# DESCRIPTION OF FATIGUE FAILURE PROCESS FOR LAMINATED COMPOSITES STRUCTURES

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**SUMMARY:** The aim of the work is to propose the numerical method of fatigue stiffness degradation and life estimations for composite structures subjected to arbitrary loading and boundary conditions. The analysis is based on three assumptions: (i) the definitions of the damage initiations, (ii) the approximations of the stiffness degradation with the number of cycles and (iii) the form of fatigue life correlations. The numerical investigations are conducted with the use of physical nonlinearities describing the stress-strain relations for each individual ply in the laminate independently (different behaviour of loading and unloading phases) and of geometrical nonlinearities. A nonlinear, finite element technique is used for the stress analysis. Based on the state of stress, different failure modes are detected by a set of fatigue failure criteria. To remove the requirement of the large experimental data base for the development of the fatigue process a simple technique is established. Finally, the numerical results are compared with the experimental ones - the example of plate with a hole, subjected to fatigue loading. The results demonstrate a very good agreement between experiments and numerical studies.

**KEYWORDS:** Fatigue analysis**,** stiffness degradation, numerical modelling, plates, experimental studies, progressive failure

## INTRODUCTION

For composite constructions a fatigue analysis is a complicated problem due to the complexity and variety of failure modes, boundary and loading conditions that may have a great influence on a fatigue life. The above problems have caused that there is a lack of a general fatigue models that are capable of accurately predicting the entire failure process. However, experimental investigations in this area have given a lot of information about the fatigue process development. Based on that it is possible to distinguish two fundamental area of interests: (i) the fatigue phenomena occurring in an individual ply of the laminate and (ii) the fatigue behaviour of the laminate on interlaminar faces.
The aim of the present paper is twofold:

1) to propose a model describing the fatigue process in an individual ply and then ,
2) to confront the model predictions with the experimental results availaible in the literature and conducted by the author.

In order to measure the development of fatigue process the residual stiffness E(N) is recorded for each cycle by a computer. Generally, the stiffness reduction during the fatigue process versus the number of cycles exhibits three different regions - see e.g. D.L. Jones *et al.* [1] ; a rapid changes at the beginning and at the end of the fatigue life and a long intermediate period of a relatively slow stiffness variations. According to the classification introduced by R. Talreja [2] the first region corresponds to a random matrix cracking, the second to an interlayer delaminations and the third one to a fibre breakage. On the contrary to the traditional approaches (see e.g. D.L. Jones *et al.* [1], J.N. Yang *et al.* [3])we propose to describe the function characterizing the stiffness reduction vs. the number of cycles as the function composed of three straight lines where the ends of the intermediate line are treated as statistical variables. In this way it is possible to determine the reliability functions for each fatigue process and in addition, it is not necessary to distinguish fibre and matrix dominated fatigue failure modes (see e.g. J.N. Yang *et al.* [3]) . The development of the fatigue damage is simply modelled by the analysis of a few cycles of loading.
The proposed model of the stiffness degradation has been verified experimentally. Tests have been performed on glass/epoxy rectangular specimens having a central hole and made of eight individual layers. Two types of specimens have been analysed oriented at $0^o$ (fibre dominated failure) and $45^o$ (matrix dominated failure). A total number of 20 specimens have been tested in fatigue at one stress level equal 60% of the static ultimate strength. The stiffness was measured at preselected cycle numbers without stopping the loading process.
The proposed in the work model of the stiffness degradation have been used to verify the correlation with the experimental results, now, especially in view of the fatigue life estimations. The agreement seems to be very good.

## MODELLING OF PHYSICAL NONLINEARITIES

Experimental studies (see e.g. [4]) demonstrate evidently that the nonlinear stress – strain relation for composites can be modelled in the following way:

$$\sigma = \beta(1 - e^{r\bar{\varepsilon}}) \tag{1}$$

where : βr is a value of Young's modulus, whereas σ and $\bar{\varepsilon}$ denote stresses and strains at the direction of the external load. The above formula is valid for one fibre direction only which is examined during an experiment. During the fatigue tests permanent elongation occurs which is expressed by an additional parameter A:

$$\sigma = A + \beta(1 - e^{r\bar{\varepsilon}}) \tag{2}$$

In terms of plasticity eqns (1), (2) describe the classical nonlinear hardening curves. However, we propose to extend the above relations (1) and (2) for an abitrary fibre direction, introducing the following assumptions:

- the value $\bar{\varepsilon}$ is not the physical strain but a strain parameter that varies with fibre orientations in the following way:

$$\bar{\varepsilon}(\theta) = \varepsilon[\cos^2(\theta) + \frac{\varepsilon_m^u}{\varepsilon_f^u}\sin^2(\theta)], \quad 0 \le \varepsilon \le \varepsilon_f^u \tag{3}$$

- The parameter r is dependent on fibre orientations as follows:

$$r(\theta)=\frac{1}{\beta}\left[E_1\cos^2\theta+\left(2G_{12}-E_1-E_2\right)\cos\theta\sin\theta+E_2\sin^2\theta\right] \tag{4}$$

- β is a constant
- the fibre elongation is the highest at the direction θ=0 and is equal to $A_0$, so that :

$$A(\theta)=A_0\cos\theta \tag{5}$$

θ denotes the fibre orientation and $\varepsilon_f^u$, $\varepsilon_m^u$ are the ultimate strains of fibre and matrix, respectively.

The mesured strain is denoted in eqn (2) by ε – as it may be noticed the maximal value of strains corresponds to the ultimate tensile strain. Inserting eqs (3) and (4) into the relation (2 (or (2)) we obtain experimentally confirmed series of the strain-stress curves for different fibre orientations. It may be easily verified that for the fibres oriented at 0 and 90 deg one can get as the limit cases the classical linearity of the curves and nonlinearity for fibres oriented at 45 deg - see Fig.1. Since the compression behaviour of composites is different than the tension one we apply the stress – strain characteristics measured in the static tests. In general, the relation is nonlinear and similar to the plotted in Fig.1. Let us notice that both tension and compression behaviour of composites is nonlinear but not always may be approximated by a strict mathematical formula, especially for compression. Those nonlinear characteristics will be used further in the numerical modelling of fatigue behaviour of structures. In general, in numerical analysis they can be approximated for each step of loading or unloading.

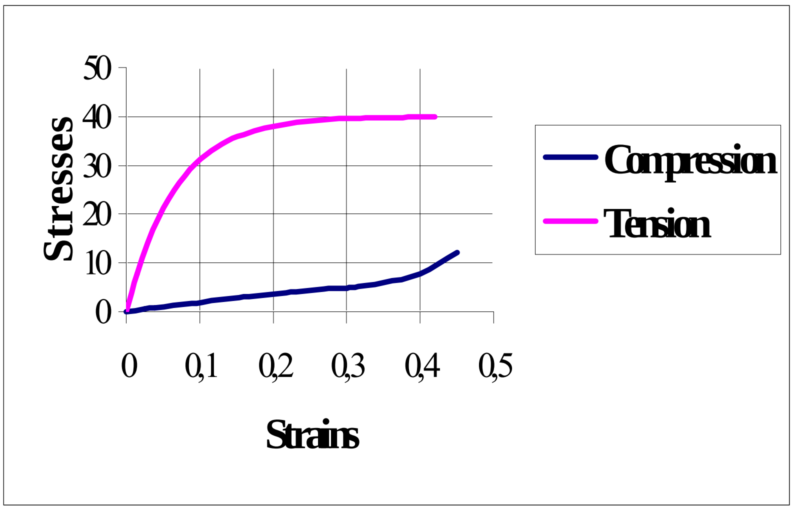


Fig. 1: The stress-strain relations for unidirectional composites

## MODELLING OF FATIGUE DAMAGE

### Fatigue failure modes of composites

Fatigue damage in composite laminates or structures is associated with different mechanisms of failure such as : matrix cracking, fiber – matrix debonding, fiber breakages and delaminations. However, it should be emphasized that fatigue failure modes are identical to static ones and in addition, the physical nonlinearities observed in the static stress-strain diagrams discussed in the previous section are the result of the progressive laminate failure development (an interlayer separation). In general, interlayer delaminations (understood in the sense of macro-cracks) are preceded by an accumulation of matrix micro-cracks. Finally, the global fatigue damage of laminated composites occurs as fibres breake. The fatigue damage mechanisms described above result in drastic changes of the strength, stiffness and other mechanical properties of composite structures. However, it has been obsreved that the specific fatigue failre modes enumerated above are directly joined with the particular behaviour of stiffness degradation – Fig.2. The stiffness degradation variations with the number of cycles N are a nonlinear process given by the following differential equation (see e.g. Jones *et al.* [1]) :

$$\frac{d\left[\bar{E}(N)\right]}{dN} = -\frac{1}{(N+1)\left[\bar{E}(N)\right]^{m-1}}, \bar{E}(N) = E(N)/E(N_f) \tag{6}$$

where $N_f$ is a fatigue life of structures, and m denotes an experimentally fitted material parameter. With a neglecting error the above relation may be modelled by three straight lines plotted in Fig.2. The first line corresponds to the matrix cracking and fibres debonding , the second to interlayer delaminations, whereas the third one to fibres breakages – see Talreja [2].

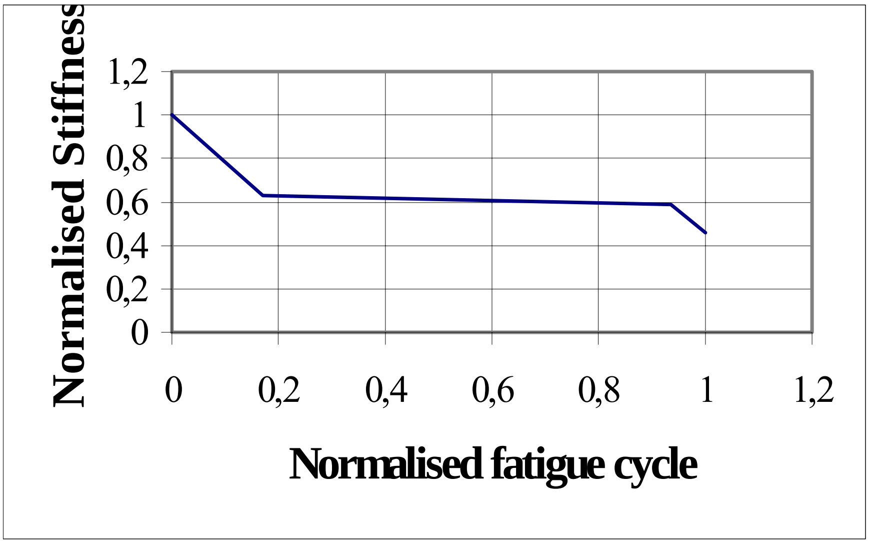


Fig.2: Variations of stiffness degradation with fatigue life.

The measurement of the stiffness degradation can be made during a fatigue process nondestructvely. However, the problem how to find the mechanistic model of that process is still open although there is a lot of different models – see e.g. the review made by Yang *et al.* [3].

## Initiation of fatigue failure

Despite of the local damage the initiation of three types of failure modes may be described sufficiently well (see Tsai and Hahn [5]) with the use of the lamination (homogenized) theory. The matrix failure is determined with the use of the Tsai- Wu criterion:

$$F_{01}\sigma_{11} + F_{11}\sigma_{11}^2 + F_{12}\sigma_{11}\sigma_{22} + F_{02}\sigma_{22} + F_{22}\sigma_{22}^2 + F_{44}\sigma_{12}^2 \le 1 \tag{7}$$

since it predicts the layer strength for stress states with relatively large transverse stresses that affects much more the matrix than fibres. The experiments demonstrate that for uni-directional composites the Tsai-Wu criterion overestimates the strength in the fibre direction. Therefore, for the analysis of fibre cracking the use of the simplest maximum strain criterion is proposed:

$$\varepsilon_{ij}^c \le \varepsilon_{ij} \le \varepsilon_{ij}^t \tag{8}$$

As it is reported in the literature (see e.g. Talreja [2]) delamination damage is caused and treated as the mode I and II of the crack growth. The interlayer separation is an equivalent to a macro-crack in metals. In the present work, the initiation of delaminations is investigated with the use of the quadratic delamination criterion (Lagace and Brewer [6]):

$$\frac{\sigma_{13}^2}{X^2} + \frac{\sigma_{23}^2}{Y^2} + \frac{\sigma_{33}^2}{Z^2} \le 1 \tag{9}$$

It shoud be mentioned that the definition of the crack (static or fatigue damage) initiation is still a subject of controversy. However, for our purposes we use the definitions given by eqns (7) – (9).

## NUMERICAL ANALYSIS

Fatigue damage propagation will be examined in pure numerical way (FE) using NISA II package [7]. In general, the analysis is divided into the following steps:

- at the beginning of the loading process the external load is increased to the stress level equal 60% of the static ultimate strength
- at each load step the initiation of the failure modes is checked in each individual ply of the laminate
- if the matrix cracking is observed the stiffness in the element is reduced to the fibre stiffness only
- if the fibre breakage is initiated the stiffness is reduced in the element to the matrix stiffness
- if delaminations are possible the interface between two neighbourhood separated plies is replaced by the contact frictionless elements
- to determine the fatigue life $N_f$ from the assumed fatigue life correlation – one of the possible form of it is given below

The proposed analysis is conducted using nonlinear physical laws presented in the previous section. The finite element analysis provides stress and strain distributions in the analysed composite structures under the prescribed boundary and loading conditions. Then, with the aid of the failure criteria (of course, assuming in advance that they are correct – it may be sometimes questionable) it is possible to determine the slope of each parts of the curve plotted in Fig.2 as well as the points of their joints. In the nemerical analysis, the slope of the curves is evaluated for ten cycles of loading.

The above numerical procedure should be supplemented by the appropriate relations dealing with the fatigue life correlations since the final fatigue life $N_f$ is still uknown. In the present approach this correlation is assumed to be in the form of the stress – life (S-N) in the following form:

$$\sigma_a = \sigma_f \left(N_f\right)^b \tag{10}$$

where $\sigma_a$ is a stress amplitude, $\sigma_f$ is a fatigue strength coefficient and b is a fatigue strength exponent. The uknown quantities are material properties and are determined through the experimental tests.

## EXPERIMENTAL RESULTS

### Experimental Procedure

The material used in this study was continuous glass fibre/ epoxy resin. Two different lay-ups were chosen in manufacturing the specimens, i.e. 0 and 45 deg. The geometries of specimens were different: for static tests "straight-side" and "straight-side" (127x25x 2 mm) with a hole having a diameter equal to 4mm and located at the centre for fatigue tests. The specimens were end-tabbed for testing, bonding aluminium tabs of 40 mm in length and 2 mm in thickness with an epoxy adhesive. The static and fatigue tests were performed using Instron 8511 machine. The static tests were conducted at a constant cross-head rate of 1.0 mm/min, whereas the fatigue tests under load control at a frequency of 10 Hz with a stress ratio of 0.12. The tests were continued until the specimen failed in order to establish S-N curves and in order to measure the residual-tensile stiffness and strength as well as to investigate damage mechanisms.

### Static and Fatigue Properties

From the stress-strain response, the static strength and stiffness of laminates with analysed lay-ups have been determined. For both fibre directions the stress-strain curves demonstrate a nonlinear behaviour that can be modelled with a good accuracy by the relation (1). For static and fatigue tests the results of the experimental analysis have been summarized in Table 1.

| Fibre orientations | $0^0$ | $45^0$ |
|---|---|---|
| Static tests<br><br>Young's modulus in GPa | 13.14 | 9.62 |
| Fatigue tests<br>Log ($N_f$) | 4.156 | 4.564 |

Table 1: Results of the experimental tests (the average values).

Plots drawn in Figure 3 exhibit the stiffness degradation during the fatigue analysis for two fibre directions analysed herein. As it may be seen the plots do not have the intermediate state corresponding to the fatigue delaminations of structures – compare with Fig.2. In addition, the final part of the line presenting the fibre breakage (Fig.2) is in our case negligibly small. For specimens oriented at 0 deg the stiffness degradation is much higher than for those oriented at 45 deg. Therefore, it may be concludeded that under both static and fatigue loading conditions the damage is mainly associated with the appearance of the transverse matrix micro-cracks and fibre debonding and at the end with the final, sudden fibre breakages observed at the hole (the stress concentration). Such a behaviour of structures is identical for both fibre orientations analysed herein, although for specimens having fibres oriented at 45 deg the transverse matrix cracking is a dominant mode of failure.

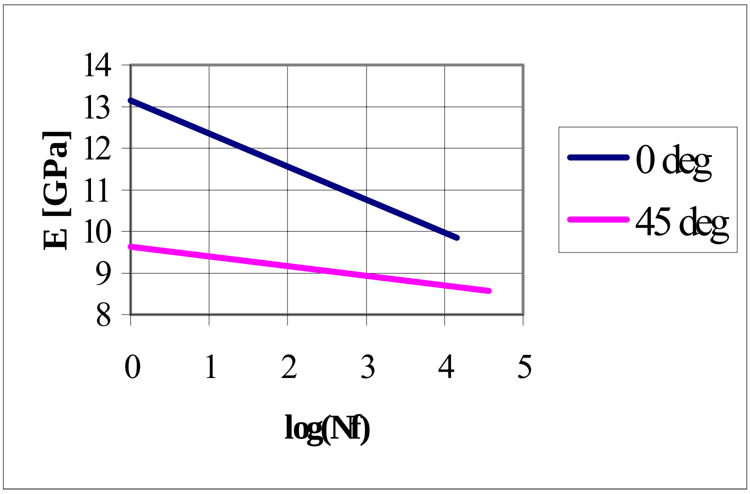


Fig.3: The stiffness degradation vs number of cycles

## NUMERICAL VERIFICATION

In order to examine the capability and the correctness of the proposed method of numerical modelling we conduct the numerical analysis for the problem of the plate having a hole that have been verified experimentally. The aim of the study is to determine the total fatigue life and the form of final fatigue damage.

The numerical analysis of plate deformations have been performed with the use of 2-D shell triangular elements (NKTP 32) having 6 nodes (NORDR = 10). Since the deformations of the structure are large, so that in the numerical analysis it is necessary to take into account geometrical nonlinearity option. A full Newton-Raphson procedure was used to solve the nonlinear equations. The results of computations are plotted in Fig.4 using the convention of the logarithmic contours. The final catastrophic fatigue failure mode have been associated with a fibre breakage in the perpendicular direction to the longer axis of the plate. The crack have appeared at the notch root and then developed to the edge of the plate. Such a situation is just plotted in Fig.4. It should be mentioned that the final fatigue life evaluated numericallay is in a very good agreement with the experimental results (see Table 1) as well as the form of the fatigue damage.

## CONCLUSIONS

The present work establishes a new modelling approach to simulate the behaviour of composite laminated structures subjected to general fatigue loading conditions. Using the idea of the traditional progressive damage model which is capable of simulating of static behaviour of composite laminates, the new fatigue model is developed. The fundamental advantages of the model consists of the following components: (1) the application of nonlinear physical laws for each individual plies in the laminate, different for loading and unloading processes and the esimations of the fatigue damage progress by the analysis of one or few cycles of loading. In this way it is possible to determine the state of damage at any load level and number of cycles. Based on the above assumptions, a computer code NISA II have been adopted to simulate the fatigue failure behaviour of structures. The capability and the correctnes of the model have been verified by the comparison with the experimental results.
We intend to extend the experimental model to include and examine also the delamination failure mode in the laminate having arbitrarily oriented individual plies. With the regard to the numerical modelling we want to analyse the residual strength.

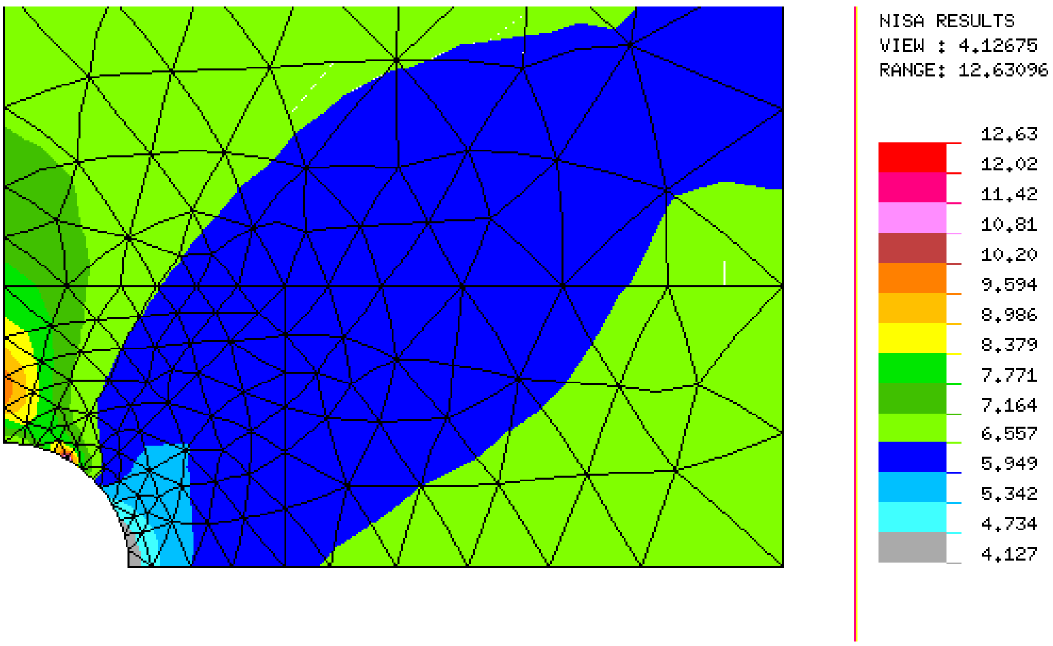


Fig.4: Logarithmic life contours of circular notch member subjected to tension – fibres oriented along the tension direction (a cut off fragment of the structure).

## ACKNOWLEDGEMENTS

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