MONTE-CARLO APPROACH TO THE CREEP-RUPTURE LIFETIME DISTRIBUTION OF FIBER-REINFORCED METAL MATRIX COMPOSITES

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SUMMARY: A Monte-Carlo simulation technique is developed to predict the creep-rupture lifetime distribution of fiber-reinforced metal matrix composites. The simulation is based on an elasto-plastic creep finite element analysis in which the fiber and matrix are expressed by an axial element and plane stress element, respectively. The axial creep-rupture process simulation of a boron-aluminum composite is carried out under several applied stresses, by taking the $r_{\text{min}}$ method into the simulation procedure. The results show that the creep-rupture process of the composite is concerned with change in fiber stress brought from the shear stress relaxation of the matrix as well as time-dependent fiber breakages. Both the creep-rupture lifetime and reliability of the composite are improved as compared to those of the weakest fiber. It is expected that, as the number of fibers in the composite increases, the reliability in lifetime is further improved.

KEYWORDS: Fiber-Reinforced Composites, Creep-Ruupture, Stress Relaxation, Lifetime, Reliability, Monte-Carlo Simulation, Finite Element Method, Weibull Distribution

INTRODUCTION

It is well-known that a fracture mechanism of fiber-reinforced composites is much different from that of engineering ceramics, governed by the weakest defect in the material. Microdamages such as fiber breakages and interfacial debonding are accumulated in the material during loading. The material fractures when the damages achieve a critical quantity. Namely this fracture mechanism is essentially a cumulative damage process, and often discussed based on stochastic point of view. A representative stochastic model for the composite strength analysis is the “chain-of-bundles model” proposed by Rosen [1]. In a fiber-reinforced composite, an axial stress acting on the fiber completely recovers in regions apart from the broken point of the fiber due to matrix shear. In other words, the disadvantage brought from such a breakage is limited only within the unrecovered stress region, called the “ineffective length” [1, 2]. If the composite structure is considered to be partially a bundle with a constant ineffective length, the whole composite could be modeled as a chain connecting bundles to each other. The composite strength analysis can then be performed by applying the
Weibull distribution to the fibers with a constant ineffective length. The analyzed composite strength is improved in scatter, as compared to the single fiber strength, with an increase in the number of fibers [3].

There is another method for predicting the relation between a composite strength and a single fiber strength. That is a Monte-Carlo simulation technique, in which the composite strength can be simulated by assigning Weibull random numbers to fiber element strengths [4]. Most of the simulation techniques are based on a mechanical model, called the “shear-lag” model [5], which consists of the tension-carrying fibers connected by the purely shear-carrying matrix. This model can easily be treated in a point that force equilibrium is one-dimensional, but it is difficult to apply it to the composite with a low volume fraction of fiber. Also it is still unknown if this model is available for the simulation technique in which the matrix indicates a creep behavior.

Fiber-reinforced metal matrix composites such as boron/ aluminum and silicon-carbide/ titanium composites are expected to be applied for high-temperature structural materials, because of their remarkable strength, rigidity and thermal properties. However, there are few reports in which the lifetime properties of the metal matrix composites are treated from the viewpoint of materials reliability engineering as mentioned above. Thus, this study develops the Monte-Carlo simulation technique for predicting the creep-rupture lifetime of fiber-reinforced metal matrix composites based on the finite element method, in which tension-carrying line elements representing the fibers and plane stress elements representing the matrix are used as constituents. The result shows that the lifetime distribution of the composite is effectively improved as compared to that of the fibers own.

ANALYSIS

Finite Element Model

A composite simulated in this study is a boron fiber reinforced aluminum matrix composite. Figure 1 shows the finite-element model of the composite. The model consists of a 2-node line element and a 4-node isoparametric element under plane stress, which represent a fiber element and a matrix element, respectively. These two elements are bonded by placing the fiber element at the two nodes along y-axis of the matrix element. Since a creep strain occurring in the fibers is negligibly small, the fiber element is assumed to behave linear-elastically until achieving a critical stress or time (i.e. a strength or lifetime). This element is also assumed to lose statically its whole deformation resistance immediately after achieving the strength or lifetime. The matrix element is assumed to be an elasto-plastic creep body without fracture, such that its constitutive equations are described by the $n$-power hardening rule during loading process and the Norton-Bailey’s law during creep deformation. A double-node system that can simulate an interfacial debonding phenomenon between the fiber and matrix [6] was not
introduced, as the interface of this composite is fabricated to be relatively strong. Thermal stresses occurring in the composite are neglected, because it is proved that they are too small to affect the strength properties [7]. The simulated composite includes ten fibers, each of which is positioned with 0.1 mm fiber spacing and axially divided into 20 elements. The width of the matrix element is adjusted to possess 15.0% fiber volume fraction. Thus, the present mesh is composed of 200 fiber elements, 180 matrix elements and 210 nodes.

**Fiber Lifetime Distribution**

Ceramic fibers such as boron, carbon and silicon-carbide have a scatter in strength which is often evaluated by the Weibull distribution [8]. Ceramic fibers are also known to be broken time-dependently with a scatter in rupture time [9]. It is assumed that, therefore, each fiber element in the present simulation obeys two kinds of independent random variables. The tensile strength, $\sigma$, one of the variables, obeys the following 2-parameter Weibull distribution

$$F(\sigma) = 1 - \exp \left\{ - \frac{L}{L_0} \left( \frac{\sigma}{\sigma_0} \right)^{m_f} \right\}$$  \hspace{1cm} (1)

where, $m_f$ and $\sigma_0$ are the Weibull shape parameter and scale parameter, respectively. $L_0$ is a standard gauge length, which was used in estimating the Weibull parameters $m_f$ and $\sigma_0$. $L$ is an extrapolated gauge length, and in this study corresponds to a fiber element length $\Delta x$. The Weibull random numbers assigned to each fiber element can be obtained by substituting uniform random numbers in the range of 0 to 1 into the inverse function of eq.(1).

On the other hand, the fiber lifetime, $t_B$, another random variable, was generated by the time-dependent Weibull distribution [9] as follows;

$$F(\Omega(t_B)) = 1 - \exp \left\{ - \alpha_0 \int_V \Omega(t_B)^\beta dV \right\}$$  \hspace{1cm} (2)

where, $\alpha_0$ and $\beta$ are the Weibull shape parameter and scale one for this distribution, respectively. $\Omega(t)$ is a cumulative damage variable accumulated in a fiber element, and becomes a critical quantity when $t = t_B$. $\phi\{\}$ is the damage rate given by the following power law breakdown rule;

$$\phi\{\sigma(t)\} = \gamma \sigma(t)^\rho$$  \hspace{1cm} (3)

where $\gamma$ and $\rho$ are positive constants. $\sigma(t)$ is a stress history, which changes with time in each fiber element. Substitution of eq.(3) into eq.(2) yields

$$F(\Omega(t_B)) = 1 - \exp \left\{ - \mu \frac{\Delta x}{L_0} \int_0^{\sigma(t)} \sigma(t)^\rho dt \right\}$$  \hspace{1cm} (4)

where $\mu = \alpha_0 \gamma^\rho$. In the simulation procedure, the term of eq.(4), $\int_0^{\sigma(t)} \sigma(t)^\rho dt$, is numerically integrated by each time increment. Once this term reaches the critical quantity $\Omega(t_B)$ in a fiber element, this element is broken. The critical quantities were also generated by substituting the uniform random numbers into the inverse function of eq.(4).

**Global Stiffness Equation Including the Effect of Matrix Creep**

An elasto-plastic creep analysis is applied to the matrix creep behavior. This analysis is based on the creep potential theory, in which the equivalent creep strain rate $\bar{\varepsilon}$ and equivalent
stress $\bar{\sigma}$ are related with the Mises’ criterion [10]. In the finite element analysis the creep strain increment of the matrix $\{\Delta e_m\}$ is described by the theory as follows;

$$\{\Delta e_m\} = \frac{2}{3} \frac{\bar{\sigma}}{\sigma} \{\sigma'\} \Delta t$$

(5)

where $\{\sigma'\}$ is the stress deviators. By applying the Norton-Baily’s equation to the creep strain, the equivalent creep strain rate is as follows;

$$\bar{\sigma} = \frac{3}{2} nk \bar{\sigma} \frac{1}{m} \frac{\sigma}{\varepsilon} \frac{m-1}{n}$$

(6)

where $k$, $m$ and $n$ are constants. In eq.(6) the strain hardening rule is applied. The creep strain increment of the matrix $\{\Delta e_m\}$ can be changed into the apparent nodal load increments $\{\Delta f\}$ as

$$\{\Delta f\} = \int \{B\}^T \{D\} \{\Delta e\} dV$$

(7)

where $[B]$ and $[D]$ are the displacement-strain and strain-stress matrices, respectively. Thus, the global stiffness equation of the present finite elements is given as follows;

$$\{\Delta f\} + \{\Delta f\} = \{K_m\} + \{K_f\} \{\Delta u\}$$

(8)

where $[K_f]$ and $[K_m]$ are the fiber- and matrix-elements stiffness matrices, $\{\Delta f\}$ is the nodal load increments and $\{\Delta u\}$ is the displacement increments.

$r_{\text{min}}$ Method

In a creep deformation process of fibrous composites, the fiber stresses distribute with complication around the fiber breakage points. To solve the complication, the $r_{\text{min}}$ method [11] is applied to the simulation procedure, which is usually used in analyzing a plastic deformation in metals. According to this method, the displacement increments $\{\Delta u\}$ necessary to a fiber element breakage can be calculated by searching the minimum value, $r_{\text{min}}$ of the ratios $r=\Delta \sigma/\Delta \sigma^*$ in all fiber elements. Where, $\Delta \sigma^*$ is a tentative stress increment and $\Delta \sigma$ means the stress difference between a previously calculated stress and a fiber element strength. By multiplying the $r_{\text{min}}$ by the tentative displacement increments $\{\Delta u\}$, the exact displacement increments $\{\Delta u\}$ immediately before the fiber element breakage can be calculated. The exact stress increments $\{\Delta \sigma\}$ can also be estimated by multiplying the $r_{\text{min}}$ by the $\{\Delta \sigma^*\}$. Then, the elasticity of the fiber element giving the $r_{\text{min}}$ is set to zero. After the set done, a load accumulated in the fiber element is released outside incrementally through the two nodes of this element under the fixed boundary condition by each load increment or time increment.

Simulation procedure

The present simulation procedure consists of two major parts. One is the loading process at loading, and another part is the creep process after loading. First, two Weibull random numbers, i.e. random strength and lifetime are assigned to each fiber element. Next, the loading process starts under the boundary condition of load increment applied at each fiber end along y-axis. The stress rate of this process is 19.6 MPa/sec. During the process, fiber element breakages are predicted using the $r_{\text{min}}$ method by each increment step. The breakages are decided by only the assigned element strengths, but the cumulative damage quantity, $\Omega(t)$ is also calculated for each fiber element. This process is continued until the sum of the load increments achieves a given load. However, if the composite stress is decreased below 70% of the maximum stress before achieving a given load, this composite is evaluated to be fractured.
### Constants used in fiber properties

<table>
<thead>
<tr>
<th>$E_f$</th>
<th>$\beta$</th>
<th>$\mu$</th>
<th>$\rho$</th>
<th>$m_f$</th>
<th>$\sigma_0$</th>
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</thead>
<tbody>
<tr>
<td>314.4 GPa</td>
<td>0.398</td>
<td>8.96x10^-5</td>
<td>22.7</td>
<td>10.7</td>
<td>3.19 GPa</td>
</tr>
</tbody>
</table>

### Constants used in matrix properties

<table>
<thead>
<tr>
<th>$E_m$</th>
<th>$v$</th>
<th>$A$</th>
<th>$n$</th>
<th>$m$</th>
<th>$K$</th>
<th>$n'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.6 GPa</td>
<td>0.33</td>
<td>3.85x10^-5</td>
<td>3.10</td>
<td>0.379</td>
<td>22.3 MPa</td>
<td>0.0411</td>
</tr>
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</table>

### Dimension of composite specimen

<table>
<thead>
<tr>
<th>Length</th>
<th>Thickness</th>
<th>Width</th>
<th>$\Delta x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 mm</td>
<td>0.4 mm</td>
<td>2.16 mm</td>
<td>0.15 mm</td>
</tr>
</tbody>
</table>

At the loading process. When the sum of the load increments achieves a given load, the simulation is changed into the creep process. In this process the above-mentioned creep analysis is carried out, on the condition that the fiber ends are all free boundaries. The calculated damage quantity in fiber elements decides their breakages in this process, and the $r_{\text{min}}$ method is then applied as follows: If $r_{\text{min}} > 1$, no fiber element fractures and the next calculation is carried out under a given time increment. If $r_{\text{min}} 1$, exact damage quantities as well as exact stresses are calculated for all fiber elements by multiplying the $r_{\text{min}}$ by tentative damage quantities and stresses. The $r_{\text{min}}$ is also multiplied by the given time increment. After these calculations done, a load of the broken fiber element is released incrementally through the two nodes of the element, as mentioned above. The calculation is repeated until 1000 hr or finished when the average stress of all fiber elements is lower than 90% of the maximum average stress, around which the composite strain increases suddenly.

### Material properties

The material constants used in this study are shown in Table 1. The assumed test temperature is 300°C in air, at which the Weibull parameters of strength of a boron fiber were measured [9]. The Young’s modulus of this fiber at 300°C was extrapolated from the modulus of room temperature, since being linearly decreased with decrease in strength at elevated temperatures [12]. Furthermore, tensile and creep tests of the 1050 pure aluminum were carried out at 300°C to determine the parameters used in the constitutive equations. The time increment given for the creep analysis is initially set to $10^{-8}$ hr, and increased by multiplying 1.1 by itself until 0.01 hr. However, after the load release calculation for broken fiber elements, this was again set to $10^{-8}$ hr.

### RESULTS AND DISCUSSION

#### Creep Curves of B-Al Composite

**Primary creep**

Figure 2 shows the simulated creep curves of the B-Al composite at the applied stresses of 343.0, 367.5 and 392.0 MPa. In the every figures the specimens exhibit a primary creep as seen in a real composite [13]. Since in axially loaded unidirectional composites the matrix deformation is restrained by the rigidity of the fibers, the normal stress of the matrix relaxes along the fiber axis. The primary creep therefore appears within the time when the fibers...
Sustain the relaxed matrix stress. In a composite with high modulus fibers and relatively low modulus matrix, the primary creep deformation occurs immediately after loading [13]. The result obtained here simulates well actual deformation behavior. At the primary creep no fiber element breakage occurred, although in some specimens the breakages were observed in the loading process.

Secondary creep

In Fig. 2(b) the slope of the creep strain is almost unchanged after exhibiting the primary creep. In the figures (a) and (c), on the other hand, a momentary increase of about 0.02% strain is observed. This is due to either of the following two effects;

1. Time-dependent fiber breakages.
2. Fiber breakages by stress concentration induced by prematurely broken fibers.

If the composite consists of large number of fibers, a continuously increasing secondary creep without a momentary change would be observed. Furthermore, in case the fiber itself indicates a creep deformation, the secondary creep curve would have a larger slope.

Fracture without tertiary creep

All the composite specimens in the Fig. 2 are characterized by indicating a fracture without tertiary creep. This is considered to be caused by accumulation of fiber element breakages in the composite. The same behavior is experimentally observed in B/Al and SiC/Ti composites [5, 6]. This may be a representative creep-rupture manner in axially loaded unidirectional composites.

Change with time in fiber stress distribution

Figure 3 shows a change with time in fiber stress distribution around a fiber element breakage, which was intentionally broken immediately after reaching the applied stress of 343.0 MPa. The stress profiles in the figures are non-dimensionalized by dividing each fiber element stress by an average fiber stress and drawn on the matrix elements for the sake of convenience. The dotted lines in each figure mean fiber axes. The result shows that the fiber element breakage provides a stress concentration to unbroken fibers around itself, as shown in the fiber stress regions more than 1.10 or 1.15. These regions indicate a small extent at $t = 0$ hr, but extend gradually with time along the fiber axis. The stress region less than 0.90 of the broken fiber, defined as the ineffective length [1, 2], also increases with time along the fiber.
axis. In the figure the ineffective length at $t = 100$ hr grows three times longer than the initial length. This change is caused by the shear stress relaxation of the matrix, which is largely delayed as compared to the above-mentioned normal stress relaxation of the matrix. Such a difference in occurrence of these relaxations are also analyzed for a SiC/Ti composite by Ohno, et al. [14]. Thus, the causes of the fiber breakages shown in the secondary creep are due to change in fiber stress brought from the shear stress relaxation of the matrix as well as time-dependent fiber breakages.

**Creep-Rupture Lifetime Distribution**

Figure 4 shows the Weibull plots by the mean rank method for the simulated creep-rupture lifetime of the B-Al composites at the applied stresses of 318.5, 343.0 and 367.5 MPa. In the figure, five simulation data fractured at the loading process of the highest applied stress are removed. The result shows that the lifetime data scatter in a wide range, in which the ratios of the minimum lifetime to the maximum are approximately $10^{-3}$. Such a range also appear in B-Al composites with a high volume fraction of fiber [15]. However, the scatter in lifetime is improved as compared to that of the single fiber. Table 2 shows the estimated Weibull parameters for the lifetime data. It should be noted that, although the scale parameter decreases with an increase in the applied stress, every shape parameters are higher than the given shape parameter, $\beta$ of the single fiber (See, Table 1).

The simulation data are not linear, but slope, as the time goes by. To compare the simulation data with a lifetime distribution of the weakest fiber, this distribution lines are estimated as follows; The applied stresses of 318.5, 343.0 and 367.5 MPa are changed into the initial fiber stresses by the axial rule of mixture as follows:

$$\sigma_c = \varepsilon_c E_f V_f + K \varepsilon_c^{n'} (1 - V_f)$$

where $\sigma_c$ is the applied stress, $\varepsilon_c$ is a strain of the composite which is equal to the fiber and matrix strains, $E_f$ is the elastic modulus of fiber and $V_f$ is the volume fraction of fiber. $K$ and $n'$
are constants shown in Table 1. The initial fiber stresses are found by multiplying by $E_f \varepsilon_c$ in eq.(9), which can be solved by an iteration method. The lifetime distribution of the single fiber can be obtained by substituting the initial stress into eq.(4) by each applied stress. Finally, the lifetime distribution of the weakest single fiber, $H_{MN[1]}(t)$, is obtained as:

$$H_{MN[1]}(t) = 1 - \{1 - F(t)\}^{MN(10)}$$

where $N$ is the number of fibers and $M$ is $L_c/\Delta x$ ($L_c$ is the gage length of specimen). The simulated data are all lower than the lifetime distributions of the weakest fiber especially in low probability regions, although some of these indicate almost the same values in high probability regions. This means that fiber breakage accumulation improves the reliability of the composite lifetime. As reported elsewhere [3], the tensile strength of fiber reinforced composites is much reduced in scatter, especially for the composite with a large number of fibers. It is expected that, therefore, the lifetime distribution of the composite is further improved with an increase in the number of fibers.

### CONCLUSION

A Monte-Carlo simulation technique for predicting the creep-rupture lifetime of fiber-reinforced metal matrix composites was developed to investigate how a scatter in lifetime of the fiber affects the lifetime distribution of the composite. The simulation was based on an elasto-plastic creep finite element analysis, in which the fiber and matrix are expressed by an axial element and plane stress finite element, respectively. The axial creep-rupture process of a boron-aluminum composite was carried out under several applied stresses, by taking the $r_{\text{min}}$ method into the simulation procedure. The results showed that the creep-rupture process of the composite is concerned with change in fiber stress brought from the shear stress relaxation of the matrix as well as time-dependent fiber breakages. The Weibull shape parameter estimated from the simulated lifetime data showed a larger value than the shape parameter of the fiber. Both the creep-rupture lifetime and reliability of the composite were increased, as compared to those of the weakest fiber. Thus, the lifetime distribution of the composite is improved, especially in a low probability region. It is expected that, as the number of fibers in the composite increases, the reliability of the composite lifetime is further improved.
REFERENCES