ULTIMATE SHEAR OF BEAMS STRENGTHENED WITH CFRP SHEETS

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SUMMARY: The role of externally applied CFRP sheets in the evaluation of the shear strength of a reinforced concrete element is analyzed. With regard to this an analytical model is developed. It represents an extension of an existing model for the limit analysis of thin-webbed reinforced concrete beams falling in shear. A numerical analysis is performed to investigate on the influence of the mechanical and the geometrical parameters of the composite sheets on the ultimate shear capacity of the RC beams.

KEYWORDS: shear, carbon fiber, ultimate strength, strut-and-tie.

INTRODUCTION

Carbon fiber reinforced plastic (CFRP) sheets are being widely used in the recent years for the flexural and the shear strengthening of reinforced or prestressed concrete structural elements such as columns, beams or slabs. The principal advantages of using CFRP sheets are their high corrosion and chemical resistance, light weight and high tensile strength. However the reason which makes this solution more convenient and competitive with respect to the traditional techniques, such as strengthening with steel plates or replacement of structural elements, is the possibility of realizing the CFRP solution with very low construction time and high simplicity of installation.

The CFRP sheet is bonded to the concrete surface by an epoxy adhesive. The flexural strengthening involves the tensile surface of the elements, which are generally placed at the bottom of beams or slabs, while the shear strengthening involves both the bottom and the lateral surface of the beam web. In the case of columns the CFRP sheets are usually wrapped around the entire surface of the elements in order to provide a better confinement.

An extensive literature is available with regards to the modeling of the flexural behavior of reinforced concrete beams strengthened by CFRP sheets; on the contrary the shear behavior still needs a better understanding especially for what concerns the prediction of the CFRP sheets contribution to the shear strength of RC beams.

There are some contributions which analyze the ultimate shear behavior of beams without any transverse reinforcement and strengthened by CFRP sheets. Among these, the work [1] discusses the analogies, before after cracking [2], between the behavior of ordinary concrete beams reinforced with stirrups, and the response of beams reinforced only with CFRP sheets.

The study shows that the shear cracking region is not affected by the application of the composite sheet and the composite strain level increases significantly after a load value close to the ultimate strength of the unwrapped beams. Two other interesting results are worthy to be observed: a) during all the tests the fabric does not debond prior to failure, b) the beams crush because of the concrete failure before the fabric reaches their tensile capacity. The test results lead to define a method able to estimate the contribution of the fabric to the shear strength taking into account the fibers inclination to the beam axis.
The strengthening of concrete beams reinforced in shear is investigated experimentally in [3]. Eight beams are tested to evaluate the increment of shear strength provided by unidirectional side strips placed either perpendicularly or diagonally to the beam axis. The influence of the fibers inclination on crack propagation, shear strength and stiffness is analyzed.
The object of the present paper is to propose an analytical model which provides the ultimate shear strength of RC beams with transverse reinforcement and strengthened by CFRP sheets. The effect of the application of CFRP sheets on the shear response of RC beams is analyzed with a special concern to the variability of the principal geometrical and material parameters, such as the fibers orientation and the composite mechanical ratio. An effective modeling of the contribution of the composite sheet should account for the above factors as well as consider the possible interactions with the resisting mechanisms of the ordinary RC beam such as the truss, the beam and/or the arch mechanisms.

THE ANALYTICAL MODEL

The analytical model proposed herein represents an extension of the truss model with crack friction [4] presented in [5-7] which allows allows to study the ultimate shear behavior of reinforced and prestressed concrete I-beams. The model starts with the assumption of fixed crack pattern and crack shape through the beam shear span. In this work the case of flexure-shear cracks is considered and the crack spacing is considered the same provided by analyzing the unwrapped beam. The model assumes a crack surface loaded by a shear and a normal stress fields dependent on the crack slip and opening. The general assumptions of the model reported in [5-7] remain unchanged. Strictly speaking the shear failure of the beam is governed by yielding of the stirrups and the fabric does not reach its tensile capacity at failure. Moreover the stiffening effect of the concrete on the fiber is not considered and the top sheet anchorage is assumed to sustain the fabric load without slip.
The model is based on six equilibrium equations. The evaluation of the various contributions requires three compatibility conditions and the definition of the constitutive laws for the basic materials and the aggregate interlock. This allows to evaluate the stress level in fibers and on crack surface at failure and hence identify the collapse condition and the ultimate shear strength by checking the sheet strain admissibility.
The proposed model does not include the compression flange contribution which has to be added to obtain the ultimate shear stress of the beam.

Equilibrium equations

In Fig. 1 are shown the forces acting on a single strut due to the different shear resistance mechanisms. The distributed couple m is given by

$$m = b\, s \sin\varphi\, (\sigma_{nt} - \sigma_{nn} \cot\varphi) \qquad (1)$$

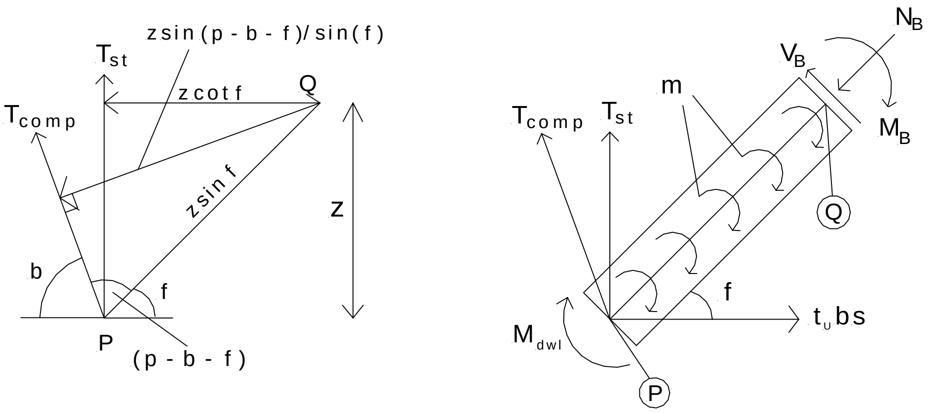

Fig. 1.: Actions on a single strut

where φ is the crack inclination to the beam axis, σ_{nt} and σ_{nn} are the shear and the normal stress fields acting on the crack surface, N_B and M_B represent the axial load and the bending moment due to the strut bending response, M_{dwl} is the couple due to the dowel action, T_{st} and T_{comp} are the stirrup and the composite action, b is the web thickness, z is the internal couple lever arm and τ_u is the nominal ultimate shear stress.
The model defines two sets of equilibrium conditions: the first referring to the equilibrium of the inclined strut, the second referring to the equilibrium of the upper and lower joints of the strut.
The following equilibrium conditions are analyzed:

- rotational equilibrium of the inclined strut with respect to the point Q;
- rotational equilibrium of the inclined strut with respect to the point P;
- longitudinal equilibrium of the inclined strut;
- equilibrium of the upper joint (Q)
- longitudinal equilibrium of the lower joint (P);
- transversal equilibrium of the lower joint (P).

The introduction of the composite sheet modifies all the equilibrium equations except the rotational equilibrium of the strut with respect to the point P and the equilibrium of the upper joint. Therefore the equilibrium equations are rewritten by taking into accounts the composite contribution.
The rotational equilibrium of the inclined strut with respect to the point Q yields:

$$\tau_u = \Delta\tau_{INT} + \Delta\tau_B + \Delta\tau_D + \Delta\tau_{st} + \Delta\tau_{comp} \tag{2}$$

where the single resistant contributions to the ultimate shear are explicitly included:

$$\Delta\tau_{INT} = \sigma_{nt} - \sigma_{nn} \cot\varphi \tag{3}$$

$$\Delta\tau_B = \frac{M_B}{b\,s\,z} \tag{4}$$

$$\Delta\tau_D = \frac{M_{dwl}}{b\,s\,z} \tag{5}$$

$$\Delta\tau_{st} = \frac{T_{st}\cot\varphi}{b\,s} = \rho_{st}\, f_{sy}\cot\varphi \tag{6}$$

$$\Delta\tau_{comp} = \frac{T_{comp}}{b\,s}\frac{\sin(\pi-\beta-\varphi)}{\sin\varphi} = \rho_{comp}\,\sigma_{comp}\frac{\sin(\pi-\beta-\varphi)}{\sin\varphi} \tag{7}$$

In Eqn. 6, ρ_{st} and f_{sy} are the geometrical stirrup ratio and the yield strength of the steel. In Eqn. 7, ρ_{comp} and σ_{comp} are the geometrical ratio and the stress in the sheet.
The horizontal equilibrium of the inclined strut provides:

$$\tau_u\, b\,s = N_B\cos\varphi - V_B\sin\varphi + \rho_{comp}\,\sigma_{comp}\, b\,s\cos\beta \tag{8}$$

Then:

$$N_B = b\,s\,(\tau_u - \frac{V_B\sin\varphi}{b\,s} - \rho_{comp}\,\sigma_{comp}\cos\beta)\frac{1}{\cos\varphi} \tag{9}$$

Let N be the axial load in the direction of the principal compression field inclined of α to the beam axis. Then the horizontal equilibrium of the joint P in Fig. 1 provides:

$$\tau_u = \frac{N}{b\,s}\cos\alpha + \rho_{comp}\,\sigma_{comp}\cos\beta \tag{10}$$

Defining $\sigma = N/(b\,s\sin\alpha)$, Eqn. 10 yields:

$$\sigma = \frac{\tau_u - \rho_{comp}\,\sigma_{comp}\cos\beta}{\sin\alpha\cos\alpha} \tag{11}$$

The transversal equilibrium of the joint P provides:

$$T_{st} = N\sin\alpha + T_{comp}\sin\beta \tag{12}$$

By using Eqn. 10, Eqn. 12 becomes:

$$\rho_{st}\,\sigma_{st} = \tau_u\tan\alpha - \rho_{comp}\,\sigma_{comp}\cos\beta\tan\alpha + \rho_{comp}\,\sigma_{comp}\sin\beta \tag{13}$$

or equivalently:

$$\tan\alpha = \frac{\rho_{st}\,\sigma_{st} - \rho_{comp}\,\sigma_{comp}\sin\beta}{\tau_u - \rho_{comp}\,\sigma_{comp}\cos\beta} \tag{14}$$

Compatibility equations

For the ordinary reinforced concrete beam two compatibility conditions are necessary: a) the overall shear-induced strain in the longitudinal direction is negligible, b) the average concrete strain in the transversal direction equals the average stirrup strain.

The application of the CFRP sheets requires to consider a further compatibility condition expressing the equality between the average composite strain in the fiber direction ε_ξ^{comp} and that of the concrete in the same direction ε_ξ^c. This condition can be expressed as follows:

$$\varepsilon_\xi^{comp} = \varepsilon_\xi^c \tag{15}$$

The average concrete strain ε_ξ^c includes a contribution, named ε_ξ^{cr}, derived from the crack pattern according to a smeared crack approach, and a contribution, named ε_ξ^d, associated to the diagonal compression field of the strut. Thus:

$$\varepsilon_\xi^c = \varepsilon_\xi^{cr} + \varepsilon_\xi^d \tag{16}$$

The expression of ε_ξ^{cr} , derived from the Fig. 2, is:

$$\varepsilon_\xi^{cr} = \frac{\delta_{tm}\cos(\pi-\beta-\varphi)+\delta_{nm}\cos(-\pi/2+\beta+\varphi)}{s\ \sin\varphi}\sin(\pi-\beta-\varphi) \tag{17}$$

where δ_{tm} and δ_{nm} are respectively the average crack slip and opening.

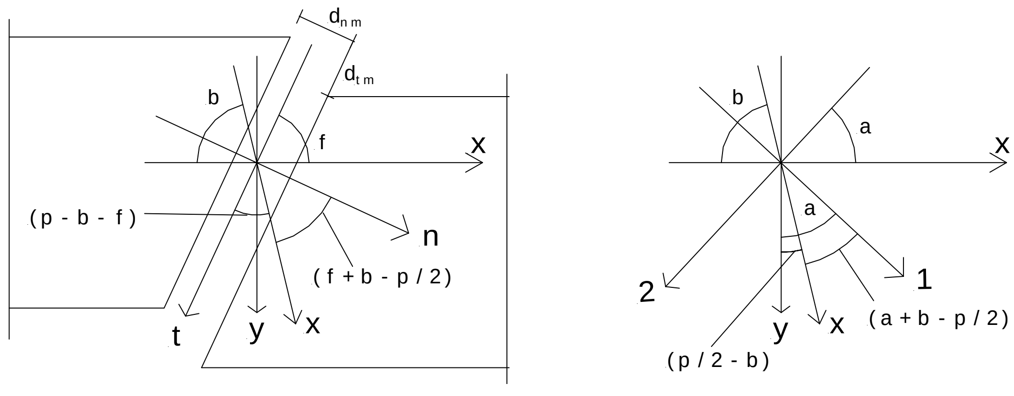

Fig. 2: Crack and strain directions

The term ε_ξ^d can be derived from the Cauchy expression:

$$\varepsilon_\xi^d = \frac{\varepsilon_1+\varepsilon_2}{2} - \frac{(\varepsilon_2-\varepsilon_1)\cos 2(-\pi/2+\beta+\alpha)}{2} \tag{18}$$

where ε_1 and ε_2 are the principal strains related to the diagonal compression field related by the expression:

$$\varepsilon_1 = -\nu\varepsilon_2 \tag{19}$$

Eqn. 19 becomes:

$$\varepsilon_\xi^d = \frac{-|\varepsilon_2|}{2}\left[(1-\nu)-(1+\nu)\cos 2(-\pi/2+\beta+\alpha)\right] \tag{20}$$

The expression (17) and (20) lead to the following expression for the average composite strain:

$$\varepsilon_{\xi}^{comp} = \frac{\delta_{tm}\cos(\pi-\beta-\varphi)+\delta_{nm}\cos(-\pi/2+\beta+\varphi)}{s\ \sin\varphi}\sin(\pi-\beta-\varphi) + \frac{-|\varepsilon_2|}{2}\left[(1-\nu)-(1+\nu)\cos 2(-\pi/2+\beta+\alpha)\right] \tag{21}$$

Constitutive laws

The constitutive laws for the aggregate interlock, the solid concrete between the diagonal cracks, the stirrup-to-concrete bond slip and the dowel action are those assumed for the ordinary reinforced concrete beam proposed by [5-7]. The introduction of the CFRP sheets introduces the further unknown σ_{comp} related to ε_{ξ}^{comp} by the CFRP sheet linear elastic constitutive law. Since in this work the tension-stiffening effect due to composite-to-concrete bond is neglected, the local value of the strain in the sheet is equal to its mean value anywhere.

NUMERICAL RESULTS

The proposed model is applied to analyze the ultimate shear response of a beam reinforced by CFRP sheets. Within this scope the parameters investigated are the fiber orientation β and the composite mechanical ratio ω_{comp} given by:

$$\omega_{comp} = \rho_{comp}\frac{E_{comp}}{E_c} \tag{22}$$

The numerical investigation concerns a reinforced concrete beam with a 300x700 mm^2 rectangular cross section with ρ_{st}=0.00335 and a stirrup diameter equal to 8 mm. The concrete strength is 30 MPa with a maximum aggregate size equal to 20 mm. The steel yield strength is equal to 440 MPa with an elastic modulus of 210000 MPa. The total area of the longitudinal reinforcement is 3118 mm^2. The angle of the inclined concrete strut is assumed to be 45° to the beam axis. The composite ultimate strain is equal to 1.1%.

The equilibrium, compatibility and constitutive conditions provide a system of non-linear equations which is solved iteratively by means of a Fortran computer program.

In Fig. 3 is reported the variation of the ultimate shear stress τ_u with the composite mechanical ratio ω_{comp}. In the figure the ultimate shear stress τ_u is non-dimensionalised with respect to the ultimate shear stress τ_{uo} of the unwrapped reinforced concrete beam. The analyses are performed for different values of the fiber orientation β; particularly in the figure are reported the results related to $\beta = 45°,\ 50°, 60°, 70°, 80°, 90°$.

The figure shows that, for each value of the fiber orientation β, the non-dimensional ultimate shear stress τ_u/τ_{uo} increases by increasing the composite mechanical ratio ω_{comp}, thus evidencing the improvement of the ultimate shear capacity of a reinforced concrete beam provided by the introduction of CFRP sheets.

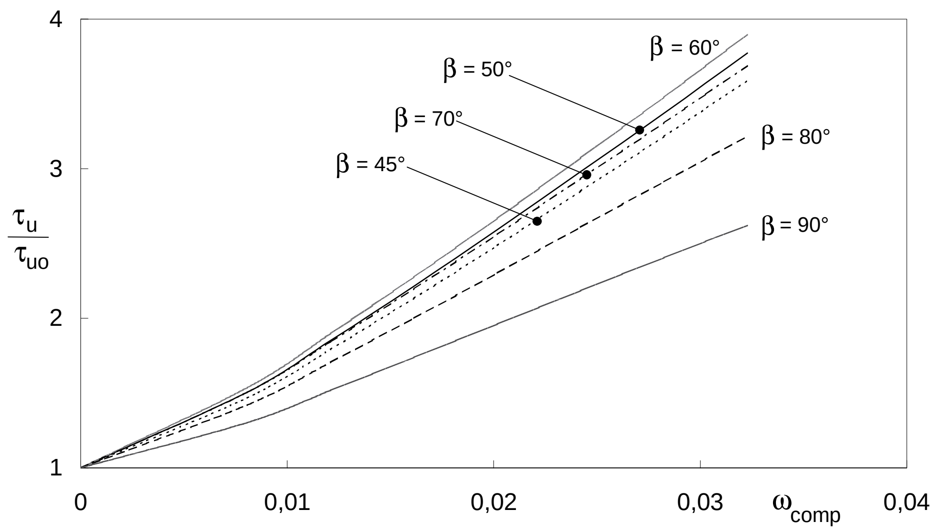

Fig. 3: Influence of composite mechanical ratio on shear strength

In Fig. 4 is reported the variation of the composite contribution $\Delta\tau_{comp}$ to the ultimate shear stress with the composite mechanical ratio ω_{comp} for $\beta = 45°$, 50°, 60°, 70°, 80°, 90°

In the figure the composite contribution $\Delta\,\tau_{comp}$ is non-dimensionalised with respect to the ultimate shear stress τ_u.

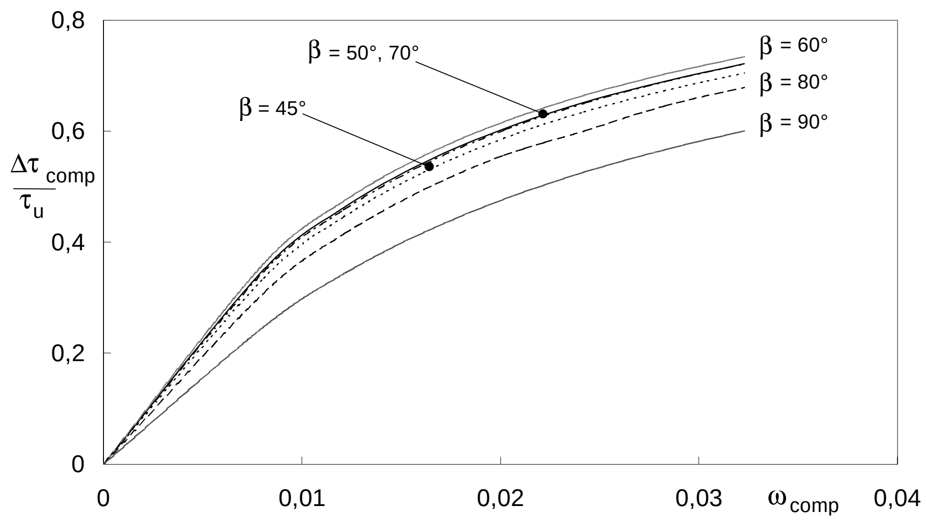

Fig. 4: Influence of composite mechanical ratio on composite contribution

The Fig. 4 shows that, for each value of the fiber orientation β, the non-dimensional composite contribution $\Delta\tau_{comp}/\tau_{uo}$ increases by increasing the composite mechanical ratio ω_{comp}. It is worthy to observe that the curves are characterized by a decreasing gradient. This means that for high values of the composite mechanical ratio ω_{comp}, the increasing of the composite contribution percentage is less than that related to low values of ω_{comp}.

Another interesting result which can be derived from the above figures is that the efficacy of the composite reinforcement is strongly affected by the fiber orientation. The values of β ranging around 60° represent the optimal orientation which definitively maximizes the ultimate shear capacity of the beam. This result can be explained by the fact that the composite performance is optimal when the fibers are oriented normal to the direction of the principal compressive stress of the concrete strut. In the case of the reinforced concrete beam herein analyzed the direction of the principal compressive stress is practically unaffected by the fiber inclination and is slightly dependent on the composite mechanical ratio. It varies from 30° to 38° in the ω_{comp} range 0.008÷0.032. This means that the direction normal to the direction of the principal compressive stress varies from 52° to 60° which represent the values of the fiber inclination maximizing the ultimate shear capacity of the beam and the composite contribution. The above result is also shown in the Figg. 5 and 6.

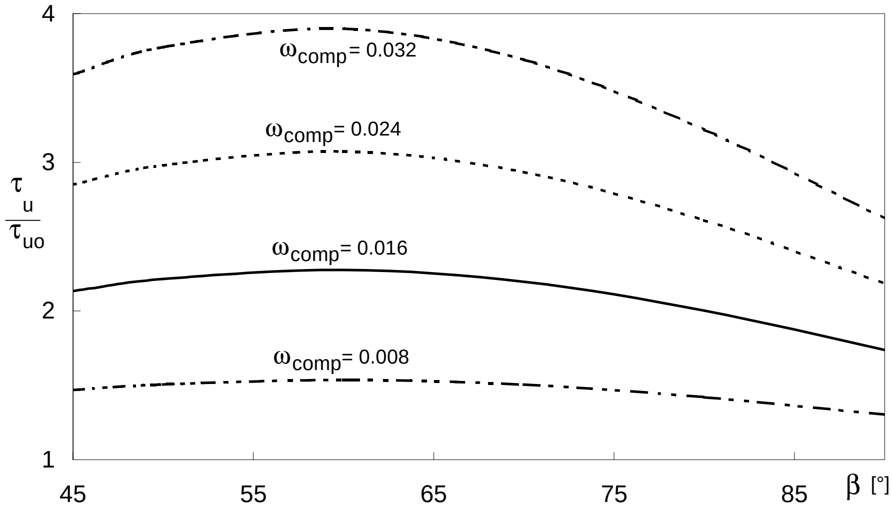

Fig. 5: Influence of fiber orientation on shear strength

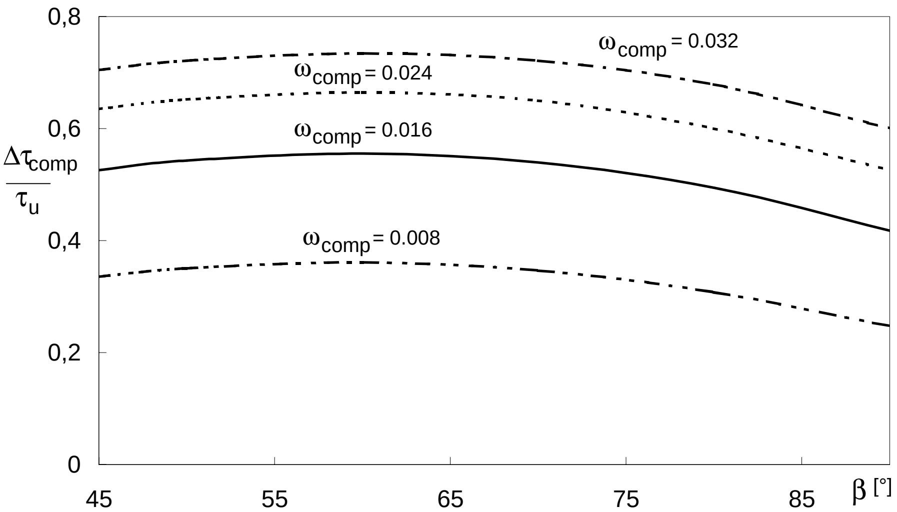

Fig. 6: Influence of fiber orientation on composite contribution

In Fig. 5 is reported the variation of the ultimate shear stress τ_u with the fibre orientation β. In the figure the ultimate shear stress τ_u is non-dimensionalised with respect to the ultimate shear stress of the ordinary reinforced concrete beam τ_{uo}. The analyses are performed for different values of the composite mechanical ratio ω_{comp}; particularly in the figure are reported the results related to ω_{comp}=0.008, 0.016, 0.024, 0.032.

In Fig. 6 is reported the variation of the composite contribution $\Delta\tau_{comp}$ to the ultimate shear stress with the fiber orientation β. In the figure the composite contribution $\Delta\,\tau_{comp}$ is non-dimensionalised with respect to the ultimate shear stress τ_u. The analyses are performed for ω_{comp}=0.008, 0.016, 0.024, 0.032.

Figg. 5 and 6 show that, for each value of the composite mechanical ratio, the ultimate shear capacity and the composite contribution percentage are maximum for values of β ranging around 60° while they assume smaller values for different values of β.

CONCLUSIONS

In this paper is proposed an analytical model to evaluate the shear strength of RC beams strengthened by CFRP sheets. The proposal represents an extension of an existing truss model for the limit analysis of thin-webbed ordinary reinforced concrete beam which account for the individual contributions to the beam shear capacity.

The proposed model includes the CFRP shear resisting contribution and accounts for the mechanical and geometrical parameters characterizing the composite sheets such as the fiber orientation and the composite mechanical ratio. The influence of these factors is analyzed by means of a numerical analysis which allows to define the design parameters which optimize the performance of the reinforced beam in terms of the ultimate shear capacity. The main result is that the composite arrangement is optimal in terms of the ultimate shear capacity of RC beams when the fibers are oriented normal to the direction of the principal compressive stress of the concrete.

REFERENCES

1. Chajes, M. J., Januszka T. F., Mertz D. R., Thomson Jr., T. A. and Finch, Jr., W.W. (1995). “Shear strengthening of reinforced concrete beams using externally applied composite fabrics”, *ACI Struct. J.*, 92(3), 295-303.

2. Park, R., and Paulay, T. (1975). *Reinforced concrete structures*; Wiley, New York.

3. Chaallal, O., Nollet, M.-J., and Perraton, D. (1998). “Shear strengthening of RC beams by externally bonded side CFRP strips”, *Journal of Composites for Constructions*, ASCE, 2(2), 111-113.

4. ASCE-ACI Committee 445. (1998). “Recent approaches to shear design of structural concrete”, *J. Struct. Engrg., ASCE*, 124(12), 1375-1417.

5. di Prisco, M., and Gambarova, P. G. (1996). “Comprehensive Model for Study of Shear in Thin-Webbed RC and PC Beams”, *J. Struct. Engrg.*, ASCE, 121(12), 1822-1831.

6. Dei Poli , S., di Prisco, M., and Gambarova, P. G. (1993). “Cover and stirrup effects on the shear response of dowel bar embedded in concrete”. *ACI Struct. J.*, 90(4), 441-450.

7. Dei Poli, S., Gambarova, P.G., and Karakov, C. (1987). “Aggregate interlock role in RC thin-webbed beams in shear”; *J. Struct. Engrg.*, ASCE, 113(1), 1-19.

8. Norris, T., Saadatmanesh, H., and Eshani, M. R. (1997). “Shear and flexural strengthening of R/C beams with carbon fiber sheets”. *J. Struct. Engrg.*, ASCE, 123(7), 903-911.

9. Hutchinson, R., Donald, D., Abdelrahman, A., and Kizkalla, S. (1998). “Shear strengthening of prestressed concrete bridge girders using bonded CFRP sheets”; *ECCM-8*, Vol. 2, pp. 43-50.

10. Ehsani, M.R. and Saadatmanesh, H. (1990). “Fiber composite plates for strengthening bridges beams”; Composite Structures , Elservier Science Publishers Ltd, England, Vol. 15, pp. 343-355.

11. Maruyama, K., and E Zhao, W. (1996). “Size effect in shear behavior of FRP reinforced concrete beams”; *Advanced Composite Materials in Bridges and Structures*, M.M. El-Badry, Editore, Canadian Soc. for Civ. Engrg., Montreal, Quebec.

12. Uomoto, T., and Ohga, H.0. (1996). “Performance of fiber reinforced plastics for concrete reinforcement”; *Advanced Composite Materials in Bridges and Structures*, M.M. El-Badry Editor, Canadian Soc. for Civ. Engrg., Montreal, Quebec.

13. Meier, U., Deuring, M., Meier, H., and Schwegler, G. (1993). “FRP Bonded sheets”; Fiber-Reinforced-plastics Reinforcement for Concrete Structures: Properties and Application, Elservier Science Publishers Ltd, England.

14. Buyukozturk, O., and Hearing, B. (1998). “Failure behavior of prestressed concrete beams retrofitted with FRP”, *Journal of Composites for Constructions*, 2(3), 138-143.