Optimization of the Performance of Gradient Composite Armor

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SUMMARY: Gradient Design Composite (GDC) was developed at Drexel University by a combination of material and geometry hybrid. Response Surface Analysis(RSA) methodology is used to optimize the specific ballistic limits of the GDC. The modified Florence's Model was used to generate virtual experimental results under a factorial design of experiment. Using this model a wide range of combinations of factors can be examined for a given objective function. The response surfaces generated in this study will provide guidance for the selection of optimum combination of design parameters for high performance composite armors.

KEYWORDS: Response Surface Analysis (RSA), Gradient Design Composite (GDC), Ballistic limit, Optimization, Composite armors

INTRODUCTION

As the velocity of impact and the nature of the impactor changes, as in the case of a very high velocity projectile (~3000ft/s), a ceramic plate is often added to the surface of the polymer composite. Lots of works have been done to understand the performance of ceramic composite armors. It is now well recognized that after impact, a conical fracture pattern develops at the impact zone, propagating towards a ceramic-backing plate interface. Also tensile fracture develops at the interface at the results of interflection of the initial compress wave. Later a conical zone of fully fragmented materials developed in front of the projectile and, as a consequence of the interaction between the projectile and the ceramic powder, projectile erosion occurs. Although as previously mentioned, the energy consumed in the fracture of the ceramic tile is very small compared with the energy of impact, the development of a zone of fractured material ahead of the penertrator seems to be of the greatest importance in defeating the projectile. Florence[1] developed a model for estimating the ballistic limit of ceramic armors, based on the assumptions that armor piercing projectiles are first shattered or blunted by the hard ceramic, and the load is then spread over large area. The composite backing deforms to absorb the remaining kinetic energy of the projectile. The ballistic limit was estimated using

$$V = \left(\frac{\varepsilon_c S}{0.91 f(a) M_p} \right)^{\frac{1}{2}} \qquad (1)$$

Where M_p is the projectile mass

a_p is projectile radius (for 7.62 NATO projectile, the radius is 3.81mm)

$a=a_p+2h_c$

h_c is the thickness of ceramic plate

h_b is the thickness of the backing composite plate

$S=UTS*h_b$

UTS is ultimate tensile strength of backing plate,

ε_c is breaking strain of backing plate

d_c is the density of ceramic

d_b is the density of backing plate

$f(a) = M_p/(M_p + (h_c d_c + h_b d_b)\, \pi a^2)\pi a$

Current small arms protective body armor systems used in the US. Army frequently consist of a ceramic face backed by fiber reinforced composite. In the interest of improving the ballistic resistance of composite armors, a Gradient Design Composite (GDC) was developed at Drexel University by a combination of material and geometry hybrid[2]. The GDC is a design concept wherein the material system consists of a harden component, an energy dissipation component and a damage containment which are organized according to a gradient of varying functions. Ceramic spheres of various sizes are embedded in a matrix of appropriate rigidity to form a face component. The backing plate is composed of fiber reinforced composite with various levels of structural integration. By proper selection of material systems and design of fiber architecture, it has been shown that a high level of damage containment can be achieved as demonstrated in the compression after impact test[3].

In past few years, a considerable research has been directed to the study of GDC [4]. It was found that modified Florence model with several simplifying assumptions provides a good prediction regarding the ballistic velocities at the experimental design stage. A novel experimental design, in which Florence model is used as a virtual experimental generator to provide the experimental data, is thus developed in this paper to exploit the GDC to its fullness and examine the performance limit of the State-of–the-Art material system.

For the GDC system, a major concern in the experimental design stage is how to achieve an optimum selection of the system parameters, such as the thickness ratio h_c/h_b (h_c is the thickness of ceramic facing plate, h_b is the thickness of composite backing plate), so that the GDC system will process the desired materials characteristics and meet given specification. As a powerful and mature design tool, the Response Surface Analysis (RSA) methodology [5] is used to optimize the specific ballistic limits of the GDC. The response surface methodology is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize the response. It is assumed that a system involves some response which depends on the input variables. The relationship between response and variables sometimes is very complicated. The success of the RSM depends on the approximation of the relationship by a low order polynomial in some of the independent variables. The polynomial coefficients are estimated by the least square procedure to minimize the root-mean-square (rms) error. In this case the response is a linear function of the coefficient, and the least square fit requires the solution of a linear system of the equations, with the process called *linear regression.* . The response surface can be reduced to canonical form, and the nature of the stationary point can be determined so that the optimal conditions can be found.

In this paper, response surface analysis methodology fits quadratic polynomial to the experimental data generated by Florence model , using least square procedure. The design optimization process then can employ the simple fitted function.

EXPERIMENTAL DESIGN

In order to optimize the ballistic properties of the ceramic armored composite, the first step is to decide the variables and response. Here the ballistic limit V and the areal density of whole composite panel are considered as responses. Experimental results show that the ballistic limit is the function of sphere size and the ratio of ceramic and composite layers. It can also be found from the Florence Model that the ballistic limit is affected by the arrangement and type of ceramic spheres, and the arrangement and type of backing composite. Simply, three independent variables, the layers of Spectra fabrics, the ratio of ceramic sphere and backing composite plate, and the density of ceramic sphere, are considered.

Table1. Properties of ceramic materials

Materials	Density (g/cm^3)	Modulus (GPa)	Hardness (Kg/mm^2)
Aluminum Oxide	3.4	227	1800
Boron Carbide	2.48	440	2790
Silicon Carbide	3.20	370	2700
Boron Carbide & Silicon Carbide	2.60	340	2750

The Second step is the design for fitting second order model. In this paper, the central composite design was selected. Box and Wilson have devised a workable alternative to the 3^k factorial system through the development of the class of the composite designs. The composite design are first order factorial designs augmented by additional points to allow estimation of the coefficients of a second order surface. The central composite designs is the 2^k factorial (the two levels of each variable coded to the usual –1, +1) augmented by the following: For $k>2$, the number of experimental points needed in central composite design are considerable less than the number required by a 3^k factorial design. The composite design can achieve a great saving in the number of the experimental runs required when the number of factors is large. According to the theory of Response Surface Methodology, for quadratic polynomial response surface with 3 independent variables, there should be 15 experiments in order to optimize the ballistic limit.

Table 2. The experimental design

Factors		Levels				
Coded variables	Actual variables	-2	-1	0	1	2
X_1	No. of layers	105	115	125	135	145
X_2	Ratio of h_c/h_b	0.4	0.5	0.6	0.7	0.8
X_3	Ceramic density	2.48	2.71	2.94	3.27	3.6

Variables: thickness of backing composite panel (h_b): from 105 layers to 145 layers
thickness ratio of ceramic plate and polymer panel (h_c/ h_b): from 0.4 to 0.8
the density of ceramic sphere (d_c) : from 2.48g/cm^3 to 3.4g/cm^3

The coded variables are:

$X_1 = (h_b-125)/10$
$X_2 = 10(h_c/ h_b - 0.6)$
$X_3 = (d_c-2.94)/0.23$

The third step is to collect the experimental data. In this paper, the Florence's Mode l was used as a *virtual experimental generator* to produce the virtual experimental results of ballistic limit. The area density A_d was calculated as the following equation:

$$A_d = d_c\times h_c + d_b\times h_b \qquad (2)$$

Where d_c , d_b are the density of ceramic sphere and backing composite panel, respectively
h_c , h_b are the thickness of these two panels, respectively.

Next, the surface response of ballistic limit and area density is calculated by using the least square procedure. Maple Software package was used to obtain the quadratic polynomial and to plot the contour of surface response.

Finally, the optimum operation conditions is determined. The optimization was performed using a generalized reduced gradient optimizer available in Microsoft EXCEL

RESULTS

The virtual experimental results of ballistic limit V_{50} generated by Florence model and area density A_d calculated by using Equation 2 are shown in Table3. Spectra 1000 laminated composite was chosen as the backing plate (density: 1000kg/m^3 , Spectra® 1000 fiber breaking strain of 3.4% and Spectra® 1000 fiber tensile strength 3.0×10^9 Pa were used in calculation).

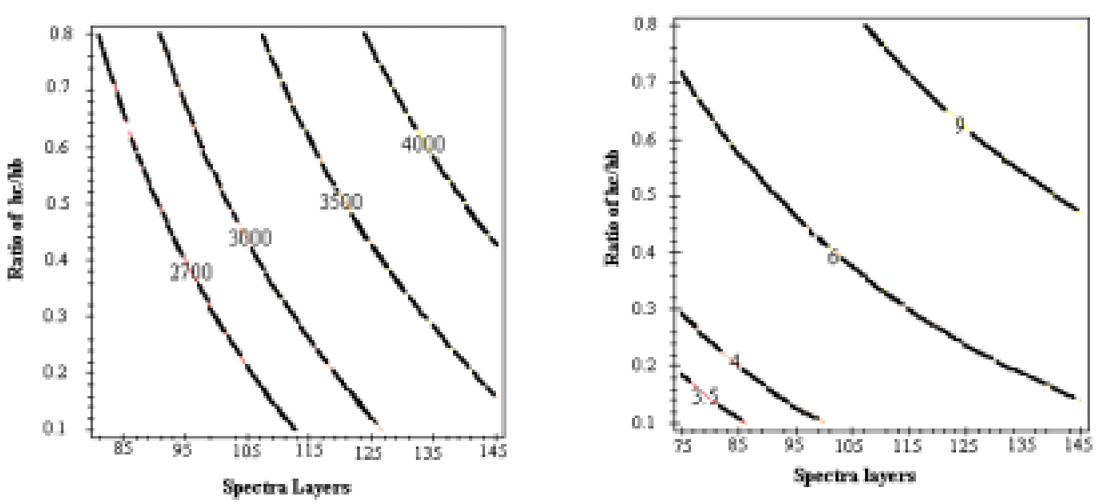
Table.3 The response of experiment design

	X1	X2	X3	V_{50} (ft/s)	Area Density (lb/ft^2)
1	-1	-1	-1	3534.3	8.070
2	-1	-1	1	4092.4	9.474
3	-1	1	-1	3852.3	9.928
4	-1	1	1	4470.3	11.655
5	1	-1	-1	3672.6	8.859
6	1	-1	1	4256.8	10.3944
7	1	1	-1	4029.3	11.03
8	1	1	1	4680.4	12.95
9	0	0	0	3734	8.649
10	2	0	0	4682.3	11.94
11	-2	0	0	3691.3	8.1055
12	0	2	0	4432.4	12.486
13	0	-2	0	3901.7	9.2678
14	0	0	2	4248.4	11.32
15	0	0	-2	4078.7	10.296

The quadratic polynomial responses surface of ballistic limit V_{50} and area density of A_d as shown below are calculated .And the contour of ballistic limit and area density were plotted by using Maple software at fixed density of ceramic sphere of $2.64/g/cm^3$

$$V_{50} = 4078.8+301.8x_1+184.9x_2+86.5x_3-2x_1^2-4.2x_2^2-9x_3^2+15.8x_1x_2+7.4x_1x_3+10.6x_2x_3$$

$$A_d = 10.2967+0.8233x_1+1.095x_2+0.5134x_3-0.0005x_1^2-0.0001x_2^2-0.0006x_3^2+0.0879x_1x_2+0.0413x_1x_3+0.0855x_2x_3$$



(a) (b)

Fig.1 The response contour of GDC
(a). The response of ballistic limit
(b). The response of area density

From the quadratic polynomial of ballistic limit, the stationary point was calculated by canonical analysis

$X_1 = 13.56$
$X_2 = 17.38$
$X_3 = 11.0$

Which are far beyond the design region (from –2, to 2). In other words, there is no stationary point in the region we are interested in.

Table 4. Optimum Design obtained using response surface

Ballistic limit (ft/s)	2582
Area Density (4lb/ft^2)*	4
Spectra layers	106
Diameter of ceramic sphere (in)	0.066
The density of ceramic (g/cm^3)	2.64

* Constraint condition

The optimization was performed using a generalized reduced gradient optimizer available in Microsoft EXCEL. The optimum design obtained, subject to the constraints of area density (Area density is less than 4 lb/ft^2) is shown in table 4

It is found from the above analysis that it is very hard to meet the required criteria using Florence model, which also means it is impossible to only use the ceramic plate as the face panel of ceramic armored composite. This is why the concept of ceramic sphere in honeycomb structure is introduced. And thus the modification of Florence model is necessary.

DISCUSSION

From the above analysis, it is impossible to meet the required criteria, which requires that the ballistic limit is more than 2700ft/s while the area density is less than 4 lb/ft^2, Some kinds of improvement would be considered.

a). The improvement of material properties of backing composite layers

Table 5 lists the properties of state-of-art materials we currently used in backing panel. It is shown that the specific energy absorption ability (which equals to UTS * ε_c / d_b) of the Spectra 1000 is the best among these currently used materials. The experimental results indicate that it is impossible to reach the specific criteria by using current material. A new question should be answered, what kind of material properties are needed to meet the specific criteria? From Florence model, it is found that the ballistic limit increases with the energy absorption ability of the backing composite layers. If a new fiber with better absorption ability can be found, or the energy absorption ability of Spectra fiber can be improved, the ballistic limit can be raised without the change of area density.

Table 5. The properties of state-of-art materials

material	Density (kg/m^3)	Ultimate tensile strength (GPa)	Broken strain (%)
Spectra 1000	1000	3	3.4
Kevlar 29	1450	2.8	4.1
PEN	1380	1.21	4
Graphite(AS4)	1800	3.58	1.54
E Glass	2540	3.45	4.8
Polypropylene	910	0.638	14

Besides the three variables use above, the fourth variable ξ_4, which is equals to the product of ultimate tensile strength and breaking strain of backing plate, is introduced.

$$\xi_4 = s \times \varepsilon_c$$

For Spectra® 1000 fiber, $\xi_4 = 3\times10^9\times3.4\% = 1.02\times10^8$. If we assume the improved absorption ability can reach 2.04×10^8, the coded variable

$$x_4 =[(\xi_4\times10^{-8})-1.53]/0.255$$

According to the theory of Response Surface Methodology, for quadratic polynomial response surface with 4 independent variables, there should be 25 experiments in order to get the optimization of the ballistic limit

Table 6. The experimental design with improved backing composite

Factors		**Levels**				
Coded variables	Actual variables	-2	-1	0	1	2
X_1	No. of layers	105	115	125	135	145
X_2	Ratio of h_c/h_b	0.4	0.5	0.6	0.7	0.8
X_3	Ceramic density	2.48	2.71	2.94	3.27	3.6
X_4	Energy absorption ability ($\times10^8$)	1.02	1.275	1.53	1.785	2.04

The experiment results are obtained by using Florence Model. The responses V_{50} and A_d are calculated. The optimum conditions can be found by using Microsoft Excel Solver.

Table 7. Optimum Design obtained using response surface

Ballistic limit (ft/s)	2716
Area Density ($4lb/ft^2$)*	4
Spectra layers	104
Diameter of ceramic sphere (in)	0.059
Density of ceramic (g/cm^3)	3.4
Ultimate strength ($\times10^9$Pa)†	3.3

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* Constraint condition

† As the upper limit

It is interesting that a small improvement in backing composite (10% increase of energy absorption ability, which is the product of ultimate strength and breaking strain) can improve the ballistic limit without increasing the area density. It provides a potential way to meet the specific criteria of GDC.

b). The introduction of ceramic sphere

With the introduction of ceramic sphere, the ceramic plate is replaced with ceramic composite panel in which ceramic sphere are the reinforcement and the epoxy as the matrix. Thus the area density of GDC composite can be decreased at a slight reduction of the ballistic limit. The area density of composite is:

$$A_d = (d_c v_{fc} + d_e v_{fe})h_c + d_b h_b$$

Where d_c: the density of ceramic sphere
v_{fc}: the volume fraction of ceramic sphere in surface composite.($\pi/6$)
d_e: the density of epoxy (1.22g/cm^3)
v_{fe}: the volume fraction of epoxy ($1-\pi/6$)
h_c: the thickness of ceramic sphere (which is assumed as the diameter of ceramic sphere)

Also the parameter *f(a)* in Florence model should be modified as:

$$f(a) = M_p/(M_p + (h_c(d_c v_{fc} + d_e v_{fe}) + h_b d_b)\pi a^2)\pi a$$

Here we assume that the other parameters in Florence Model are not changed with the introduction of ceramic sphere.

By using the same experimental design in table 2, the responses were calculated by the modified Florence model.

The responses V_{50} and A_d as shown below are calculated. The optimum conditions can be found by using Microsoft Excel Solver.

Table 8. Optimum Design obtained using response surface

Ballistic limit (ft/s)	3330	3053
Area Density (4lb/ft^2)*	4	3.5
Spectra layers	110	97
Diameter of ceramic sphere (in)	0.063	0.055
Density of ceramic (g/cm^3)	3.4	3.4

* Constraint condition

Table 9 lists all optimum conditions to reach the specific requirements:

Table 9 . The comparison of optimum results of three experimental designs

	Spectra layers@	Ratio of h_c/h_b@	Density of ceramic @ (g/cm^3)	Ultimate strength@ ($\times 10^9$ Pa)	V_{50} (ft/s)	A_d* (lb/ft^2)
Exp#1	106	0.1	2.645	fixed	2582	4
Exp#2	104	0.1	3.4	3.3	2716	4
	93	0.1	3.4	4.12	2769	3.5
Exp#3	110	0.1	3.4	fixed	3330	4
	97	0.1	3.4	Fixed	3053	3.5

@: the optimum conditions
*: the constraint condition

FUTURE WORK

Obviously more tests need to be done. There are only three variables, the layers of Spectra fabrics, the ratio of ceramic sphere and backing composite plate, and the density of ceramic sphere, are considered in the analysis above. In reality, there are other variables, such as the geometry of backing composite plate, the structure of ceramic sphere and so on, affecting the response surface. And also the economic consideration should be another response. In our design the ceramic layer is made of ceramic sphere in honeymoon structure and the conical fracture pattern is changed with the geometry of ceramic sphere.

CONCLUSIONS

The RSA can be used effectively to generate contour diagrams showing the optimum combination of design parameters for the response of interest. In this study, the response, ballistic limit V_{50} and area density A_d can be generated with the Florence's Model according to the experimental design. The ballistic limit and area density for various types of ceramic sphere and geometric combinations can be obtained by the response surface analysis. It is found that the current material system can not meet the specific requirement of ballistic limit with the certain constraint of area density. The search for better material and improvement of structural design are needed.

REFERENCES

1. Box, G.E.P. and K.B. Wilson. *On the Experimental Attainment of Optimum Condition*. J.Roy. Statist. Soc., B13,1, 1951
2. Ko, F.K., Yu, J.Z., And Song, J.W., "Characterization of Multifunctional Composite Armor", *Proceedings of the America Society for Composite* , Atlanta, GA. October 8, 1996
3. Ko, F.K., *Three dimensional Fabrics for Composite in Textile Structural Composites*, edited by Chou, T.W. and Ko, F.K., pp129-171, Elsevier Science Publishers B.V., Amsterdam, 1989
4. Ko, F.K., Geshury, A. And Song, J.W., "Behavior of gradient Designed Composites under ballistic Impact". *Proceedings, ICCM*11, Gold Coast, Australia, 1997
5. Florence, A.L., "Interaction of Projectile and Composite Armor", Part II, *Stanford research Institute Report*, AMMRG-CR-69-15, 1969

ACKNOWLEDGEMENTS:

This study is supported by the USA Army Research Office under the Multidisciplinary University Research Initiative (MURI) with Dr. Iqbal Ahmads as the program monitor.