A GENERALISED CONTINUUM MODEL FOR TEXTILE

C. Jung[1], J.F. Ganghoffer[2], B. Durand[1]

[1] *Laboratoire de Physique et Mécanique Textiles de Mulhouse. ENSITM. 11 rue Alfred Werner, 68093 Mulhouse cedex, France.*

[2] *Institute for Chemistry and Physics of Interfaces - CNRS - 15 rue Starcky, BP 2488, 68057 Mulhouse cedex, France.*

SUMMARY : Woven textile materials exhibit interesting and new features, compared to standard materials, such as negative Poisson's ratios, revealed by measurements of the variations of fabric lateral dimensions, using an optical extensometer. Micromechanical models of fabric have been elaborated, following two different lines of thoughts. In the first approach, the mere yarn is modelled as an undulated beam ; a perturbation method allows to obtain an equivalent representation of the yarn as a straight beam, the equivalent coefficients of which keeping a trace of the initial undulations. In the second approach, a unit cell is defined as one undulation of the yarn, and the rotation of the unit cell with respect to a fixed basis defines a microrotation, which enters into the kinematical description of fabric as an additional rotational degree of freedom at an intermediate scale.

KEYWORDS : mechanics of textile, negative Poisson's ratio, micromechanics, micropolar material, thin plate model.

INTRODUCTION

Woven structures like textile find an extensive use nowadays in advanced applications (even in the biomedical area) due to their very high flexibility and their adaptability to nearly arbitrary imposed shapes : these are certainly the principal reasons of their success as cloths realisations.

Woven textiles surfaces are considered as two-dimensional structures. Their small thickness compared to the transversal dimensions are generally not taken into account. Yet, they are very often used for three-dimensional realisations. The first step of the fabrication of such an object consists in a definition of the two-dimensional elements, which shall give birth to the volume after assembly. Optimisation of this plane shape -initial element- and that of the behaviour of the three-dimensional structure -the final element- requires a good understanding and knowledge of the mechanical behaviour of textile structures. A good knowledge of the mechanical behaviour of sewed multistructures would be of first importance, even before such a structure is produced. Simple mechanical testing already exhibit their peculiar behaviour : negative Poisson's ratio (or above unity), and large stretches (uniaxial extension, fig. 1). The force-extension curve can be divided in three domains, corresponding to three mechanisms : the first is a homogenisation of the fabric pattern (loss of geometrical undulations at the macroscopic scale), the second is a transfer of the undulations (loss of undulations of the yarn) and the third is due to the stretch of the yarn itself.

Increasing the applied strain then leads to solicit the fabric material on a finer and finer scale. It is clear that the geometrical and mechanical coupling between the two chains of yarns has an important role in the specific behaviour of textile. Among the existing models in the literature, very few of them consider this coupling. The aim of this work is to establish a continuous model for woven structures, starting form the microscopic scale. This work resumes part of the thesis work [1].

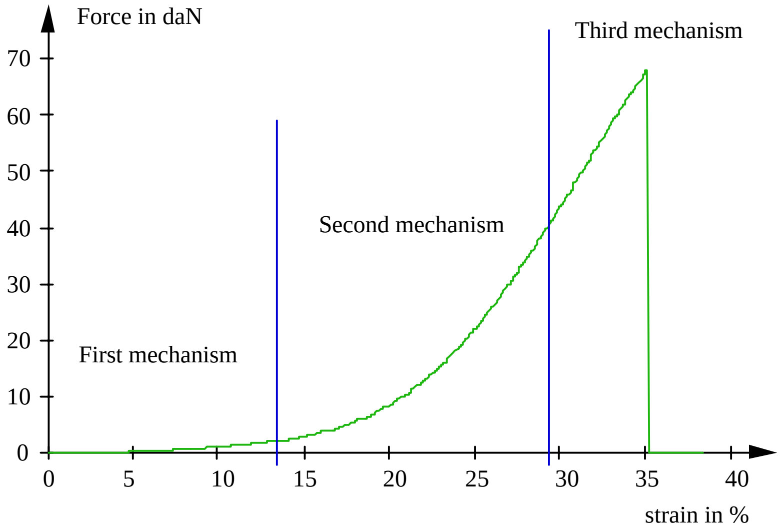
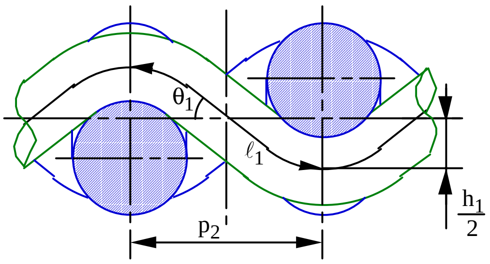


Fig. 1 : The three deformation mechanisms of fabric.

A REVIEW OF SOME EXISTING MODELS

In the literature, we find three types of models : the geometrical models, the mechanical models and the energy methods. The first geometrical model due to Peirce (1937) (fig. 2). The yarns are intertwined and rolled one each other ; the cross section is circular. Relationships between the different geometrical parameters are established (fig 2). The central question concerning any modelling attempt is : what is the nature of the efforts and their distribution on the contact area ? An other question : how is the cross section of the yarns being modelled ? As a matter of fact, many models assume an elliptic section, no symmetrical section, thus rely on simplified assumptions regarding either the geometry or the deformation behaviour of the yarn.



$$p_1 = (\ell_2 - D\theta_2)\cos\theta_2 + D\sin\theta_2$$

$$h_1 = (\ell_1 - D\theta_1)\sin\theta_1 + D(1 - \cos\theta_1)$$

Fig. 2 : The Peirce's geometrical model.

The Kawabata model assumes a simple geometry [2]: the threads have no undulations but the simple geometry is completed with traction and compression moduli (fig. 3). This model allows to calculate the yarns deformation ratio.

The mechanical energy of a textile structure has four possible origins :

- E_b : the bending energy due to the undulations,
- E_r : the twisting energy,

- E_c : the compression energy due to the contact point between the yarns,
- E_t : the extension energy due to the yarn stress.

The total energy is obtained as the sum of these four internal energies.

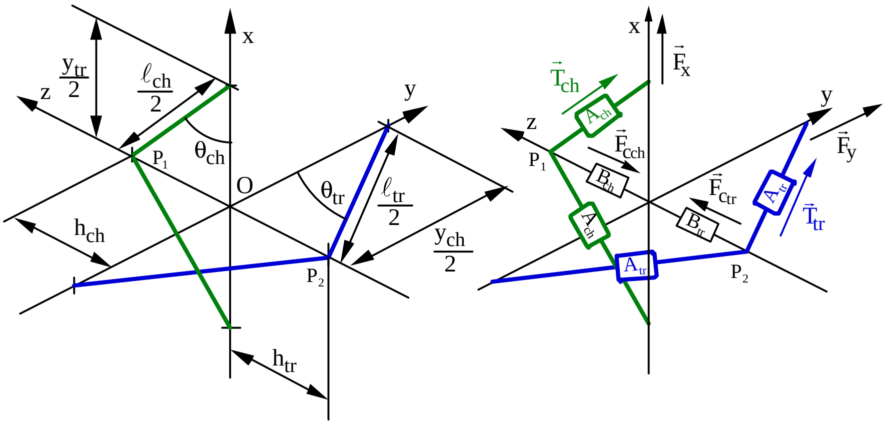


Fig. 3 : The Kawabata's model.

The bibliographical review of the mechanical studies on textile structures shows that none of these correctly describes their complexity. This might explain why experimental tests are limited to unidirectional traction, and why their interpretation is essentially statistical for more complex loading. Ideally, an approach intended to be exhaustive should consider both the geometrical, mechanical and energetical aspects.

MEASUREMENT OF POISSON’S RATIO

A fabric is not a continuous homogeneous isotropic material, but a system composed of two yarns families having more often an orthotropic behaviour, with a cohesion due to the opposite undulations of the yarns of each family. The structure, although coherent, is not blocked, as it is for a composite material or a coated fabric. These specific structural behaviours led us to realise new experiments, in order to increase our understanding of the mechanical behaviour of fabric. With an optical (laser) extensometer, we measure changes in fabric lateral dimensions during an uniaxial tensile test. The results are presented in figure 4, in terms of the variations of the Poisson's ratio. These studies show that the Poisson's ratio take a value above unity in the width, namely 3.8 and 165.6. The first value (3.8) is explained by a transfer of the undulations : a yarn in a direction looses amplitude of its undulation and the other yarn in the transverse direction gains amplitude of its undulation.

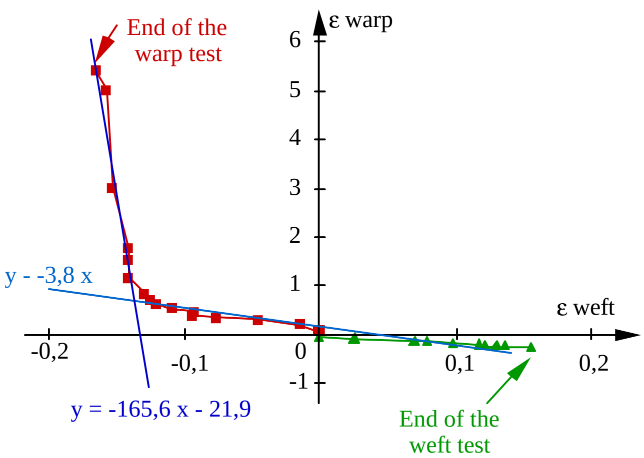


Fig. 4 : The variations of the Poisson's ratio.

The second value (165.6) is the result of a limit (fig. 5) : the extended yarn has lose all undulations and no transfer of undulations is possible any more. These Poisson's ratio values which exceed the classical limit of 0.5, are not really new. R. Postle did also find Poisson's ratio values above the unity [3].

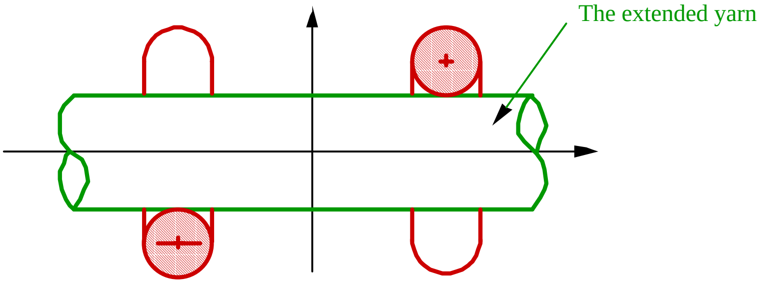


Fig. 5 : The limit behaviour of the structure.

The development of the extensometer has opened interesting and promising perspectives, but important improvements are still needed. A more sophisticated equipment based on the analysis of the temporal signal would enable to analyse at the cell scale the transmission of the local stresses within fabric.

STARTING FROM UNDULATED BEAMS

We focus on the behaviour of one yarn, which is the basic element of fabric. Since we are interested in formulating a model at an intermediate scale of description, it is relevant to homogenise the yarn undulations. The threads is modelled as an undulated beam, and we look for the behaviour obtained when the undulations are very pronounced. We rely on the work by M. POTIER-FERRY [4] on the geometrical homogenisation of an undulated beam, to build a model of undulated yarns. The limit behaviour is obtained using an asymptotic expansion of the forces and moments. We analyse the behaviour of the yarns within a unit cell (after a scaling with a $1/\eta$ ratio ; fig 6) and we characterise the kinematics and the static of fabric at the scale of this cell.

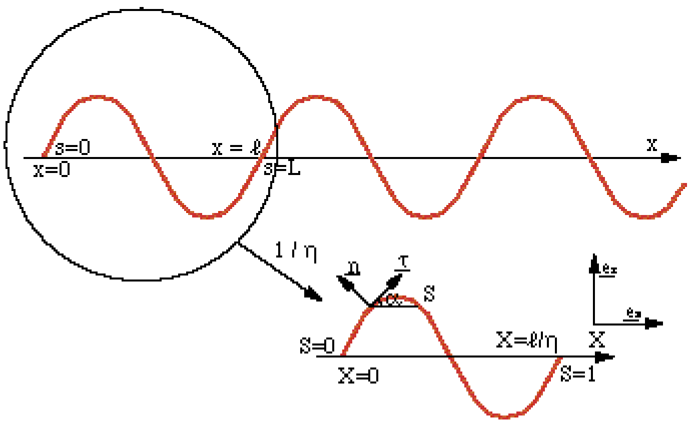


Fig. 6 : Zoom of one undulation.

We have the elastic behaviour law in a small perturbations context :

$$\underline{N}(s).\underline{\tau}(s) = E\ A\ \varepsilon(s) \quad (1)$$

As a matter of definition, the gradient of the displacement decomposes into the deformation and the rotation field, as

$$\frac{d\underline{U}(s)}{ds} = \varepsilon(s)\underline{\tau}(s) + \beta(s)\underline{n}(s) \tag{2}$$

It can be shown that the displacement field $\underline{U}(s)$ can be approximated by its normal component $v(s).\underline{N}$. As a consequence, the elastic uniaxial strain is

$$\varepsilon(s) = \frac{d\underline{U}(s)}{ds}.\underline{\tau}(s) = \frac{d\left(v(s)\underline{N}\right)}{ds}.\underline{\tau}(s)$$

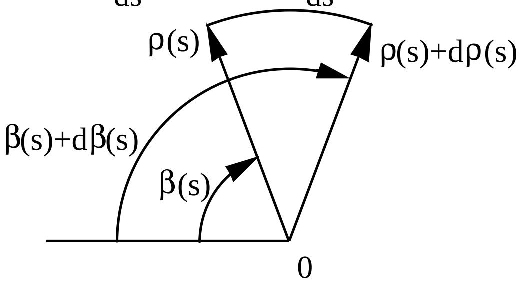


Fig. 7 : The rotation field of the beam.

The rotation variation can be approximated by (fig. 7) :

$$ds = \rho(s)\, d\beta(s) \Rightarrow \frac{d\beta(s)}{ds} = \frac{1}{\rho(s)}$$

Also, we write the curvature along the beam:

$$\frac{1}{\rho(s)} = v_{,ss} \tag{3}$$

According to this, equation (1) becomes :

$$\underline{N}(s).\underline{\tau}(s) = E\ A\ v\ v_{,ss} \tag{4}$$

We further need estimations for the norms of the variables (force and moment), so that we know the leading term of their asymptotic expansions. From the variational equation of the equilibrium problem, and using the tools of functional analysis, we first obtain the following estimate for the displacement :

$$\left\| v(s) \right\|_{L^2(0,L)} \leq C\, L^{3/2} \leq C\, \eta^{3/2},$$

with $L^2(0,L)$ the space of square integrable functions.

From that, the following estimation is deduced for the moment :

$$\left\| M \right\|_{L^2(0,L)} \approx L^{1/4} = \eta^{1/4}$$

We thus get the asymptotic expansion :

$$M(x,s) = M_0 + \eta\, M_1 + \eta^2\, M_2 + \cdots \tag{5}$$

We next use the equilibrium equation :

$$\frac{dM}{ds} + \underline{N}.\underline{n} = 0$$

and the derivation rule for two variables S and x :

$$\frac{dM}{ds} = \frac{1}{\eta}\frac{\partial M}{\partial S} + \cos\alpha(s)\frac{\partial M_0}{\partial x} \tag{6}$$

to get the first term of the asymptotic expansion of $\underline{N}^{\eta}(x,S)$:

$$\underline{N}^{\eta}(x,S)=\frac{1}{\eta}\underline{N}^{0}(x,S)+\ldots \tag{7}$$

The homogenisation techniques can then be applied to characterise the leading terms of the undulating beams, inserting the asymptotic expansions :

$$\underline{N}^{\eta}(x,S)=\frac{1}{\eta}\underline{N}^{0}(x,S)+\underline{N}^{1}(x,S)+\eta\underline{N}^{2}(x,S)+\ldots \tag{8}$$

$$\underline{M}_{G}^{\eta}(x,S)=\underline{M}_{G}^{0}(x,S)+\eta\underline{M}_{G}^{1}(x,S)+\eta^{2}\underline{M}_{G}^{2}(x,S)+\ldots \tag{9}$$

in the equilibrium equations. In order to obtain the equivalent moduli of the beam, the constitutive law is written in terms of the asymptotic expansion of the displacement field. The average operator is apply to the different coefficients of the successive powers of the small parameter η : for any quantity *a(s),* the symbol $\langle\ .\ \rangle$ means the average of *a(s)* over one undulation. The leading term describes a straight beam, with effective traction and flexion moduli given by :

$$\left(\frac{1}{EA}\right)_{eq}=\frac{\left\langle\cos^{2}\alpha\right\rangle}{E\ A\langle\cos\alpha\rangle}+\frac{\left\langle z^{2}\right\rangle}{E\ I\langle\cos\alpha\rangle}\quad;\quad(E\ I)_{eq}=E\ I\langle\cos\alpha\rangle \tag{10}$$

with E the traction modulus, I the inertia momentum, A the cross-section of the beam, $\alpha(s)$ the rotation field along the beam, and *z(s)* the variation of the altitude along the beam. The equivalent moduli keep a trace of the initial undulations of the beam, through the averages of the geometrical parameters involved in (10). The function *z(s)* describes the shape of the undulation of the beam.
This first model is potentially able to describe the behaviour of the woven structure, provided the coupling between both chains of threads in taken into account.

This model focuses on the behaviour of a single yarn ; the fabric however consists of a network of intertwined yarns (the weft and the warp, fig. 8). Therefore, in the subsequent model, the fabric is approached as a three-dimensional structure from the onset.

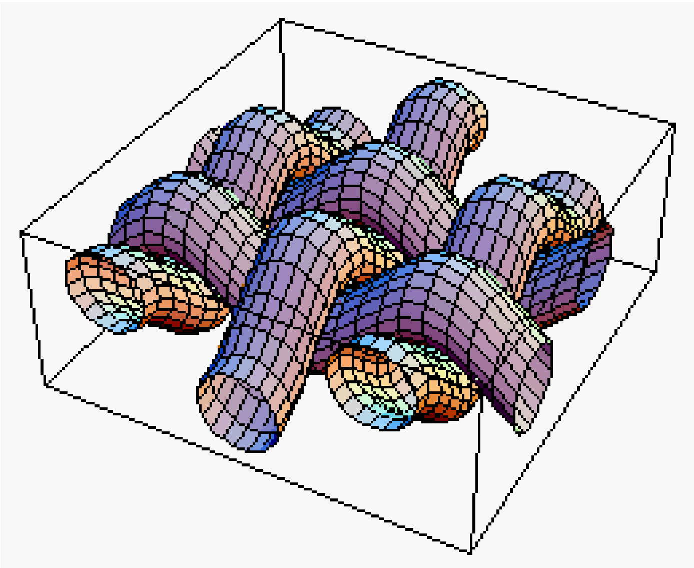


Fig. 8 : Three dimensional structure of fabric.

WOVEN FABRIC AS A MICROPOLAR MATERIAL

In this approach, the woven structure is modelled as a continuous micropolar material : the microrotation is defined as the averaged rotation of the unit cell (one undulation), around the axis x and y, thus the components ω_x and ω_y respectively (fig. 9). The thread within the unit cell is modelled as a succession of portion of circles and straight lines (fig. 9). We distinguish three scales : the microscopic scale (the scale of the threads), the mesoscopic scale (one period of undulation), and the macroscopic scale (scale of the structure).

A coupling between the two networks of threads is obtained, from the theorem of mutual actions on the contact zone :

$$E_\ell I_\ell \kappa(s) + E_t I_t \kappa'(s') = 0 \tag{11}$$

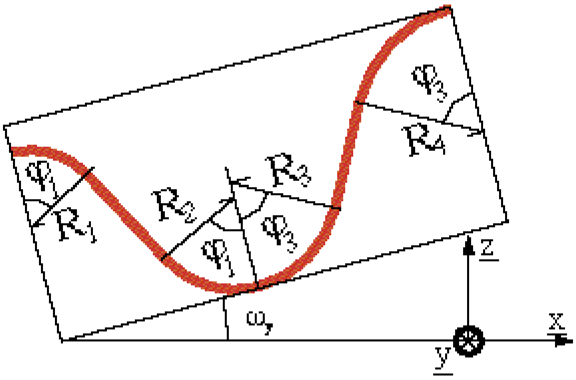


Fig. 9 : Microrotation of the unit cell (ω_y). Plane (x, z).

The indices ℓ and t are relative to the longitudinal and transversal directions respectively. When integrated over the whole contact area, and considering the geometrical modelling of the threads, the following relationship between the microrotations ω_x and ω_y is obtained :

$$\frac{E_\ell I_\ell \omega_y}{L_\ell} + \frac{E_t I_t \omega_x}{L_t} = 0 \tag{12}$$

with L_ℓ, L_t the true lengths of the threads within the cell.

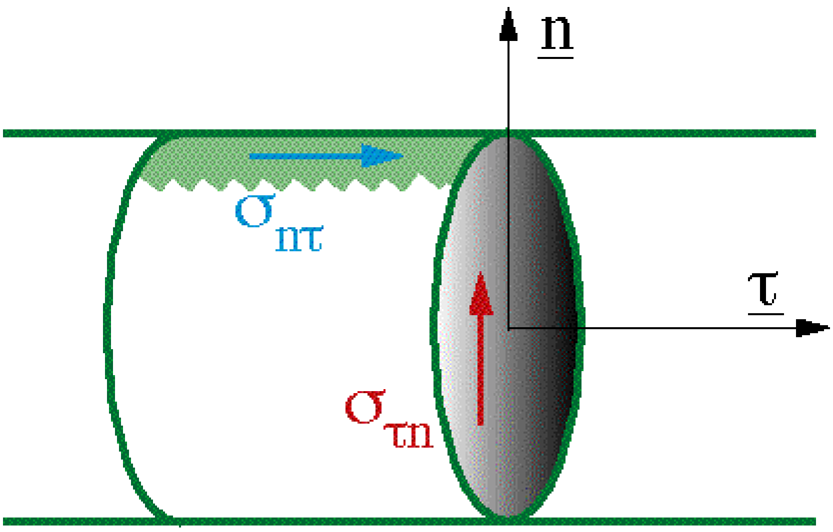


Fig. 10 : The stresses in the yarn.

We write the local stress tensor $\underline{\underline{\sigma}}_\ell$ in the Frenet basis (fig. 10), selected as:

$$\underline{\underline{\sigma}}_\ell = \begin{pmatrix} \sigma_{\tau\tau} & 0 \\ \sigma_{n\tau} & 0 \end{pmatrix} \tag{13}$$

Since the section of the yarn is vanishing, we assume a shear component of the stress which is only due to the rotation field, and not to an internal shearing within the section of the yarn, thus the non symmetrical form. Similarly, the shear part of the strain tensor is due only to the rotation field (anti symmetrical part). In doing this, we ignore the internal structure of the yarn (threads), which is being idealised as a plain material (although it is discrete on the nanoscopic level). Furthermore, the continuous beam is approached by its mean line, according to the vanishing section.

Going back to the global Cartesian basis, a simple orthogonal transformation R gives the stress tensor as :

$$\underline{\underline{\sigma}} = \left\langle R\,\underline{\underline{\sigma_\ell}}\,R^t \right\rangle \tag{14}$$

The translational part of the constitutive law equation (1) is written in the corotational frame (the unit cell rotates in fact with the angular velocity $\omega = \left(\omega_x, \omega_y, 0\right))$ as a matter of objectivity ; the anti symmetric rotation matrix W dual to the vector ω is constructed. Corotational derivatives of deformations and stresses are employed : for instance, we define the corotational derivative of the stress :

$$\overset{\nabla}{\underline{\underline{\sigma}}} = \underline{\underline{\sigma}}\,W - W\,\underline{\underline{\sigma}} \tag{15}$$

The corotational derivative of the stress tensor divides in a term which is to the yarn and another term that characterises the interaction between the yarns. Using in a similar way the corotational derivatives of the strain tensor, we can consequently write for the first yarn :

$$\overset{\nabla}{\underline{\underline{\sigma_1}}} + \overset{\nabla}{\underline{\underline{\sigma_{i_1}}}} = E_1\left(\overset{\nabla}{\underline{\underline{\varepsilon_1}}} + \overset{\nabla}{\underline{\underline{\varepsilon_{i_1}}}}\right) + E_1\ \omega_x\ \omega_y \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \overset{\nabla r}{\underline{\underline{\sigma_{i_1}}}} \tag{16}$$

and for the second yarn :

$$\overset{\nabla}{\underline{\underline{\sigma_2}}} + \overset{\nabla}{\underline{\underline{\sigma_{i_2}}}} = E_2\left(\overset{\nabla}{\underline{\underline{\varepsilon_2}}} + \overset{\nabla}{\underline{\underline{\varepsilon_{i_2}}}}\right) + E_2\ \omega_x\ \omega_y \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \overset{\nabla r}{\underline{\underline{\sigma_{i_2}}}} \tag{17}$$

Summing up the equations (16) and (17), we obtain :

$$\overset{\nabla}{\underline{\underline{\sigma}}} = E_1\left(\overset{\nabla}{\underline{\underline{e_1}}} + \overset{\nabla r}{\underline{\underline{\varepsilon_{i_1}}}}\right) + E_2\left(\overset{\nabla}{\underline{\underline{e_2}}} + \overset{\nabla r}{\underline{\underline{\varepsilon_{i_2}}}}\right) + \overset{\nabla r}{\underline{\underline{\sigma_{i_1}}}} + \overset{\nabla r}{\underline{\underline{\sigma_{i_2}}}} \tag{18}$$

with the following decomposition of the stress :

$$\overset{\nabla}{\underline{\underline{\sigma}}} = \overset{\nabla}{\underline{\underline{\sigma_1}}} + \overset{\nabla}{\underline{\underline{\sigma_{i_1}}}} + \overset{\nabla}{\underline{\underline{\sigma_2}}} + \overset{\nabla}{\underline{\underline{\sigma_{i_2}}}}$$

The term $\overset{\nabla r}{\underline{\underline{\sigma_{i_1}}}}$ represents a residual stress, which is attributed to the undulations of the fabric ; the term $\overset{\nabla r}{\underline{\underline{\varepsilon_{i_\ell}}}}$ represents a residual deformation.

The rotational part of the constitutive law on the mesoscopic scale is next obtained, starting from the local constitutive equation :

$$\underline{m} = E\,I\,\underline{\kappa} \tag{19}$$

with the microcurvature tensor in a local basis (superscript ℓ) given by

$$\underline{\underline{\kappa}}^{\ell} = \begin{pmatrix} \kappa_{ss} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Using the same transformation rule as before (rotation and averaging), the microcurvature tensor is obtained in the global basis for each yarn (weft and warp). Summing up the two constitutive equations, the micropolar law can be written :

$$\underline{\underline{m}} = 2\,\mu\,\ell_c\,\underline{\underline{\kappa}} \qquad (20)$$

κ is a representation of the microcurvature, which integrates the coupling between both yarns (equation (11)). Comparing equations (19) and (20) shows that the bending length ℓ_c can be explicitly expressed versus the mechanical and geometrical parameters of the yarn.

The complete constitutive law of fabric on the mesoscopic scale comprises then the translational part (equation (18)) and the rotational part (equation (20)). Although the fabric material is rather anisotropic (orthotropic), it appears that the representation given by these equations is isotropic. The bending length that enters the constitutive equation (20) has been evaluated, and appears to be of the same order of magnitude than the size of the unit cell ; therefore, the microscopic scale and the mesoscopic scale can not be separated : in other terms, the microscopic behaviour shall directly influence the behaviour at the next scale (mesoscopic scale). This is particularly important with respect to homogenisation techniques, which usually rely on such an assumption.

The simulation of a simple traction experiment of a single yarn undergoing a pure loss of undulation (no extension) shows that the model exhibits some trends of the observed J-shaped curve characteristic of fabric. However, it is clear that a large perturbation analysis is needed, considering the large rotations and strains observed in real fabric materials.

In the micropolar constitutive law established, the microrotation appears as an additional kinematic variable, which characterises the undulations of fabric on a mesoscopic scale. Fabric acquires thereby the status of a generalised continuous material (endowed with a microstructure). A simple calculation shows that the microrotation explains the negative (or very high) value of the measured Poisson's ratio.

CONCLUSIONS AND PERSPECTIVES

We have engaged a mechanical modelling of fabric, which appears to us rich in potential developments. We shall mention three important perspectives. One aim of this work was to establish a continuous model of fabric at the mesoscopic scale, starting from the microscopic scale ; since the mesoscopic equivalent material is continuous, the impact of the discrete nature of fabric can be questioned. The transition to the structural scale remains to be done. At this ultimate scale, additional phenomena emerge, such as the changes of the geometrical curvature (more generally speaking, the formation of wrinkles), which require the elaboration of shell models. In this respect, the microrotation plays the role of an internal parameter, characteristic of the microstructure, whereas the curvature characterises and determines macroscopic undulations. In another contribution [5], the micropolar plate model has been extended towards considerations of these curvature effects : textile shall be modelled as a thin elastic shell endowed with internal rotational degrees of freedom. Limit models for a thin shell can then be obtained depending on the ratio between two small parameters : the thickness of the shell, and the inverse of the radius of curvature [5].

Secondo, sliding occurs at the contact zones between the yarns, which is responsible for a macroscopic dissipate behaviour, and strongly influences the structural behaviour of fabric.

Tertio, efficient experimental devices need to be conceived and elaborated, the role of which is to quantify in a precise manner the deformation mechanisms of textiles, both at the microscale and at the macroscale. The area of textile suffer from a crucial lack of knowledge of the local strains and stresses (at the yarn scale, or averaged over one undulation) ; existing systems only propose averages of the mechanical fields for a tested sample. The modelling effort has to be combined in a parallel way with an experimental characterisation (to evidence phenomena specific to textile ; for a validation purpose).

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