A Strength Analysis Method of Composite Plate with Multiple Bolted Joints

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SUMMARY: In this paper, a very useful method for design calculations is proposed to analyze the strength of composite plates with multiple bolted joints. The method first transforms the problem into a complex potential value problem in anisotropic plane elasticity and then utilizes the Faber series expansion and the least squares boundary collocation techniques to obtain the series solutions of the stress concentration around holes in a finite composite laminated plate. Yamada-Sun point stress criterion and the characteristic length are used to predict the hole strength, so the calculations of the contact stress and the effect of material non-linearity, large displacement and stiffness reduction on the strength, which are complicated and yet very difficult to be simulated numerically, are not required. The method is then used to study the effect of the ply ratio and distance between bolts on the pin load distribution and bearing strength after the it is tested.

KEYWORDS: laminate, pin load distribution, joint strength, joint parameter

INTRODUCTION

The pin load distribution and bearing strength analysis method for composite plates with multiple bolt joints is one of the main means to shorten the design parameter study period and fully utilize the advantage of composite materials which can be designed as per the requirements of applications. Thus, developing such a method has been received a lot of concern by many composite structure designers and researchers. Several researchers have tried to utilize the FEM analysis combined with the damage mechanics approach to precisely model the damage process around the pin hole[1-7]. This approach is hardly used in design process since it needs a lot of computational effort to incorporate the complex effect due to the contact between pin and hole, the non-linearity behavior around holes, and stiffness reductions. Determining the design parameters requires a large amount of calculations, and needs iteration to find out the optimized parameter within infinite solutions. Therefore, the method to be used in the design process should be simple and can yield reasonable accurate results with minimum computational effort. In this paper, such a method is proposed. It is shown that the proposed series solution method combined with the point stress failure criterion are not only simple to use but also yield accurate results with minimum computational effort, thus suitable for analyzing the strength of mechanically fastened composite plates with multiple pins. Comparisons are made between the predictions by utilizing the proposed method and experimental data. It is found that the calculated

results agree well with the experimental data. Therefore, the calculated results can be directly used in the designing of composite plates with multiple bolted joints .

Pin Load Distribution and the Complex Potential Method

Consider two plates connected by multiple pins (totally L pins), shown in Fig.1.

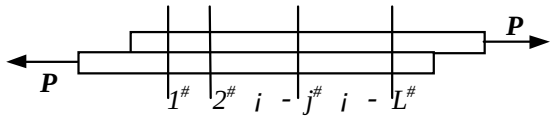


Fig. 1. Mechanically fastened by multiple pins

The compatibility and the equilibrium equations around holes are, respectively,

$$(\omega_j^D - \omega_i^D) - (\omega_j^s - \omega_i^s) = \delta_j - \delta_1 \quad (j = 2,3,...L) \tag{1}$$

and

$$\sum_{j=1}^{L} X_j = P_x \qquad \sum_{j=1}^{L} Y_j = P_Y \qquad (j = 1,2,...,L) \tag{2}$$

Equation (1) is complex equation and $\omega_j^D, \omega_{j,}^S, \delta_j$ are displacements of the upper plate, lower plate and the pin around the jth hole. Eq. (2) is real equation and X_j, Y_j are the in plane force components in the x and y direction around the jth hole, P_x, P_y are the external forces, respectively. There are totally 2L equations.

Consider a finite composite plate S (S is the domain occupied by the plate) weakened by multiple elliptical holes. Using affine transformation, mapping transformation, Laurent series and Faber expansion, any analytical function in S_j (Domain S is transformed onto the domain S_j by the affine transformation) can be written as [8]

$$\phi_j(Z_j) = \sum_{m=1}^{\infty}(B_{jm} \ln \xi_{jm} + \sum_{k=1}^{\infty} b_{jmk} / \xi_{jm}^k) + \sum_{k=1}^{\infty} a_{jk} Z_j^k \tag{3}$$

where B_{jm} can be expressed in terms of X_j and Y_j, b_{jmk} can be obtained by using the force and displacement boundary conditions, a_{jk} are determined by using the least squares boundary collocation method, $\ln \xi_{jm}$ and ξ_{jm}^k can be expanded in Fourier series.

The displacements and stresses in the composite plate can be uniquely determined by[9]

$$u = 2\,\mathrm{Re}\sum_{j=1}^{2} p_j \phi_j(Z_j) - \omega_y + u_0$$

$$v = 2\,\mathrm{Re}\sum_{j=1}^{2} q_j \phi_j(Z_j) + \omega_x + v_0 \tag{4}$$

$$\sigma_x = 2\,\mathrm{Re}\sum_{j=1}^{2} \mu_j^2 \phi_j'(Z_j), \sigma_y = 2\,\mathrm{Re}\sum_{j=1}^{2} \phi_j'(Z_j), \tau_{xy} = -2\,\mathrm{Re}\sum_{j=1}^{2} \mu_j \phi_j'(Z_j) \tag{5}$$

where p_j, q_j, μ_j are constants relative to the flexibility and anisotropy of the composite plate, and $\omega_x, \omega_y, u_0, v_0$ are rigid body displacements, respectively.

Substituting Eq. (3) and Eq. (4) into Eq. (1) and solving the resulting Eq. (1) and Eq. (2) simultaneously yield the pin load distribution. The stress at any point within the plate can then be obtained by using Eq. (5). Since the assumed complex potential function is analytic within the domain, the inner boundary conditions can be satisfied exactly, while the outer boundary conditions are satisfied approximately (error is less than 1 percent) by properly choosing the collocation points.

The Failure Criterion of the Pin Hole

Chang[10] extended the concept of characteristic length of holes without loading proposed by Whitney[11] and proposed the characteristic length of holes with loading as follows

$$\gamma(\theta) = \frac{1}{2}D + R_t + (R_c - R_t)\cos\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

where D, R_c, R_t are the hole diameter, compressive characteristic length and tensile characteristic length, respectively. It is recognized that the effect of contact between pin and hole, and the nonlinear behavior have been taken into account by the constants R_c, R_t determined by experiments[10-17]. Applying the Yamada-Sun point stress criterion on the characteristic curves layer by layer and point by point yields the failure points which determine the failure mode.

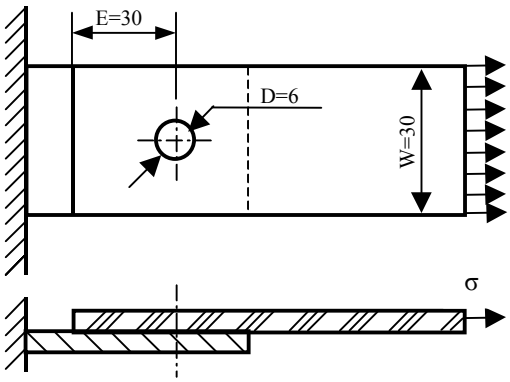
Experimental Verifications

Several experiments, including tensile tests on the specimen fastened by single pin, four pins in double rows, and eight pins in four rows, and shear tests on the specimen fastened by twelve pins in double rows , have been performed to verify the proposed method. The experimental results are compared well with the theoretical predictions shown in Figs. 2 to 4 and Tables 1 to 3, respectively. The relative errors in Tables 1-3 are calculated by :

$$\left| \frac{\text{Experimental Data - Calculated Data}}{\text{Experimental Data}} \right|$$

Table 1 Comparisons of the experimental results with the predictions for single pin connected specimen under uniaxial tension.

	Ultimate Strength (kN)	Failure Stress (Gpa)	Failure Mode
Experimental Data	29.05	188.03	Bearing Failure
Calculated Results	28.29	183.11	Bearing Failure
Error (%)	2.6	2.6	0.0



SPECIMEN
Material: T300/5222 (epoxy resin)
Ply: $0_{10}\oplus 45_{28}90_{6}$
Laminate thickness: 5.15 (mm)
Pin diameter (D): 6 (mm)
Metal material: aluminium
Metal plate thickness: 6 (mm)

Fig. 2. Computational model for the specimen fastened by single pin.

Table 2. Comparisons of the experimental results with the predictions for four pins (double rows) connected specimen under uniaxial tension.

	Pin Load Distribution		Ultimate Strength (kN)	Failure Stress (Gpa)	Max. Pin Load of a pin	Failure Mode	Max Stress Concentration	
	1st row	2nd row					+	-
Experi. Data	55.2%	44.8%	78.99	159.65	20.42	Bearing failure		
Calcul. Results	56.6%	43.4%	76.15	153.85	19.69	Bearing failure	5.20	5.40
Error (%)	2.5	3.2	3.6	3.6	3.6			

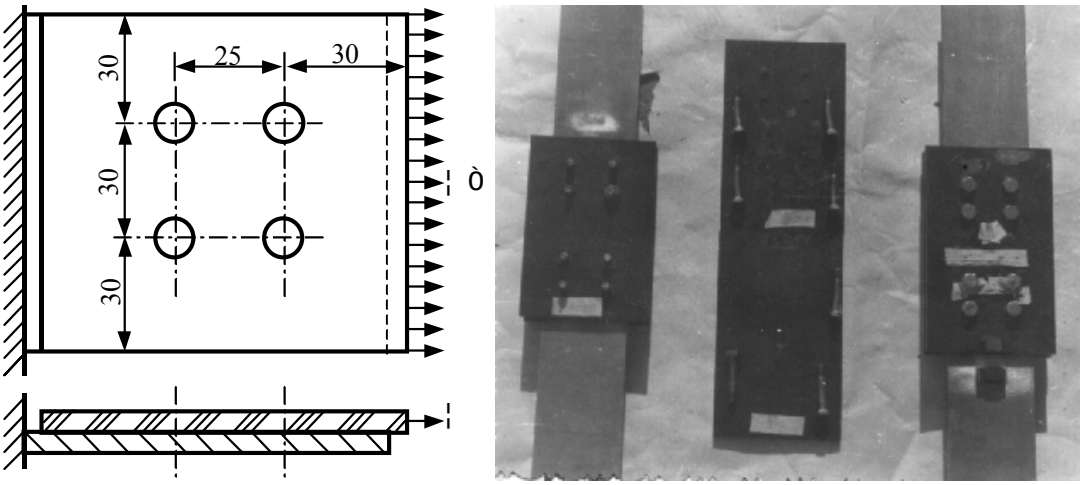


Fig. 3. Computational model for the specimen fastened by four pins in two rows under uniaxial tensile loading.

Table 3. Comparisons of the experimental results with the predictions for twelve pins (double rows) connected specimen under shear loading.

	Pin Load Distribution		Ultimate Strength (kN)	Failure Stress (Gpa)	Max. Pin Load of a pin	Failure Mode	Max Stress Concentration	
	1st row	2nd row					+	-
Experi. Data	67.5%	32.5%	140.6	136.5	23.5	Combined Tension & Shear		
Calcul. Results	65.35%	34.65%	132.5	128.6	22.14	Combined Tension & Shear	6.6	8.7
Error (%)	3.2	6.6	5.8	5.8	5.8			

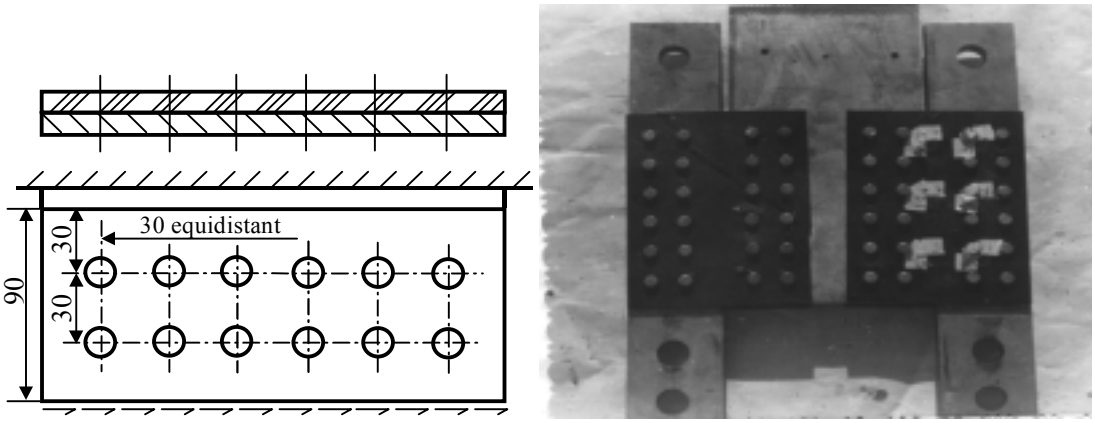


Fig. 4. Computational model for the specimen fastened by twelve pins in two rows under shear loading.

Design Parameter Study

A computer program is written based on the proposed method to study the effect of the joint design parameter on the connection strength. Only parts of the results are given herein. The input parameters are the same as given in the previous section except those explained in this section.

The variations of stress concentration and the bearing strength with the ply ratio for single pin are show in Figs. 5 and 6. It can be seen that increasing the ply of $\pm 45^o$ can reduce the stress concentration around the hole and raise the bearing strength if the ratio of the 0° layer keeps unchanged. However, increasing the 0° layers will increase both the stress concentration around the hole and the bearing strength if the ratio of the $\pm 45^o$ layer keeps fixed.

The variations of the bearing strength with the width and the pin loading direction for single pin are show in Figs. 7 and 8. It can be seen that width has little effect on the bearing strength while the pin loading direction has large effect on the bearing strength when $W / D \geq 5$. This effect should be considered during design process.

Figures 9 and 10 show the predictions of the effect of the ply ratio and row spacing on the bearing strength for a tensile specimen fastened by four pins in double rows. It is

shown that the bearing strength is the highest for a $\pm 45^o / 90^o / 0^o$ laminated plate with corresponding ply ratio of 50:40:10. It is also show that the row spacing has little effect on the bearing strength when the spacing is greater than 2.5 times of the hole diameter.

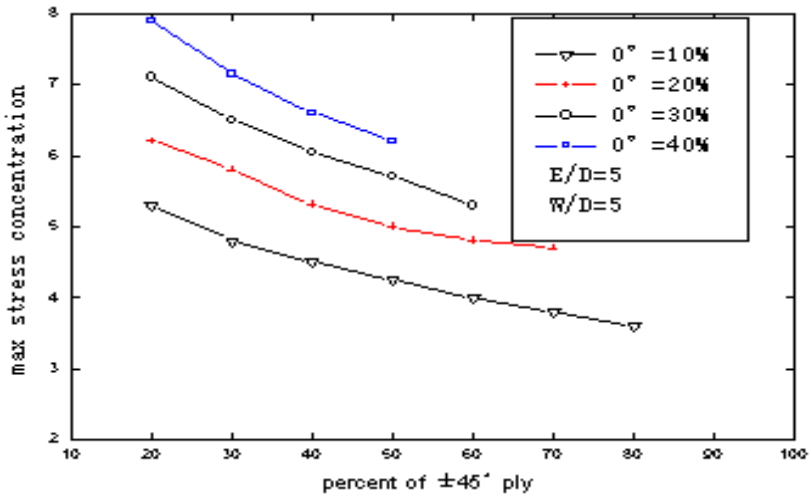


Fig. 5 The effect of ply rations on the stress concentration.

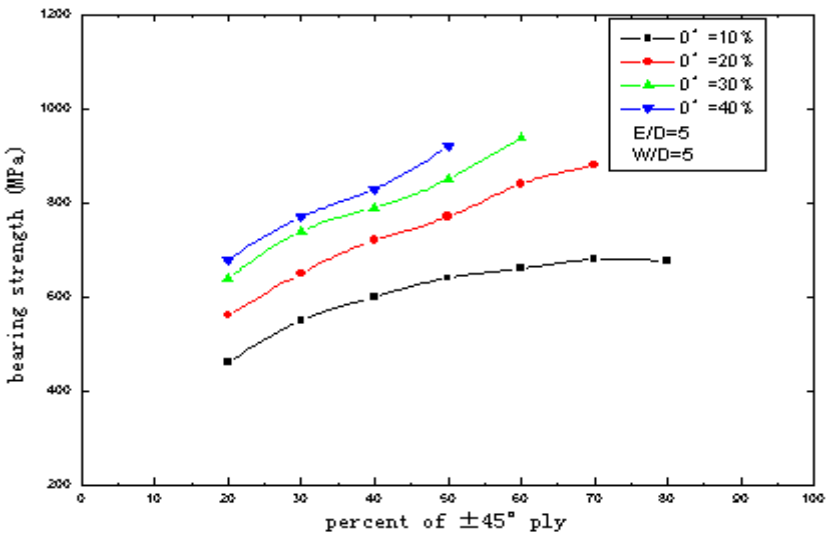


Fig. 6 Variations of bearing strength with ply ratio.

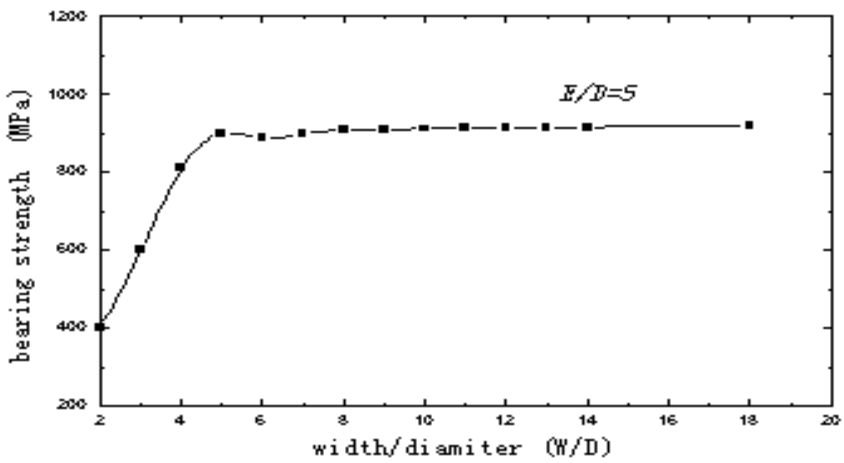


Fig. 7 Variation of bearing strength with plate width.

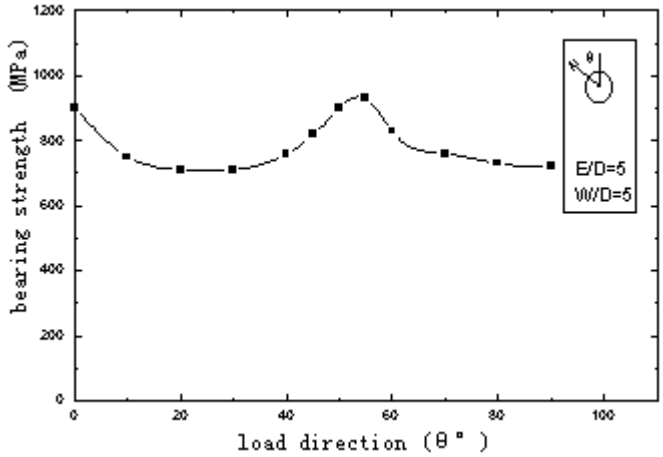


Fig. 8 Variation of bearing strength with pin loading direction.

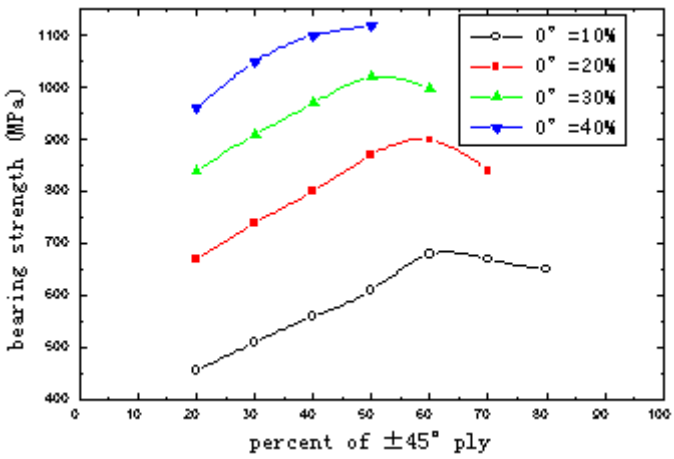


Fig. 9. Variation of bearing strength with ply ration for the specimen fastened by four pins in two rows under uniaxial tensile loading.

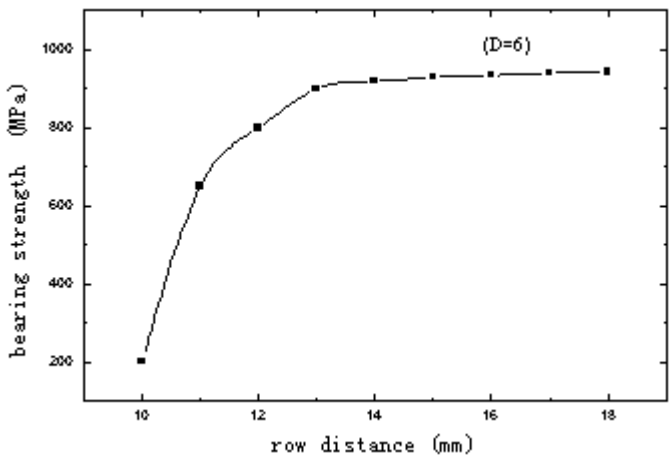


Fig. 10. Variation of bearing strength with row spacing for the specimen fastened by four pins in two rows under uniaxial tensile loading.

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