

# **CREEP BEHAVIOUR OF KEVLAR 49 INOKUCHI MATHEMATIC ANALYSE**

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## **ABSTRACT :**

Composite materials which are highly used in high technology industries such as aeronautic and space are nowadays introduced in building industry.

Because of their good physicochemical characteristics, composite cables can be used as substitute for steel in prestressed concrete. However, their long term mechanical behaviour is still not fully understood. Therefore, a rheological model describing the creep behaviour of cords of Kevlar type 49 is presented in this study.

The obtained simulations are in good agreement with the experimental results.

**KEYWORDS :** Kevlar, Prestress, Creep, Mechanical Behaviour, Viscoelasticity, Rheological Model, Simulation, Building Industry.

## **INTRODUCTION**

Composite cables, because of the fact that they are not subject to corrosion and, in addition, in the long run and at room temperature, show prestress losses due to weak flow and relaxation, can be used as prestress tools. Also their low density as well as their highly performing tension features incite researchers and industry related people to use them to build bridges and other structures, as well as for restoration purposes in damaged structures. Indeed, these composite materials are being used in many industries such as aeronautics and car manufacturing, and they are the subject of many research works (1,2, 3, 4, 5). However, to use these Kevlar reinforced cables as means of prestress, it is necessary to get a good knowledge tendon (6, 7, 8, 9), helped to know the evolution with time of its relaxation modulus.

Within the scope of this article, the results of the research which has been carried out on the study of the creep behaviour under tension of composite cables ( Kevlar high modulus, 0.95 TWA2 ) are presented.

## **INOKUCHI MATHEMATIC ANALYSE (Application to creep behaviour of Kevlar 49)**

The material tested is a braided high modulus Kevlar cord, impregnated with a heat-hardening resin of Polyester urethane; The composite, thus obtained, has a cylindrical shape of diameter 0.95 mm (11).

Using existing experimental results obtained on this cords of Kevlar type 0.95 TWA2 (12,13), a rheological model, based on the method of Inocuchi (14), for the long term behaviour of this material is presented.

The general model which correspond to a creep function of a viscoelastic material is made of an association in series of a Maxwell cells and a certain number of Kelvin Voigt solids depending on the viscoelasticity degree of the material.

The total strain is the sum of the elementary strains. Therefore, the general expression of the creep function is given by:

$$f(t) = J_0 + \frac{t}{\eta_0} + \sum_{i=1}^n J_i (1 - e^{-t/\theta_i}) \quad (1)$$

Now, the problem consists in obtaining the different parameters ( $\eta_0$ ,  $J_0$ ,  $J_i$ ,  $\eta_i$  and  $\theta_i$ ).

## DETERMINATION OF THE PARAMETERS ( $J_0$ , $\eta_0$ , $J_i$ , $\eta_i$ and $\theta_i$ ).

### Determination of $J_0$ and $\eta_0$

The creep experimental curve of the viscoelastic material is represented on figure 1.

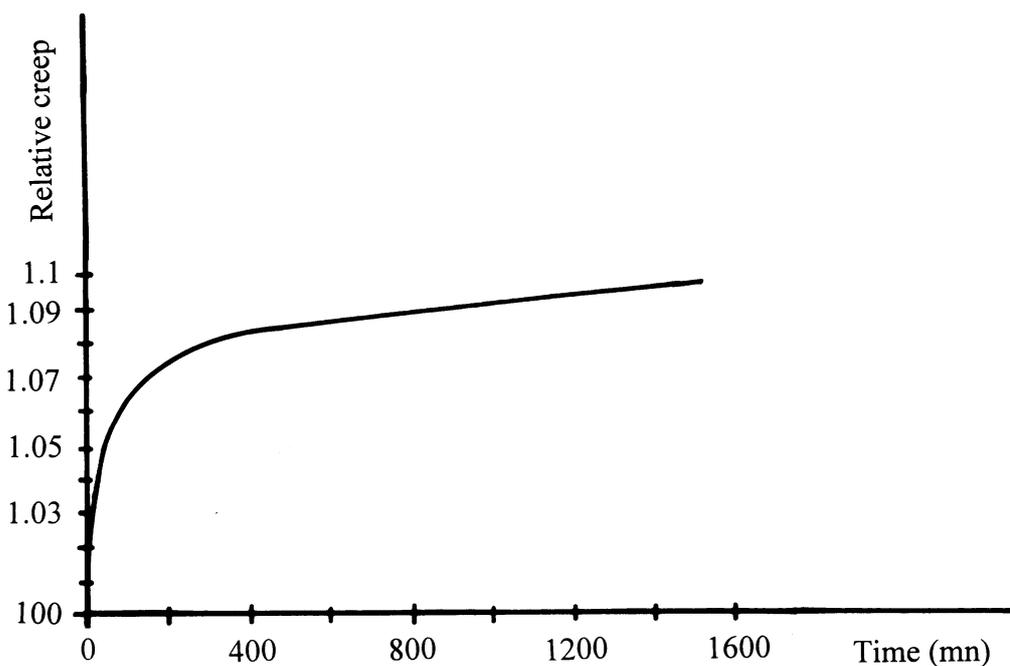


Figure 1. Experimental creep behaviour of Kevlar 49 (12)

From this curve, the value of  $J_0$  and  $\eta_0$  can be obtained because:

$J_0$  (which is the instantaneous elastic compliance) represents the ordinate at the origin of the curve  $f(t)$ .

$\frac{1}{\eta_0}$  (which is the coefficient of Newtonian viscosity) represents the slope of the asymptotic the curve.

$J_0$  and  $\eta_0$  are respectively measured as:

$$J_o = 1.02 * 10^{-11} \text{ Pa}^{-1}$$

$$\theta_0 = 10^5 \text{ mn}$$

And  $\eta_o = 9800 \text{ GPa.mn} \approx \infty$  (considered as infinite).

Next, there is still to determine the number  $n_i$  of the Kelvin Voigt cells and, therefore, the parameters  $\eta_i$  and  $J_i$  corresponding to each cell.

### Determination of the parameters $J_i$ , $\eta_i$ , and $\theta_i$

The parameters are determined by means of the mathematical analysis developed by Inocushi (14).

### Determination of the parameters $J_1$ , $\eta_1$ and $\theta_1$

Given a function such that:

$$Q(t) = \sum_{i=1}^n J_i e^{-t/\theta_i} \quad (2)$$

Considering that  $t$  is sufficiently big, it is clear from a first analysis that the function to be considered is:

$$Q(t) = J_1 e^{-t/\theta_1} \Rightarrow \text{Ln}Q(t) = \text{Ln}J_1 - t/\theta_1 \quad (3)$$

It follows that the graph of  $\text{Ln}Q(t) = f(t)$  is a non linear curve, figure 2.

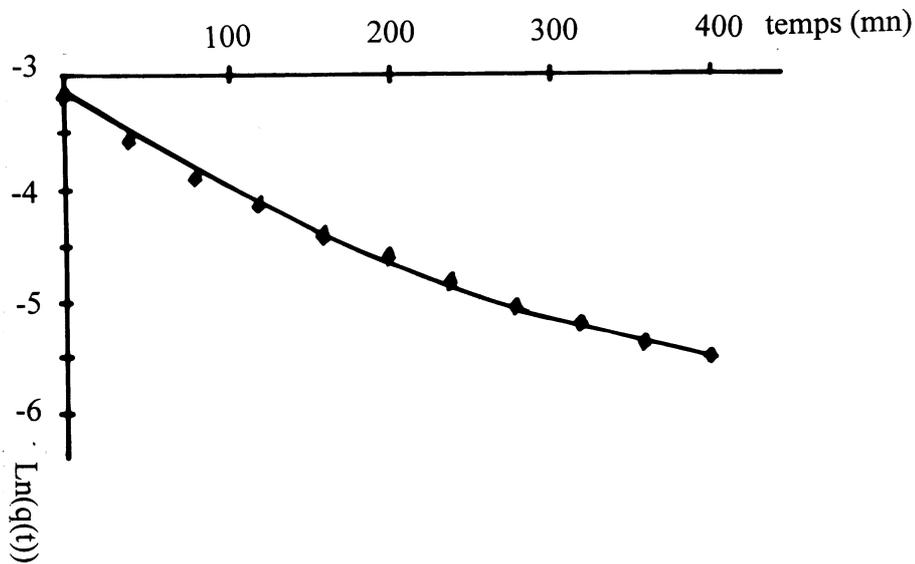


Figure2. Representative curve of the function  $\text{Ln} Q(t)$

From this figure, it can be seen that  $\text{Ln}(J_1)$  is the ordinate at the origin. Therefore, the values of the parameters  $J_1$  and  $\eta_1$  can be obtained as :

$$J_1 = 1.23 * 10^{-11} \text{ Pa}^{-1}, \theta_1 = 400 \text{ mn} \text{ and } \eta_1 = 32.5 \text{ GPa.mn.}$$

## Determination of the parameters $J_2$ , $\eta_2$ and $\theta_2$

As a second analysis, the function  $Q'(t) = q(t) - J_1 e^{-t/\theta_1}$  is considered. Taking the natural logarithm, it follows:

$$\text{Ln}Q'(t) = \text{Ln}(q(t) - J_1 e^{-t/\theta_1}) \quad (4)$$

From the graph  $\text{Ln}(Q'(t))$ , figure 3, the values of the parameters  $J_2$ ,  $\theta_2$  and  $\eta_2$  are obtained.

$$J_2 = 3.02 * 10^{-11} \text{ Pa}^{-1}, \quad \theta_2 = 80 \text{ mn} \quad \text{and} \quad \eta_2 = 2.649 \text{ GPa.mn.}$$

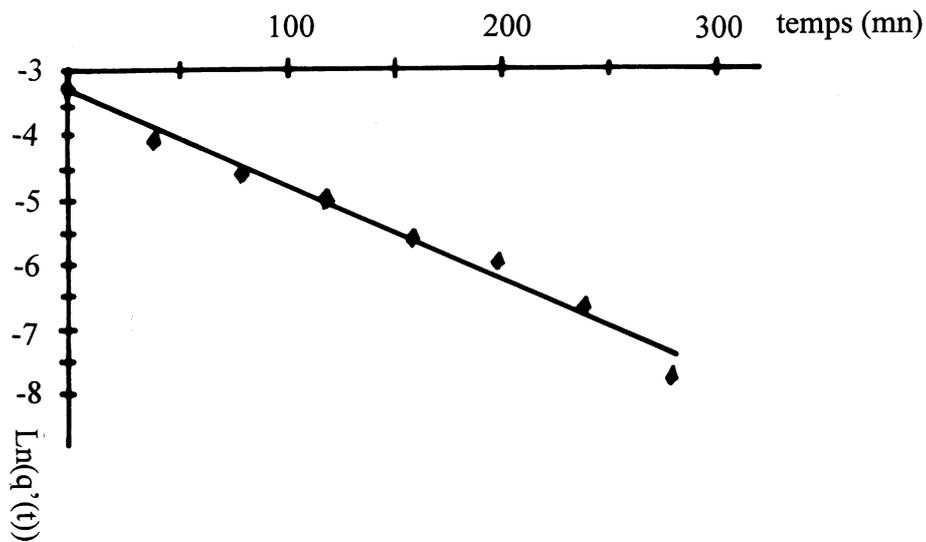


Figure 3. Representative curve of the function  $\text{Ln} Q'(t)$

Since the graph of  $\text{Ln}(Q'(t))$  is a linear curve, there is no need to do a third analysis. Therefore, the material Kevlar 49 can be represented adequately with two Kelvin Voigt solids.

## CONCLUSION

The long term behaviour of Kevlar 49 can be represented by a model composed of a spring (compliance  $J_0$ ) and two Kelvin Voigt cells (figure 4). The creep function can be expressed as follows:

$$f(t) = (1.02 + 1.23(1 - e^{-t/400}) + 3.02(1 - e^{-t/80})) * 10^{-11} \quad (5)$$

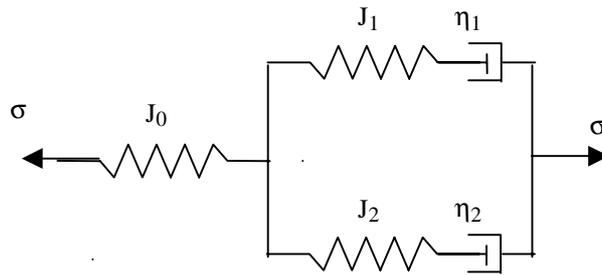


Figure 4. Rheological model of Kevlar 49 cable (type 0.95 TWA2)

The obtained results are in good agreement with experimental ones as shown on figure 5.

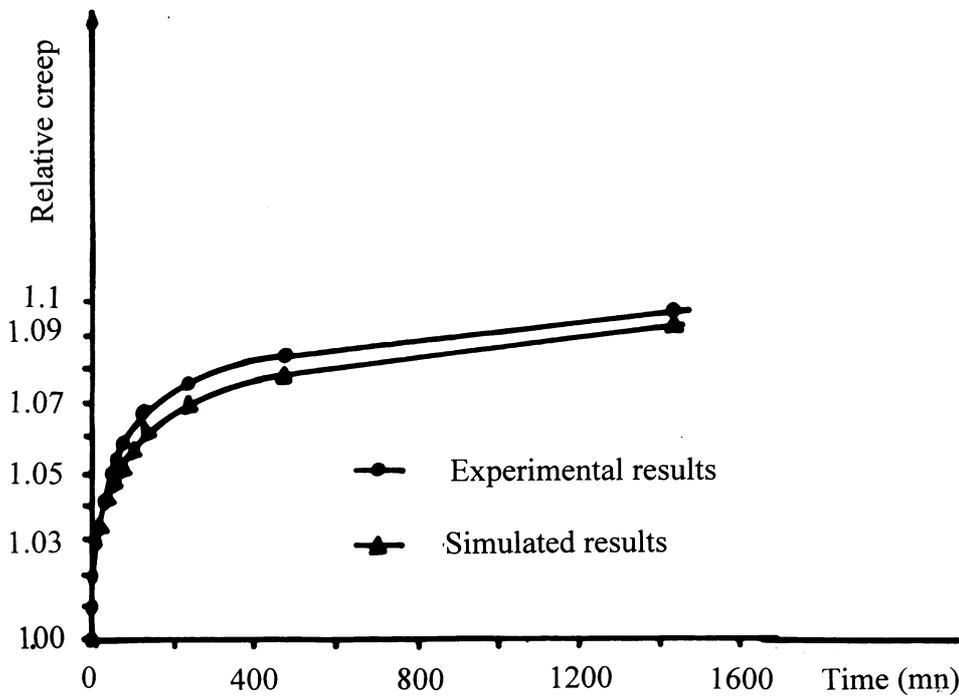


Figure 5. Creep behaviour of Kevlar 49 cable (type 0.95 TWA2)

## SYMBOLS

- $f$  : creep function of the viscoelastic material
- $t$  : time
- $J_0$  : instantaneous elastic compliance
- $\eta_0$  : coefficient of newtonian viscosity
- $\theta_0 = J_0 * \eta_0$  : retardation time
- $J_1, \eta_1, \theta_1$  : parameters of the Kelvin Voigt cell number 1
- $J_2, \eta_2, \theta_2$  : parameters of the Kelvin Voigt cell number 2

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