PLY WAVINESS ON IN-PLANE STIFFNESS OF COMPOSITE LAMINATES

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SUMMARY: A model was developed to determine the effect of ply waviness on the in-plane stiffness of composite laminates. On this model, plies containing waviness were mathematically described as in-phase sine-waves. Several configurations were included as well as generalization for any symmetric lay-up. The effective properties were calculated using both weighted average and springs in series approaches. A test program was conducted to validate the model for quasi-isotropic lay-ups. Data from the literature was also compared to model predictions for unidirectional and cross-ply laminates. These predictions correlated fairly well to test data. Model can be used as part of a framework to quantitatively access the effects of regions containing ply waviness on the global in-plane stiffness of structural composite parts. This information can be used to control quality or to impose operational limitations on such parts.

KEYWORDS: waviness, defect, effective properties.

INTRODUCTION

Ply waviness is a type of manufacturing defect that can occur in fiber reinforced composites. This type of anomaly is characterized by out-of-plane undulations of an otherwise straight ply or group of plies. Although more commonly found in filament wound cylindrical structures, ply waviness can also occur in flat laminates, especially thick ones, or in more complex RTM (Resin Transfer Molding) parts. In the filament winding process it appears to be a result of local buckling of underwrapped layers under the pressure exerted by the overwrapped ones [1][2]. On laminates it may be related to the built up of residual stresses during the cure process [3][4]. For the RTM case, it often occurs in parts geometrically complex and may be caused by dry-fiber positioning and/or tooling mounting [5]. Four sections of composite laminates containing different configurations of ply waviness are shown on Fig. 1.

Although the origin of ply waviness is still under investigation, sometimes it can be identified for specific cases. With this information available, changes can be introduced to prevent it to happen at its origin. However, ply waviness can not always be avoided and must be tolerated, in many applications, as part of the structure.

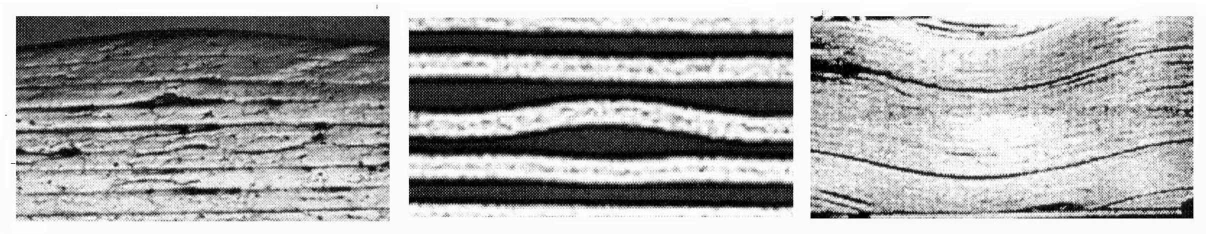

Fig. 1: Ply waviness

This problem became of particular interest for the composite industry due to its close relation to economic issues. There is a great ongoing push to drive composite materials costs down. The competition is severe and materials are being selected each time more by cost driven standards. The current qualitative approach for supporting accept / reject / repair decisions is no longer acceptable. There is a need to do this quantitatively, so that realistic and consistent assessment can be made.

Research conducted in recent years clearly showed that ply waviness reduces both the stiffness and the strength of composite structures, particularly under compressive loads. This reduction can be catastrophic depending on the severity and extension of the waviness. Thus, when designing composite structures, engineers should be aware of the implications introduced by this kind of defect.

ANALYSIS

Mathematical model

It was considered a symmetric composite laminate of thickness t composed of a sequence of unidirectional fiber reinforced plies. The material was assumed to behave linearly and elastically. The laminate was assumed to include a defined defective region containing ply waviness. This region was characterized by the type of ply waviness, its maximum amplitude and its wave-length.

A mathematical model was then developed to capture the local mechanical behavior of the region containing ply waviness. The model requires input of three major sets of data: material properties, laminate geometry and defect (ply waviness) geometry. Five engineering elastic constants are necessary to define the material of the ply, assumed to be a transversely isotropic lamina: longitudinal Young's modulus E_x, transverse Young's modulus E_y, in-plane shear modulus E_s and Poisson's ratios ν_{xy} and ν_{yz}. The laminate geometry is defined by the number of plies, ply thickness, lay-up stack sequence and number and position of the waved plies. The maximum amplitude of the waviness δ and its wave-length λ are the most important parameters to characterize the region containing the waviness. However, for convenience, these two parameters can be merged into a new one, non-dimensional. This new parameter is called here severity factor and it is defined as the ratio between the amplitude and the wave-length, as in Eqn. 1. It can also be understood as an aspect ratio of the sine-wave, or how “spiky” or “flat” it is.

$$Severity\ factor \equiv \frac{\delta}{\lambda} \quad (1)$$

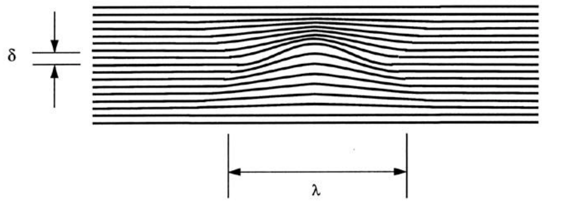

Fig. 2: Severity factor

The model is limited by the following assumptions:

- Laminated Plate Theory;
- in-plane stiffness;
- linear analysis;
- symmetric lay-ups;
- waved plies on defective region represented by in-phase sine-waves (same wave-length), as seen in Fig. 2.

Configurations

The model is capable to deal with five different configurations, divided in two major groups: uniform ply waviness and non-uniform (or graded) ply waviness. These configurations are shown in Fig. 3.

In the case of uniform ply waviness, all plies have the same amplitude and wave-length. They can be formed as a wave or they can be embedded. Although not found in manufactured composites, uniform ply waviness can be generated in controlled lab environment. However, its importance is associated to the severe penalty they impose on the stiffness defective region, since all plies are waved with the maximum (constant) amplitude.

For non-uniform or graded ply waviness only one of the plies is considered to have the largest amplitude, which can generate three different configurations: hump, indentation or embedded. The amplitude decays linearly from a maximum on the top, bottom or middle ply to zero on the outer plies. The wave-length is assumed to be constant, as mentioned in the model hypothesis.

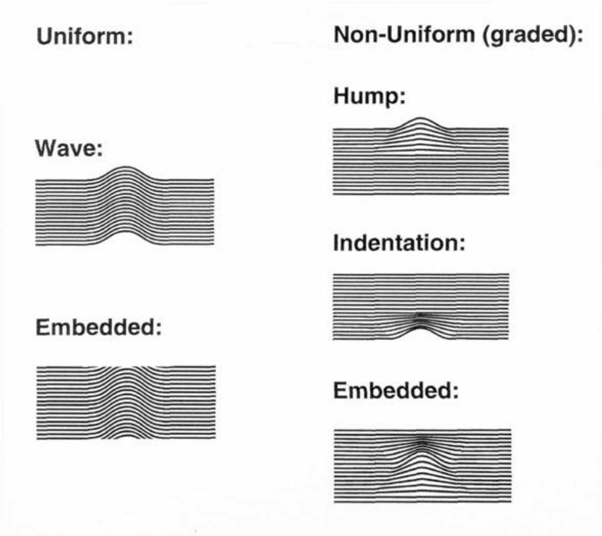

Fig. 3: Configurations considered

Coordinate systems and their transformations

In order to describe the orientation of the fibers in space, a set of three coordinate systems is necessary. Moreover, two stiffness transformations are essential to adequately describe the stiffness of an specific ply with respect to the global coordinate system.

The coordinate systems, here called On-Axis/In-Plane, Off-Axis/In-Plane and Off-Axis/Out-of-Plane, are depicted on Fig. 4.

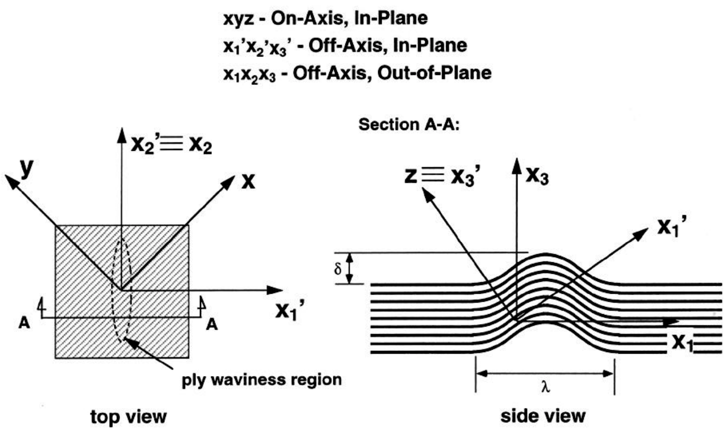

Fig. 4: Coordinate systems

The On-Axis/In-Plane *xyz* coordinate system is oriented on the plane of the ply with the *x* axis aligned with the fibers. It corresponds to the On-Axis system on the traditional Laminated Plate Theory. However, for the waved plies this system is tangent to the sine-wave described by them on the defective region.

The Off-Axis/In-Plane $x_1'x_2'x_3'$ coordinate system is used as an intermediate system between the first and second stiffness transformation. This coordinate system has the x_3' axis aligned with the *z* axis from the previous On-Axis/In-Plane coordinate system. This means that it is also tangent to the sine-wave described by the waved plies on the defective region. Also it has the x_2' axis aligned with the x_2 axis of the Off-Axis/Out-of-Plane coordinate system, further described.

The Off-Axis/Out-of-Plane $x_1x_2x_3$ coordinate system (also called global coordinate system) is used to describe the effective properties of the defective region globally. It coincides with the Off-Axis coordinate system for the non-defective region of the laminate. This system is originated by an out-of-plane stiffness transformation from the Off-Axis/In-Plane coordinate system. This is done over the common axis (x_2', x_2, respectively) in such a way that the previous coordinate system is brought back from its tangent-to-waviness position to the laminate plane.

The two coordinate systems transformations necessary to position the waved fiber on space, On-Axis/Off-Axis and In-Plane/Out-of-Plane, are shown in Fig. 5.

The first stiffness transformation is performed from the On-Axis/In-Plane coordinate system to the Off-Axis/In-Plane coordinate system. It is carried over the *z* axis on the first system which corresponds to the x_3' axis on the second one. It is equivalent to the traditional Laminated Plate Theory On-Axis to Off-Axis stiffness transformation which uses the lay-up angle, θ. For this reason is called here On-Axis/Off-Axis stiffness transformation, done on the plane of the ply, which is tangent to the sine-wave in the defective region.

The second stiffness transformation is performed from the Off-Axis/In-Plane coordinate system to the Off-Axis/Out-of-Plane coordinate system. It is carried over the x_2' axis on the first system which corresponds to the x_2 axis on the second one. It results bringing the first system from its position tangent to the sine-wave in the defective region to the plane of the laminate. For this reason it is called here In-Plane/Out-of-Plane stiffness transformation. The angle φ is constant through-the-thickness for a determined x_1 position in the uniform waviness configuration and varies from ply to ply in the non-uniform (or graded) waviness configuration.

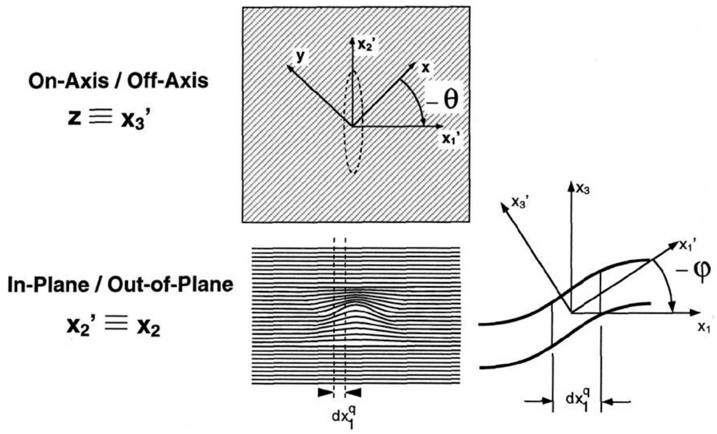

Fig. 5: Coordinate systems transformations

Local out-of-plane angle φ

In order to obtain the effective properties of the region with ply waviness, a numerical approach is used. Taking advantage of the problem symmetry, half of the wave-length is divided into a finite number *ns* of sections with constant length dx_1^q. Each one of these sections is considered to be a particular laminate with common in-plane ply orientations and different out-of-plane ply orientations as seen in Fig. 6.

The out-of-plane angle φ is calculated for each ply in each q^{th} section assuming the waviness is in the form of a sine-wave. Eqns. 2 and 3 give the mapping of the referred sine-wave on a local coordinate system $X_q^w Y_q^w$, as shown in Fig. 7. The out-of-plane angle φ is then given by Eqn. 4.

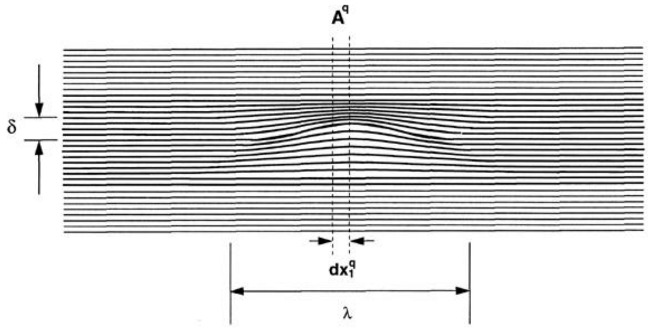

Fig. 6: Local numerical approach

$$X_q^w = \frac{\lambda q}{2ns} \tag{2}$$

$$Y_q^w = \frac{\delta}{2}\left\{1 + sin\left[\frac{\pi}{2}\left(\frac{X_q^w}{\lambda/4} - 1\right)\right]\right\} \quad q = 0,1,2,\cdots,ns \tag{3}$$

$$\varphi_q = \arctan\left(\frac{Y_q^w - Y_{q-1}^w}{X_q^w - X_{q-1}^w}\right) \tag{4}$$

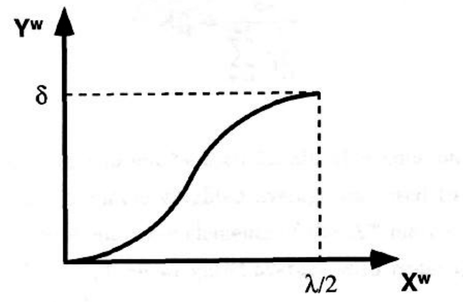

Fig. 7: Local sine-wave mapping

For each one of these segments q, an $\mathbf{A}^q$ matrix can be evaluated using the transformed $(\overline{\mathbf{Q}})_k$ matrices for each ply on the traditional Laminated Plate Theory fashion as in Eqn. 5. In this equation, h_k represents the thickness of the k^{th} ply and the integration is performed on the total number of plies, np.

$$\mathbf{A}^q = \sum_{k=1}^{np} \left(\overline{\mathbf{Q}}\right)_k \left(h_k - h_{k-1}\right) \tag{5}$$

Once these $\mathbf{A}^q$ matrices are obtained, the next step is to "smear" them to obtain the effective $\mathbf{A}^w$ matrix which represents the waved region.

Springs in series

Two different approaches are used to obtain the effective $\mathbf{A}^w$ matrix, according to its elements. First, the A_{11}^w component can be represented as a train of linear springs in series in the *11* direction. This is an intuitive approach and it is depicted on Fig. 8. On this figure, the segment laminate is transformed in a spring on the train of springs in series. The component A_{11}^w of the matrix $\mathbf{A}^w$ is then given by Eqn. 6.

$$A_{11}^w = \frac{ns}{\sum_{q=1}^{ns} \frac{1}{A_{11}^q}} \tag{6}$$

However, this reasoning can not be used for all other components of the $\mathbf{A}^w$ matrix. In those cases, a simple weighted average was used to obtain the referred effective matrix. Therefore, all other elements of the $\mathbf{A}^w$ matrix ($A_{12}{}^w$, $A_{16}{}^w$, $A_{22}{}^w$, $A_{26}{}^w$ and $A_{66}{}^w$) are obtained through an weighted average with respect to the wave-length λ, according to Eqn. 7.

$$\mathbf{A}^w = \frac{1}{\lambda}\sum_{q=1}^{ns}\mathbf{A}^q\left(X_q^w - X_{q-1}^w\right) \tag{7}$$

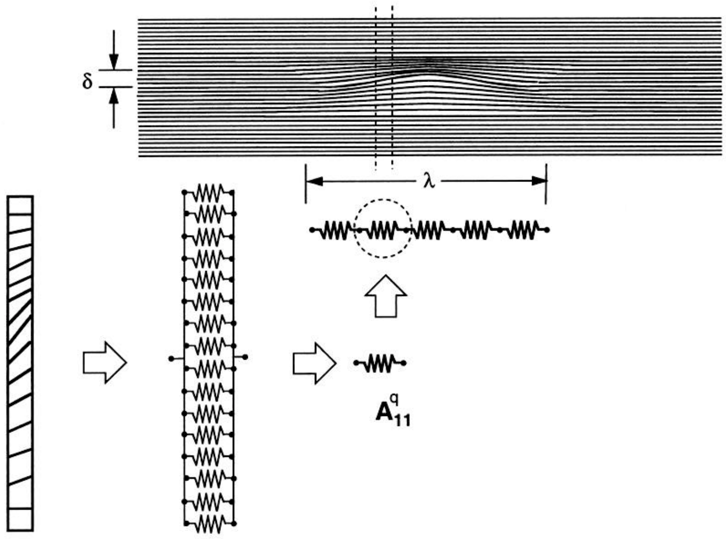

Fig. 8: Springs in series approach

RESULTS

Model predictions were compared to test results. Data available in the literature were also used in this process. Standard compression test data were obtained for quasi-isotropic lay-up in the test program performed as part of this work. Unidirectional [6][7] and cross-ply [8] compression test data from the literature were also used in the model validation. Standard *stress vs. strain* plots were used to compare test data to model predictions. Figures depict these plots where the circles represent test data, the solid line represents the model and the dashed line represents the Laminated Plate Theory prediction for a non-defective specimen.

Quasi-isotropic

Standard compression specimens in the graded embedded configuration were manufactured and tested. The material carbon/epoxy (T800/3900-2) prepreg and a quasi-isotropic lay-up $[[0/+45/-45/90]_S]_S$ were used in the specimen fabrication. Specimens were instrumented with "back-to-back" strain gages, respectively referred as "top" for the strain gage bonded to the face close to the crest of the waviness and "bottom" for the strain gage bonded to the opposite face. Waviness amplitude and wave length was measured resulting in a severity factor of 0.075 for all specimens. Fig. 9 compares test data with model prediction, for both top and bottom strain gages. It is important to point out that the top strain gage showed stiffer results than the bottom one, which is consistent with the expected asymmetric behavior.

Unidirectional

Hsiao, Daniel and Wooh [6][7] published test data for compression tests on unidirectional laminates containing uniform and graded ply waviness. They also used a carbon/epoxy system (IM6G/3501-6) in unidirectional 0^o lay-up. Two configurations were used: uniform embedded, with 150 plies (thickness of 0.75 in, 19.05 mm) and severity factor of 0.085, and graded embedded with 72 plies (thickness of 0.36 in, 9.14 mm) and severity factor of 0.040. These test results are compared to the model predictions for the uniform embedded and graded embedded ply waviness configurations on Figs. 10 and 11, respectively. The uniform embedded configuration does not present asymmetry which makes test data for the top and bottom strain gages coincide. However, for the graded embedded configuration, there is such asymmetry and the presented test data is actually an average for the top and bottom strain gages.

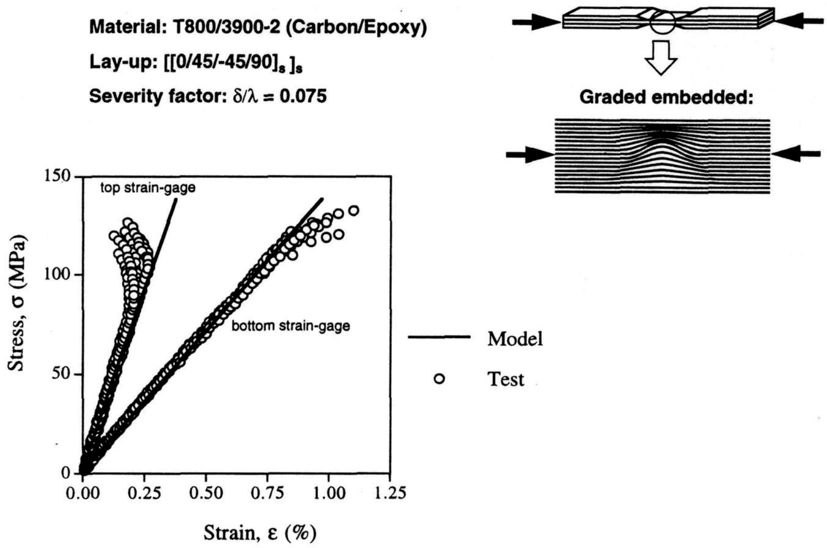

Fig. 9: Compression test data and model correlation for quasi-isotropic lay-up

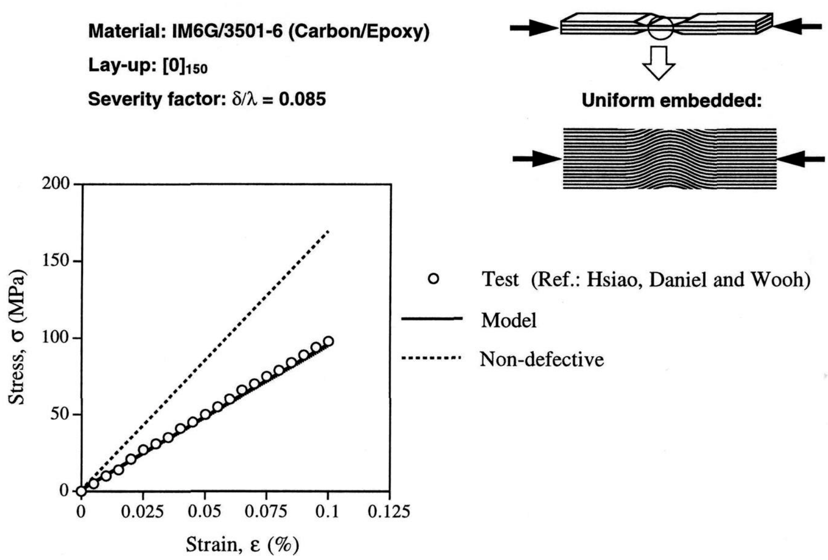

Fig. 10: Compression test data and model correlation for uniform unidirectional lay-up

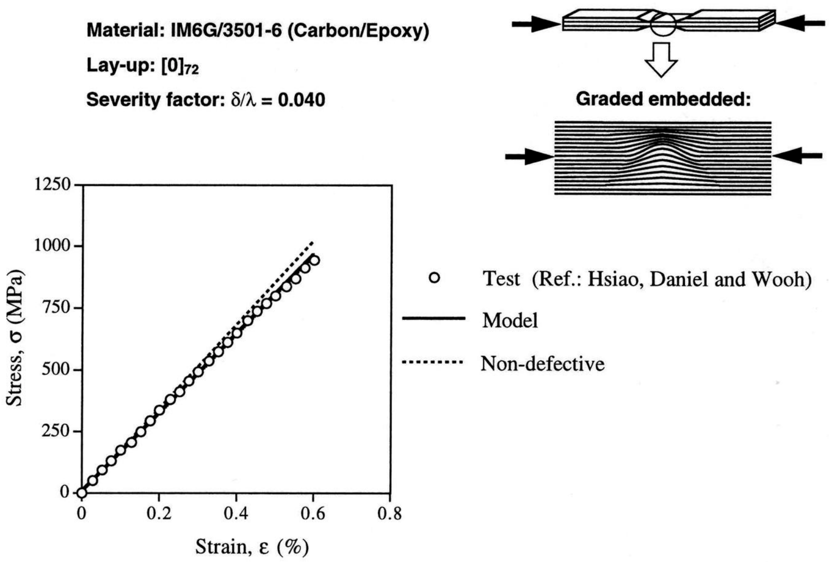

Fig. 11: Compression test data and model correlation for graded unidirectional lay-up

Cross-ply

Adams and Hyer [8] published an extensive test database on the subject of cross-ply laminates containing ply waviness. Although they did not publish *stress vs. strain* plots for such tests, Prof. Adams was kind enough to send us some of his plots. His specimens were manufactured using carbon/epoxy (T300/PR1700) with a cross-ply lay-up ($[90_2/0_2/90_2/0_2/90_2/\bar{0}_{2w}]_s$), where the bar over the last 0^o ply group on the sequence means that it is not included in the laminate lay-up symmetry. In other words, this specific ply group stands alone “sandwiched” by the remaining symmetric ply groups. Also, the subscript *w* means that this ply group contains waviness. The waviness was configured as graded embedded with a severity factor of 0.037. Test data and model predictions are plotted on Fig.12. Again, these results are an average for the top and bottom strain gages.

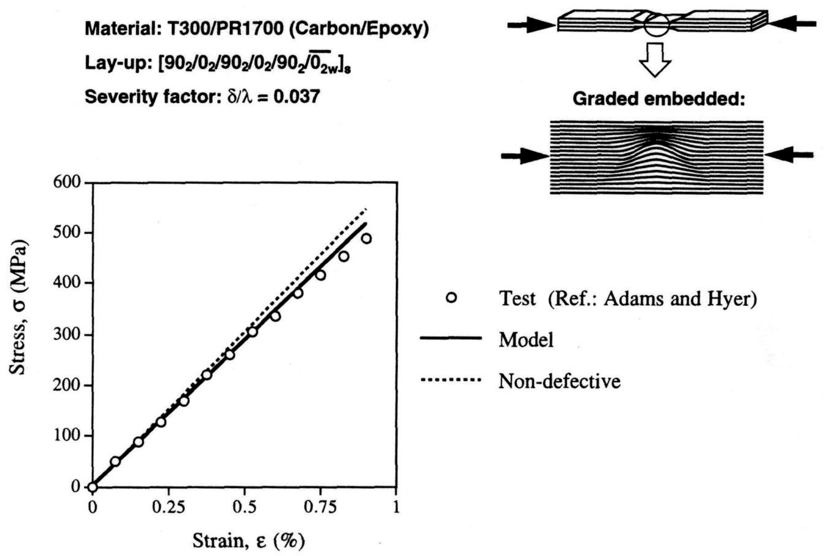

Fig. 12: Compression test data and model correlation for cross-ply lay-up

CONCLUSION

The proposed model predictions agreed fairly well to test data, for different lay-ups and materials. The model was also capable to predict the asymmetric behavior for quasi-isotropic lay-up. The uniform unidirectional lay-up was the worst case for in-plane stiffness reduction since it exhibited the greater severity factor for all plies. The model can be used as part of a framework to quantitatively access the effects of regions containing ply waviness on the global in-plane stiffness of structural composite parts. Global analysis can be performed using the effective properties evaluated on the defective region. Such information can be used to control quality or to impose operational limitations on those parts. Although in most cases ply waviness did not cause important decreases on the in-plane stiffness of the laminates, its effects on their strength can be more severe and should be further evaluated.

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