi) & 0 \\ 0 & 0 & G(\xi) \end{pmatrix}, \ \Gamma(\xi) = (\alpha_1,\alpha_2,0)^T, \ \overline{E} = \overline{E}[\xi(\sigma,T)], \ \Gamma = \Gamma[(\sigma,T)]$$

and

$$\overline{\sigma}(\xi) = \overline{E}(\xi)(\overline{\varepsilon} - \overline{\varepsilon}^{ph} - \Gamma(\xi)T), \ \overline{\sigma} = (\sigma_1,\sigma_2,\tau)^T, \ \overline{\varepsilon} = (e_1,e_2,e_{12})^T, \ \overline{\varepsilon}^{ph} = (e_1^{ph},e_2^{ph},e_{12}^{ph})^T,$$

$$\sigma_2 = E_{21}e_1^e + E_{22}e_2^e - (E_{21}\alpha_1 + E_{22}\alpha_2)T, \ \sigma_1 = E_{11}e_1^e + E_{12}e_2^e - (E_{11}\alpha_1 + E_{12}\alpha_2)T, \ \tau = Ge_{12}$$

$$E_{21} = \frac{E_2\mu_{21}}{1-\mu_{12}\mu_{21}}, \ E_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \ E_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}},$$

$$E_1 = E_1[\xi(\sigma,T)], E_2 = E_2[\xi(\sigma,T)], G = G[\xi(\sigma,T)], \ \Delta\overline{\varepsilon}^{ph} = (A\overline{\sigma} + B\overline{\varepsilon}^{ph})\Delta\xi,$$

$$\Delta\xi = \frac{\Delta T}{\Delta M} - \frac{k}{\Delta k}I^{T}_{,\sigma}\Delta\overline{\sigma}, \ \Delta M = M_f - M_s, \quad \overline{I}_{,\sigma} = (I_{,\sigma_1}, I_{,\sigma_2}, I_{,\tau}),$$

Constitutive Eqns 10,11 are written in analytical form, allowed to solve linear elastic problem in implicit form on each temperature layer and can be used for description of the phase transformation in SMA and orthotropic bodies with the shape memory elements. Assume that numerous refinements of the stress state have been introduced on each step of the variations in temperature. These constitutive equations can be used conjunction with a Finite Element Method in the same manner as it was received by D. C. Lagoudas and J.C. Boy [14] but they are modified because here constitutive equations are used that allow to describe oriented transformations phenomena.

Refined SMA beam, plate and shell theories

The numerical results show that solutions for SMA structures based on the classic beam and plate theories lead to errors into strain-stress state of the SMA structures for forward transformation process. The problem of using refined theories for SMA is no less important than for the composite materials. Thus, to study stress-strain state, coupled thermomechanical problem must be solved based on the refined methods of the stress analysis and refined beam, plate and shell theories. It is known that the problem of constructing the applied theories of the two-dimensional objects is equivalent to the kinematics formulation, which makes it possible to give a two-dimensional complete description of the deformational model. We assume to use a general approximation of the displacement field which provides consistency of the refined displacement-based theory [15]. We introduce the following displacement field for the SMA beams referred to the Cartesian coordinates *x, z,* ($|z| \le h/2$*, h* is thickness of the beam):

$$u = u_0(x) + u_1(x)z + u_i(x)\varphi_i(z), \quad v = v_1(x) + v_i(x)\chi(z)_i, \quad \chi_i = \frac{d\varphi_i}{dz}, \qquad i = 1,2,...I$$

Generalizing displacement field for plates referred to the Cartesian coordinates *x,y,z* can be written in the following form:

$$\begin{aligned} &u = u_0(x,y) + u_1(x,y)z + u_i(x,y)\varphi_i(z), \quad v = v_o(x,y) + v_1(x,y)z + v_i(x,y)\psi_i(z), \\ &w = w_0(x,y) + w_i(x,y)\chi_i(z), \quad \varphi_i = \psi_i, \quad \chi_i = \frac{d\varphi_i}{dz}, \qquad i = 1,2,...I \end{aligned} \tag{12}$$

For shells referred to orthogonal curvilinear coordinates *x,y,z,* approximation functions $\varphi_i, \psi_i, \chi_i$ entering Eqn 12 should be connected by following relationships:

$$\left(1 + \frac{z}{R_1}\right)\frac{\partial \phi_i}{\partial z} - \frac{\varphi_i}{R_1} = \chi_i, \quad \left(1 + \frac{z}{R_2}\right)\frac{\partial \psi_i}{\partial z} - \frac{\psi_i}{R_2} = \chi_i,$$

where R_1 *and* R_2 are the principal radii of curvature of the shells middle surface.

So, to obtain solution of the couple thermomechanical problem for SMA structures, we can use constitutive equations in the form (7), (8) and equilibrium equations in association with natural boundary conditions that are found taking into account approximations (12) based on the Lagrangian functional. To solve in-connected problem taking into account phase transformations in the SMA structures we must consider quasi-elastic problem, introducing quasi-modulus of elasticity in place of the elasticity modulus of the SMA layers or filament of the composite structures with the aid of the relationships (9). The quasi-elastic problem are formulated in Laplace transform space in respect to parameter ξ based on the refined plate and shell theories. Finally, solutions of in-connected problem can be found using operator of the inverse Laplace transform.

To illustrate the importance of the refined SMA structures theories, we computed problem of the pure bending for the SMA beam, which is loaded by normal stresses on the transversal edges $x=\pm 1$. Assume that stresses are distributed as anti-symmetric cubic dependence and are created bending moment with amplitude equal to unit. The forward transformation process is derived during cooling. Assume the following parameters: M^0_s=18^{0}C, M^0_f=3^{0}C, k=0.2(^{0}C/MPa), E_A=70000 MPa, E_M=30000 MPa, $E_{A,M}$ are elastic modulus for austenit and martensite state; $E=E_A-\xi(E_A-E_M)$, $0<\xi<1$, $l/h=10$, $b^+=b_0^+(1-\xi)$, $\gamma^+=\gamma_0^+(1-\xi)$, $b_0^+=25\cdot 10^{-4}(1/MPa)$, $\gamma_0^+=1$. Solution is found based on the refined theories with approximation of the displacement field that are given by relationships (12). It was accepted that $I=2$. Distributions of the beam curvature alone longitudinal coordinate x for forward transformation ($\xi=0{,}5$ and $\xi=1{,}0$) are presented in Figure 1 by broken lines. Solid lines correspond to classic solution of pure bending of beam. Classic beam theory leads to large errors into the calculations of the beam curvature. This fact is connected with appreciable variations of the shear and normal stresses across the slab near edges of the SMA beam.

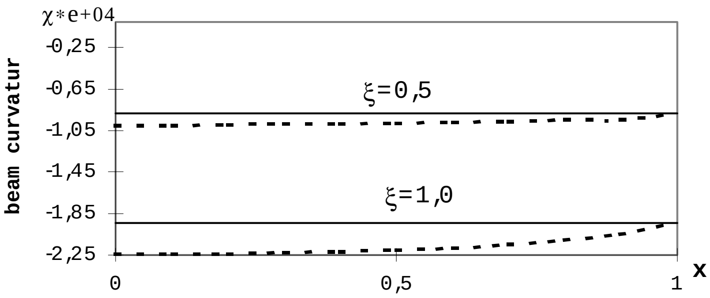

Fig.1 Distributions of beam curvature with forward transformations

ACTIVE SMA COMPOSITES

The active composites (AC) that reinforced with SMA fibers or players are discussed and composite deformation problems are solved for linear elasticity of matrix. Fully controlled active composites denote that their shape in a certain areas may vary within wide limits in a controlled manner under the action only of the temperature variation of the SMA elements. The shape memory alloy deforms without mechanic loading only under heating (process of reverse martensite transformation). In order to close the hysteresis loop in cooling it is necessary to apply some mechanical loading (process of forward martensite transformation). This full recovery during cooling is given by forces that are generated in the matrix in the process of deformation during the heating time of the shape memory elements.

Consider briefly the unidirectional composites and try to describe behavior of these structures under temperature loading. For simplicity assume that thermal expansion can be neglected. During temperature increase and decrease composite beam length, the reactive stresses arise in the fibers, useful employment may be executed by composite structures. In the first stage the phase deformation in SMA filaments are aroused from load σ_0 as a result of the forward transformations process. This deformation can be calculated by integrating constitutive Eqn 7 with initial condition $\varepsilon^{ph}{}_0 \,\big|_{\xi=0}=0$. Thus filaments are in martensite phase and couple with matrix. During heating the volume part of martensite phase decreases and deformation in fibers is generated because of shape memory effect. These deformations may be found as solution of Eqn 8. The composite under consideration is deformed during heating. Stress strain state is described with the aid of the Hooke's equations for fibers and matrix, equilibrium equations and compatibility equation

$$\begin{aligned} &\varepsilon_f^e = \sigma_f / E_f(\xi), \quad E_f(\xi) = \xi E_M + (1-\xi)E_A, \quad \varepsilon_m = \sigma_m / E_m, \\ &P/f = \upsilon\sigma_f + (1-\upsilon)\sigma_m, \quad \upsilon = F_f / F, \quad \varepsilon_e + \varepsilon_{ph} = \varepsilon_m \end{aligned} \tag{13}$$

where P is useful load, $E_{f,m}$ are modulus of elasticity fibers and matrix, F_f and F are cross section area filaments and active composite, υ is non-dimension fraction of fibers in composite material.

Consider stage of composite cooling that rank below the stage of heating. Assume that this stage begin when volume part of martensite phase is equal to ξ_0 and correspondingly phase deformation is equal to $\varepsilon_-^{ph}(\xi_0)$. Thus the phase path of the fibers deformation during cooling may be found as solution of Eqn 7 with initial condition $\varepsilon_+^{ph}\ \big|_{\xi=\xi_0} = \varepsilon_-^{ph}(\xi_0)$. For cooling through interval of austenit-martensite transformation phase deformation increases in direction to operating stresses and fibers are elongated (forward transformation). In a general case the phase deformation is the function of fibers and matrix properties, non-dimension fiber fraction, force P acting on the prior stage of martensite-austenit transformation, properties of SMA. During the temperature decrease through interval of the forward transformation the active composite comes back to its new form. The next steps of temperature loading (increase and decrease) provide the closed hysteresis loop. So, we have two way shape memory effect. The optimal conditions of hysteresis loop closure under heating and force loading at the first stage and cooling at the second stage can be written as

$$\varepsilon_-^{ph}(\xi_1) = \max_{\alpha_j}(\varepsilon_-^{ph}(\xi_1)), \qquad 0 < \xi_1 < 1. \tag{14}$$

where α_j are mechanical, geometric and strength parameters of SMA composite, ξ_1 is parameter of the phase transformation at the second step. The optimal design problems arise. This task is solved exactly for in-connected phase transformation problem.

In the same way one of the variants of the active composite structure with two-way effect may be considered in the form of a sandwich plate. The active elements are top and bottom layers. Let as consider this active structure taking into account couple problem. Figure 2 shows distributions of the full deformations of the layers (SMA layers properties mentioned above, k=0.5(^{0}C/MPa)), alone the temperature. At first active layers are loaded in austenit phase and cooled through interval of the austenit-martensite transformation (represented by the lines AB, BC, CD, DE). Points B, C show martensite start and finish temperatures. Point D is unloaded point. Then layers are embedded into the flexible matrix and are loaded by heating (represented by the line FK). The bending deformation appears even when external loading is absent only as a result of shape memory effect. At this step of deformation the plate can produce the useful work. The process of the layers cooling through interval of forward transformation and of the heating for reversible transformation can be considered on the second stage (represented by the lines KNM, MLK). Conditions of two ways shape memory effect may be established by making the full deformation of plate for complete process of heating and cooling equal to zero. The optimal design problem was solved to give the greatest amplitude deformations for the hysteresis loop (KNMLF).

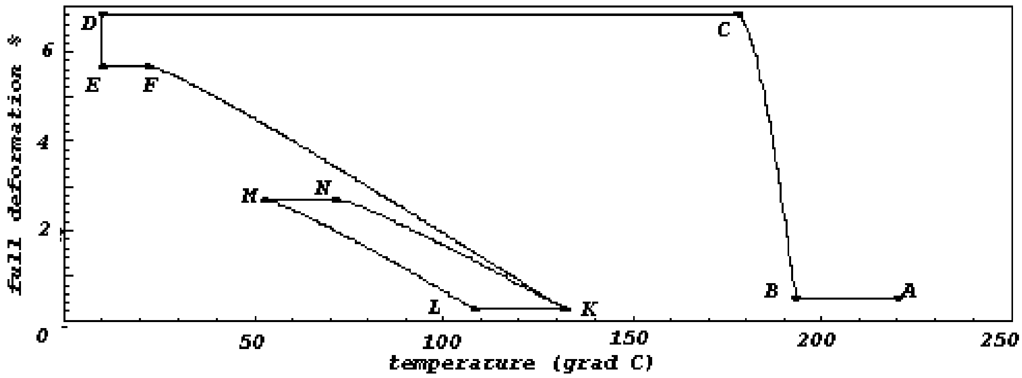

Fig. 2 Distributions of the layers deformations in the sandwich panel along temperatures

Let as consider now other type of SMA composite structure with two-way effect. Assume the sandwich panel has the constant bending curvature. Only top layer is from SMA. At the first stage this layer is austenit phase and is loaded by normal stresses, because bending is caused by curvature. The phase deformations arise in the top layer under cooling (forward transformation). The sandwich plate is recovered its original shape under heating because shape memory effect on the next stage of the temperature loading. This plate may be used as anti-icing structure. Figure 3 shows phase strain distribution (curve 1) and full strain distribution (curve 2) for forward transformation alone the parameter ξ. Assume the following parameters: $H = 0.005m, E_M = 30000MPa, E_A = 70000MPa,\ E = 2000MPa$,

$h = 0.001m$, where h and H are thickness of SMA layers and middle ply, E is modulus of the middle ply.

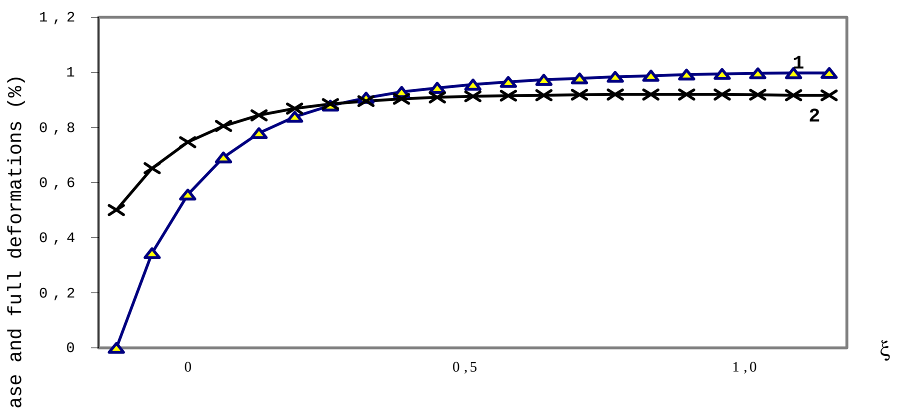

Fig.3. Phase and full deformations of the sandwich panel with the SMA layer.

Note that initial assumption of stress state homogeneity in composites with SMA active filaments (Eqns 13) may be non-corrected when deformations are generated by fibers. The transmission problem of deformations from fibers or layers to composite is to be at the sacrifice of shear stresses in matrix. To appreciate this was consider a contact problem for SMA beam is joined with orthotropic strip on the edge, y=1. The orthotropic plate referred to non-dimensional coordinates x and y normalized to plate length L and width H was considered. Longitudinal edge, y = -1 and transverse edges, x = ± 1 were free. Initial strain stress state in the system is defined by constant deformation ε_0, that was generated in the SMA beam. This problem was solved [15] based on the Laplace transforms with respect to the martensite volume part parameter ξ. As a result normal stresses in the beam and normal and shear stresses in the matrix were found [16]. It was defined that normal stresses vary with forward transformation during cooling and change their sign when volume part of martensit phase exceeds the value $\xi \approx 0.5$ due to oriented transformation phenomenon. Note also that normal stresses are not close to constant value and approach the quadratic function when the beam is in austenit state ($\xi \approx 0$). Figure 4 shows distributions of the compound beam curvatures with length-thickness ratio *L/H*=10, ε_0=0.01 along its length after forward transformation $\xi = 1.0$. Assume the following parameters: $F_b / F = 0{,}1$, $E / G = 3$, $E_b / E = 1$ (lines –1); $F_b / F = 0{,}1$, $E / G = 5$, $E_b / E = 0.8$, (lines -2); $F_b / F = 0{,}1$, $E / G = 5$, $E_b / E = 5$ (lines -3). Here *E* and *G are* elasticity modulus of plane strip, E_b is Hook's modulus of the SMA beam, F_b *and* F are squares of the cross section area of the SMA beam and orthotropic strip. The air lines on the figure 4 correspond to curvatures that are found based on the classical beam theory. Thus refined methods of the solution become important.

Fig. 4. Distributions of the compound beam curvatures along the length ($\xi = 1{,}0$)

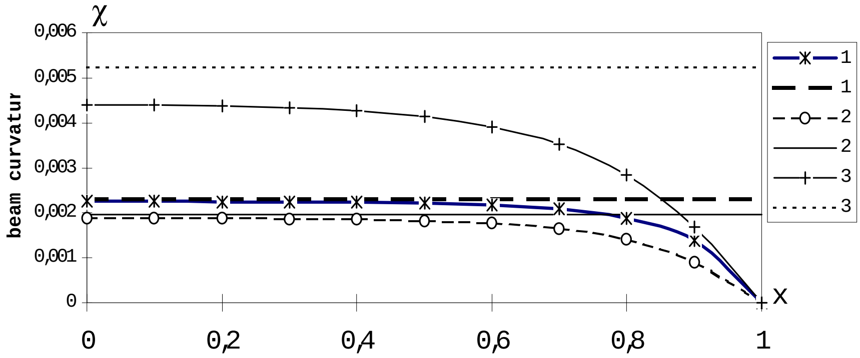

Figure 5 shows the deformation distributions of the compound strip along temperature during closed process of the cooling and heating loading. Assume the following parameters: *E/G=0.3*, $M_s = 49^0 C, M_f = 33^0 C, A_s = 66^0 C, A_f = 83^0 C$; curve 1-*h/L=0.5*, $E_b / E = 1$, $F_b / F = 0{,}1$ curve 2-*h/L=0.1*, $E_b / E = 0{,}8$, $F_b / F = 0{,}1$; curve 3-*h/L=0.1*, $E_b / E = 1$, $F_b / F = 0{,}3$

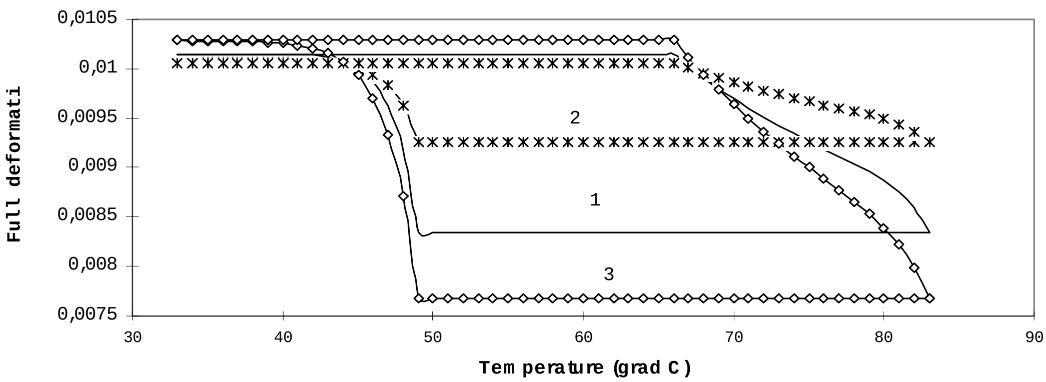

Fig. 5 Deformation distributions for closed process

The other types of the active composite structures including layered shells with SMA layers and fibers were considered under other types loading. The problems of the optimal design are also studied.

CONCLUSIONS

The general form of thermodynamic constitutive equations for shape memory materials had been established. Stress strain states of the active composites with SMA elements were found with the aid of the refined plate and shell theories. Contact problem for SMA system was solved. It was established that to study SMA Stress-strain State the coupled thermoelastic boundary problems must be solved and refined theories must be used. The solutions, which are based on the non-connected statement of the boundary problem lead to errors into Strain-stress State of the shape memory, alloy structures. SMA fully controlled active composites are described and optimal parameters are established for such materials.

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