TRANSIENT RESPONSE OF SANDWICH AND LAMINATED COMPOSITES WITH DAMPING UNDER IMPULSE LOADING

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SUMMARY: The transient response of sandwich and laminated composite structures with viscoelastic layers under impulse loading is studied using the finite element method. On this reason, the finite element models of sandwich and laminated composite beams and plates have been developed. The viscoelastic material behavior is represented by the complex modulus model. An efficient method using fast Fourier transform has been proposed. This method is based on the trigonometrical representation of the input signals and matrix of the transfer functions. The present implementation gives the possibility to preserve exactly the frequency dependence of the storage and loss moduli of viscoelastic materials. Numerical examples are given to demonstrate the application and validity of the approach presented.

KEYWORDS: transient response, fast Fourier transform, finite element method, viscoelastic damping, complex modulus model.

INTRODUCTION

Recent decades, an important area of computer simulation in mechanics is the dynamic analysis of sandwich and laminated composite structures with viscoelastic layers. This is connected with further improvements of commercially available composite materials with high damping properties and it’s increasing applications in a various fields of engineering: aerospace technology, shipbuilding, automotive industry, mechanical engineering, etc. However, mostly analyses are performed in the frequency domain [1,2] and only small quantity of investigations is devoted to a solution in the time domain [3-6]. This can be explained by considerable difficulties arising in the analysis.

The sixth-order differential equation of motion in terms of the transverse displacement for a three-layer sandwich beam with a viscoelastic core and for a variety of beam boundary conditions was derived for the first time in the paper [3]. This equation was analysed successfully by using a special class of forced, uncoupled and complex modes of vibration. But these complex modes can exist only, when the beam is externally excited by specific “damped normal loadings”, which are also complex and proportional to the local transverse inertia loading of the beam. The time domain behaviour of machine members made from composite materials and sandwich panels was obtained from the frequency domain response by the Fourier transform technique in the paper [4], using for this purpose the equation of motion presented in a form appropriate for a non-complex notation. The complex modulus model was used to describe the behaviour of viscoelastic material in these papers and the material loss factors and storage moduli were approximated as constant values.

The fractional derivative model was used to examine the viscoelastic behaviour of a damping layer in a simply supported beam in the paper [5]. The beam was analysed by using both a continuum formulation and a finite element formulation to predict the transient response to a step loading. In the paper [6], authors added to this model the modulus degradation and the termal effect and carried out a non-linear dynamic analysis in the time domain for the multilayer sandwich beam subjected to dynamic loading. It was shown in the paper [7], that the fractional derivative model is more accurate and appropriate for both a rubbery and a glassy viscoelastic materials than a more simple models presented as a lumped parameter systems. However, if to use a material data in the frequency domain, it is possible to apply experimental data without any transformations as explain below.

The objective of the present study is to obtain the transient response of sandwich and laminated composite structures with viscoelastic layers under impulse loading using the finite element method. The present implementation gives the possibility to preserve the frequency dependence for the storage and loss moduli of viscoelstic materials exactly. Some finite element examples are given to illustrate the application of the method.

TRANSIENT RESPONSE ANALYSIS

One of the most used models to describe the rheological behaviour of viscoelastic materials in the finite element analysis is the complex modulus representation [8]. Since this model is solvable, that is making use of existing computing facilities, and the results of such a theoretical analysis show sufficiently good agreement with experiments [9]. Using this model, the constitutive relations will be expressed in the frequency domain as follows

$$\sigma_0 = E^*(\omega)\varepsilon_0 = E'(\omega)\left[1 + i\eta(\omega)\right]\varepsilon_0 \quad , \quad \eta(\omega) = \frac{E''(\omega)}{E'(\omega)} \quad ,$$

where σ_0 and ε_0 are the amplitude of the harmonically time-dependent stress and strain respectively, E^* is the complex modulus of elasticity, E', E'' are the real and imaginary parts of the complex modulus of elasticity, η is the loss factor and ω is the frequency. It is necessary to note that the storage and loss moduli in this case are defined directly in the frequency domain by experimental technique for each material and can be used without any transformations in the numerical analysis.

The forced vibration equation of a structure with viscoelastic damping using the complex modulus model appears as follows in matrix form

$$\mathbf{M}\ddot{\mathbf{X}}^* + \mathbf{K}^*(\omega)\mathbf{X}^* = \mathbf{F}(t) \quad ,$$

where $\mathbf{M}$ is the mass matrix; $\mathbf{K}^*(\omega) = \mathbf{K}'(\omega) + i\mathbf{K}''(\omega)$ is the complex stiffness matrix. $\mathbf{K}'(\omega)$ is determined using the elastic $E'(\omega)$ and shear $G'(\omega)$ moduli, while $\mathbf{K}''(\omega)$ is found using the imaginary parts of the complex moduli $E''(\omega) = \eta_E(\omega)E'(\omega)$ and

$G''(\omega) = \eta_G(\omega)G'(\omega)$, where $\eta_E(\omega)$, $\eta_G(\omega)$ are the material loss factors in tension and shear respectively, and ω is the frequency. $\mathbf{X}^*, \ddot{\mathbf{X}}^*$ are the complex vectors of displacements and accelerations; $\mathbf{F}(t)$ is the load vector. The transient response of the system, described above, can not be obtained effectively applying direct integration methods or modal superposition method [10], because in this case it is not possible to determine the variation of the material properties $E^*(\omega)$ and $G^*(\omega)$ with respect to time. The time domain behaviour of a structure may be obtained from the frequency domain response by the Fourier transform technique.

The method proposed is based on the assumption that any complex input signal can be interpolated by trigonometric polynomials. It is more convenient to use for this purpose the Fourier transform to find the frequency spectra of excitation

$$\mathbf{F}^*(\omega_j) = F[\mathbf{F}(t_k)] = \frac{\Delta\omega}{2\pi}\sum_{k=0}^{N-1}\mathbf{F}^*(t_k)\,e^{-i\frac{2\pi jk}{N}} \quad ,$$

where t_k is a set of discrete times for the excitation $\mathbf{F}(t)$ and for the response $\mathbf{X}^*(t)$; ω_j is a set of discrete frequencies for the frequency spectra of excitation $\mathbf{F}^*(\omega)$ and for the frequency response $\mathbf{X}^*(\omega)$. The response of the structure for each trigonometric component is calculated exactly using the matrix of transfer functions. Incidentally it is necessary to solve the following system of complex linear equations:

$$\left[\mathbf{K}^*(\omega_j) - \omega_j^{\,2}\mathbf{M}\right]\mathbf{X}^*(\omega_j) = \mathbf{F}^*(\omega_j) \quad .$$

The displacements of structure in the time domain can be obtained by the inverse Fourier transform

$$\mathbf{X}^*(t_k) = F^{-1}\left[\mathbf{X}^*(\omega_j)\right] = \Delta t\sum_{j=0}^{N-1}\mathbf{X}^*(\omega_j)\,e^{i\frac{2\pi jk}{N}} \quad .$$

Numerical realisation of the Fourier transform is performed by the routine using a variant of the fast Fourier transform algorithm [11] known as the Stockham self-sorting algorithm [12], which takes advantage of the cyclic repetition of the complex exponentials in the discrete Fourier transform and drastically reduces the number of calculations required. Obviously, the accuracy of the discrete Fourier transform depends on the number of samples N and the sampling interval Δt. The choice of $\Delta\omega$ and N depends on the frequency response shape, the accuracy needed and the computing capacity available. The frequency interval $\Delta\omega$ for the inverse transform must be the reciprocal of the total time record length and equals to $\Delta\omega = 2\pi / N\Delta t$.

It is necessary to note that the value of function at a discontinuity must be defined as the midvalue if the inverse Fourier transform is to hold. Moreover, using discrete Fourier transform, it is necessary to remember that it is based on the assumption about periodicity of load applied. For periodic functions with known periods, it is necessary to choose $N\Delta t$ interval equal to a period or integer multiple of a period. For those cases where the period of a periodic function is not known, the concept of a data-weighting function or data window must be employed [13]. For the non-periodical loads, the period of load can be expanded by addition of long interval for a zero loading.

FINITE ELEMENT MODELS

The Timoshenko's beam and Mindlin-Reissner plate finite elements lie in a basis of the sandwich and laminated composite beams [14,15] and plates [16,17] finite element models. The widely known expressions of displacements in the first order shear deformation theory have the following form:

$$u = u_0 + z\gamma_x \quad ; \quad v = v_0 + z\gamma_y \quad ; \quad w = w_0 \quad ,$$

where u_0, v_0, w_0 are the displacements in the reference plane, z is the co-ordinate of the point of interest from the reference plane, γ_x, γ_y are the rotations connected with the transverse shear deformations. This hypothesis is applied in two different ways: integrally for all layers of a laminated composite finite element model (Fig. 1) or separately for each layer of a sandwich finite element model (Fig. 2). In the first case, transverse shear stiffness is obtained by means of shear correction factors. The second case corresponds to the broken line model and satisfies, on this reason, to the following displacement continuity conditions between the layers:

$$\begin{aligned} u^{(1)} &= u^{(2)}\Big|_{z=z_1} \quad ; \quad u^{(2)} = u^{(3)}\Big|_{z=z_2} \quad ; \\ v^{(1)} &= v^{(2)}\Big|_{z=z_1} \quad ; \quad v^{(2)} = v^{(3)}\Big|_{z=z_2} \quad ; \\ w^{(1)} &= w^{(2)}\Big|_{z=z_1} \quad ; \quad w^{(2)} = w^{(3)}\Big|_{z=z_2} \quad , \end{aligned}$$

where in the brackets, the numbers of layers are given.

NUMERICAL RESULTS

Some test problems and simple finite element examples are given in the paper [18].

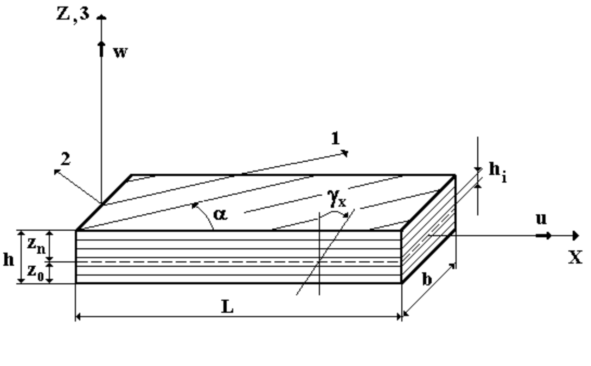

Fig. 1: The laminated composite beam

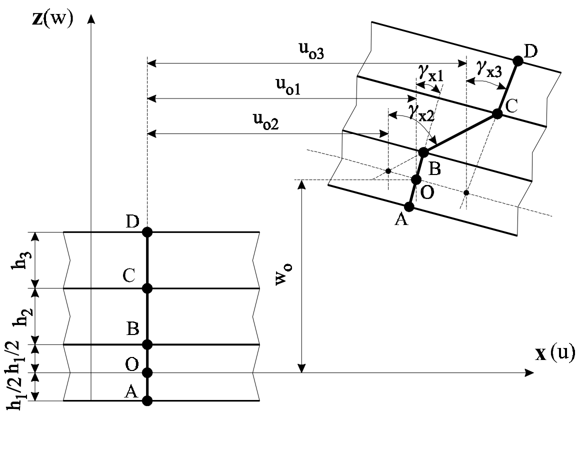

Fig. 2: Kinematic assumptions for the sandwich plate in the ZX-plane

Sandwich plate under impulse loading

As a numerical example, illustrating application of the method, the transient response analysis of a sandwich plate with the edges length a=1 m and b=0.5 m and simply supported boundary conditions for the opposite sides has been presented. The thickness of layers: h_1=0.0065 m, h_2=0.0025 m, h_3=0.001 m. The external layers are aluminium and have the following properties: E'=71 GPa, υ=0.32, ρ=2800·Ns²/m⁴. The damping material C-1002 (EAR Corporation) with υ=0.49 and ρ=1300·Ns²/m⁴ is chosen from the literature [9] to simulate the sandwich core. Expressions describing the shear modulus and loss factor of this material for the frequency range f=0,...,2000 Hz under temperature 23.9^{0}C are

$$G' = 44.4 - 17.6/z \quad \text{MPa},$$

where $z = 0.4 + 0.0003 f$;

$$\eta_G = \eta_E = 3.718 - 2.637z + 1.030z^2 - 0.5957 \cdot 10^{-9}/z^4 - 1.054/z^{0.25} + 0.3062 \cdot 10^{-4}/z^2,$$

where $z = 0.005 + 0.0004975 f$.

Dependence of these values on frequency is shown also in Fig. 3 and 4. The excitation and response measurement points are located at the centre of the plate. The rectangular shape impulse is applied. Since the symmetry exists, only quarter of the plate is taken into consideration (Fig. 5) and discretized with 64 sandwich plate finite elements.

In the numerical treatment, N=5000 and Δt=0.0005 s are chosen resulting in $\Delta\omega = 0.8\pi$ rad/s. The time interval is selected in order that the response of the system vanishes at this time. The load and the transient response are shown in Fig. 6 and 7. It is necessary to note that the static component ($\omega_j = 0$) is deleted from the results presented.

Sandwich beam under multiple impulse loading

Another example shows the possibility of the present methodology to analyze sandwich and laminated composite structures with viscoelastic damping under multiple impulse loading. For this purpose, a beam with width b=0.05 m, length L=1 m and thickness of layers h_1=0.007 m, h_2=0.002 m, h_3=0.001 m is examined. Material properties of layers are the same as in the

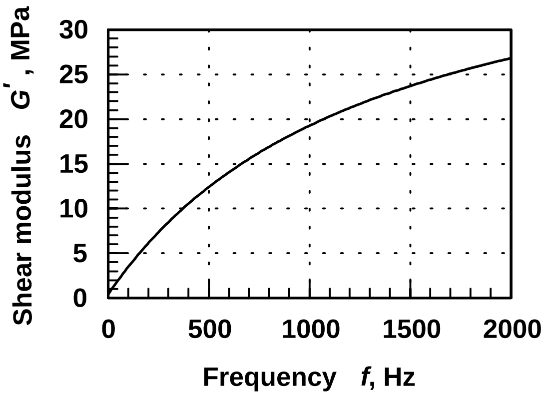

Fig. 3: Dependence of the shear modulus on frequency for the damping material

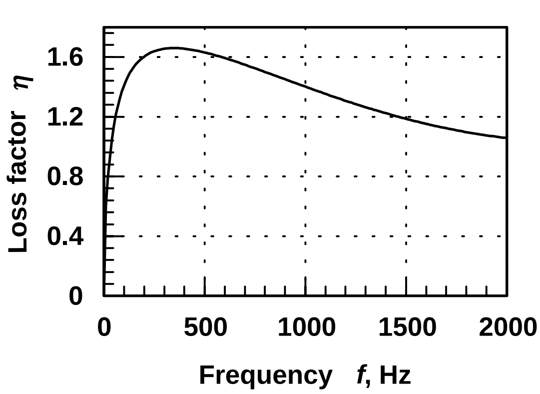

Fig. 4: Dependence of the loss factor on frequency for the damping material

previous example. The clamped boundary conditions are applied from one side of the beam (Fig. 8). The excitation points are located at the center and at the quarter of the beam length from the free end, the response measurement point – at the end of the beam. The rectangular shape impulses with different amplitude are applied. The sandwich beam is discretized with 4 sandwich beam finite elements.

The time interval is selected in order that the response of the system vanishes at this time. On this reason: N=10000, Δt=0.0005 s, $\Delta\omega = 0.4\pi$ rad/s. The load and the transient response are shown in Fig. 9 and 10. It is necessary to note that the static component ($\omega_j = 0$) is deleted from the results presented also as in the previous case.

CONCLUSIONS

The present approach was developed with the aim to use his as a universal tool in the transient finite element analysis of sandwich and laminated composite beams and plates with viscoelastic layers, which applied widely in the aeronautical, ship and automobile structures.

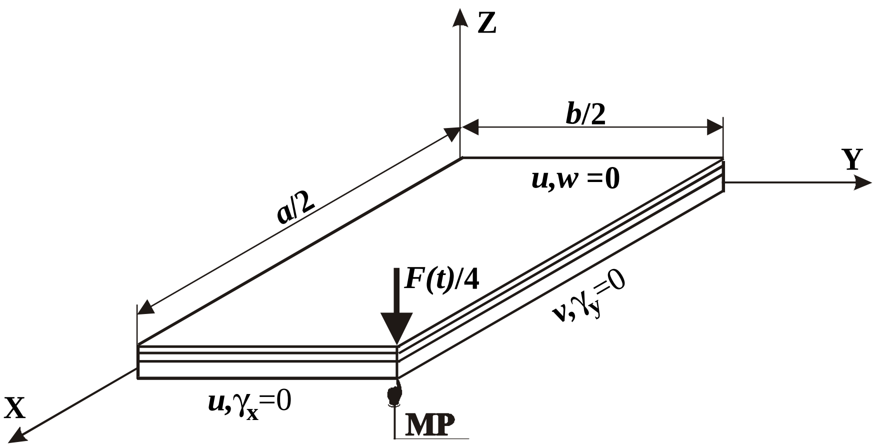

Fig. 5: The sandwich plate tested (MP – measurement point)

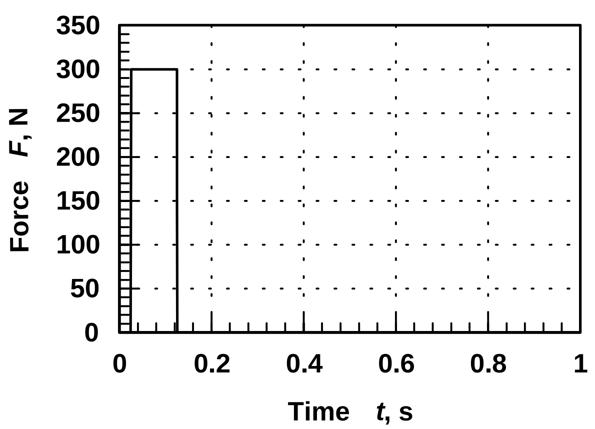

Fig. 6: Applied load

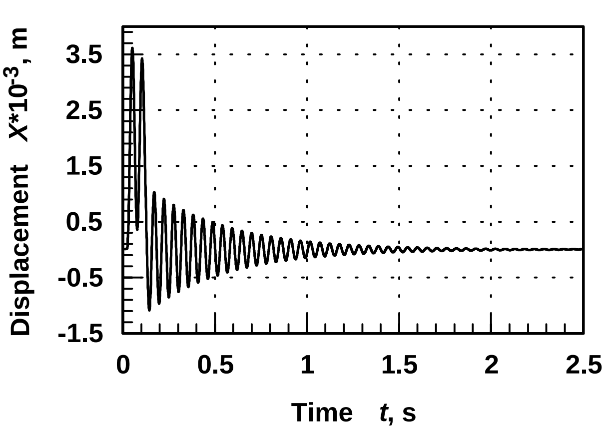

Fig. 7: Transient response of sandwich plate

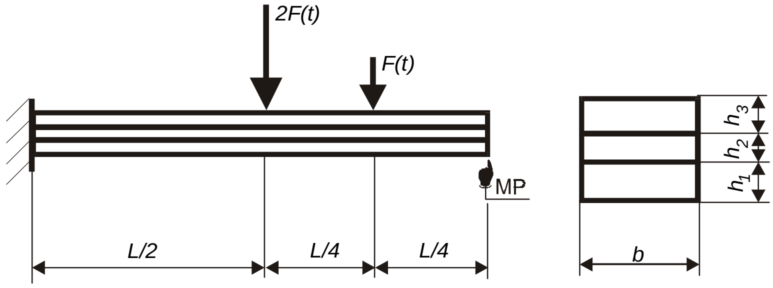

Fig. 8: The sandwich beam tested (MP – measurement point)

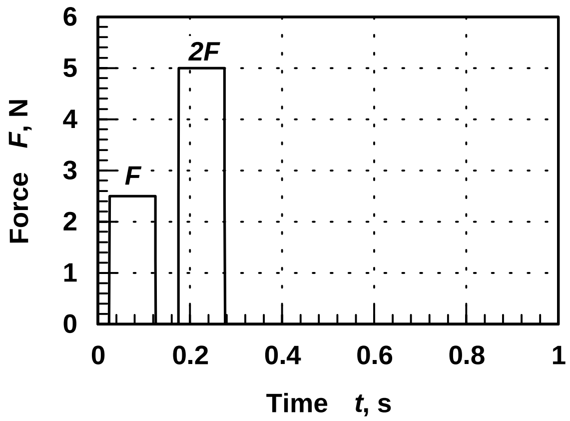

Fig. 9: Applied load

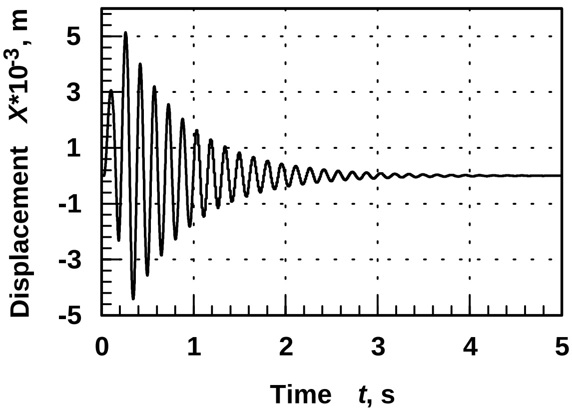

Fig. 10: Transient response of sandwich beam

This technique gives the possibility to preserve the exact mathematical formulation for the damping model examined and to calculate structures with high damping. Material data in the frequency domain are taken into consideration that is why the data from experiments are used without any transformations in the finite element analysis. Numerical results demonstrate the validity of the present implementation.

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