ANALYTICAL DISCRETE MODEL OF NONSYMMETRIC SANDWICH PANELS AT BENDING

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SUMMARY: The purpose of this paper is to present an applied theory based on an explicit analytical solution of the bending problem of three-layer sandwich-like plate of an arbitrary asymmetric structure under point loading. Local effects are brought out within the scope of the discrete model considering specific character of filler elastic properties. Local characteristics were shown to be varied several fold according to the asymmetry parameters of a structure.
The solution degeneration peculiarities stemming from sliding of layers and their rigid link in the context of the Kirchhoff-Lowe hypothesis as well as with modification of the model allowing for transverse shear and compression of a normal have been examined in a close agreement with degeneration of geometric and physical parameters of the discrete model adopted. The results have been illustrated as the curves and surfaces with a proper selection of the solution characteristics governing a local effect.

KEYWORDS: three-layered plate, cylindrical bending, non-symmetric structure, discrete model, point forces, local effects, moment stresses.

INTRODUCTION

In spite of extensive studies, that have been thus far carried out on the bending problem of sandwich structures, there are narrow gaps still left in understanding and consideration of the effect of structural features on bending characteristics. Among these are the local effects occurring due to point loading and special edge fixing of a structure. Structural asymmetry of a sandwich complicates a bending process and presents basic difficulties in its analysis.
Refinement of the design and the detection of the effect of structural features on design characteristics is occurred within the context of an applied theory by the rise in deformation freedom degrees and thus the increase in equations order. The discrete models, therewith, for study of local characteristics are more preferable than the global ones.
Not excluding from consideration the different variants of the model description of deformation of layered plates proposed by many known authors it must be yet stated the lack of the analytically simple and rather exact procedure of prediction of local characteristics that suitable to engineering design. This procedure can practically be derived from "screening" the results of qualitative theory of differential equations obtained by the variational methods for the process considered. However, in view of the high order of equations, the solution is classified by a certain numerical algorithm. This is identical in providing the final result with the initial application of a numerical method, such as FEM, developed for the model adopted. In this manner a great body of matters involving the problem of sandwich bending was

considered by Frostig Y., Tomson O.T. and others. The more complex variant of the model in polar coordinates was recently suggested by Frostig Y. [1]. The action of point forces was simulated by "spring inserts" at a sandwich section. The higher order of the model found by the author at the expense of complication of midlayer kinematics and extending the variation procedure with two additional shear stress functions along the lines of layer connections do not fundamentally change the numerical realization of the solution. The qualitative analysis of the solution procedure previously suggested by the author [2] for point force action is restricted to the stage of obtaining eigenfunctions in the conditions of constancy of deformations across the thickness of a midlayer.

Objective of the Article

With necessity of qualitative parametric analysis of the local bending characteristics of a sandwich panel the analytical approach to the solution of variational equations by the operational Laplace method allowing for point forces as Dirac's functions has been suggested. For one-dimensional problem of cylindrical bending this leads to the direct way of reduction the boundary value problem to the Cauchy problem with normalization of fundamental functions at a panel centre. The convergence problem of displacement derivatives (and thus stresses themselves) for discontinuous loading and point forces is automatically taken off using the operational method. The asymmetry of a sandwich structure if not makes it possible to separate the solution forms into classical shear and compression along a normal, can not, however, interfere with the representation of the final solution in a closed form as the rank of the matrix for the set of boundary conditions with normalized functions is essentially decreased. Basing on [3] let us formally outline the initial relationships of the problem, broaden the qualitative analysis of the solution procedure for local curvatures and conduct the parametric investigation of normal stresses for comparison with a classical model.

BENDING OF ASYMMETRIC SANDWICH PANEL

To consider different features of deformation of three-layer plate of asymmetric structure in greater detail it is necessary to elucidate the specific character of a solution reasoning from the qualitative theory of differential equations for the applied model adopted. We shall restrict our consideration to the one-dimensional problem of cylindrical panel bending from the surface loads on the upper and lower face planes of width b . The turn of a normal in bending of each rigid layer, which is modeled on Kirchhoff- Love, with uniform compressive and shear strains across the thickness of a midlayer is adopted. (The elastic characteristics of a soft midlayer will be henceforth defined by modules E_z and G_{xz}). The kinematic hypotheses of applied theory introduced for a separate layer exclude from consideration the characteristics of stress-strain state solely at the places of a point force application, however, in this case the mechanism of a load transfer between layers modifies only slightly. It is appropriate to consider the localization of surface loads and edge conditions in bending of the panel by a discrete model possessing more number of freedom degrees as against a global model.

Basic Notation and Equations

Thus the discrete model possesses four degrees of displacement freedom: deflections w_i and longitudinal displacements u_i of face layers $i=1,2$. Introducing the local coordinate system half-way of an either face layer thickness, see Fig.1, the layer kinematics can be readily deduced from the four functions on the interval $-l \le x \le l$ and respectively over the intervals $-h_i/2 \le z^{(i)} \le h_i/2$, $i=1,2$, and the problem equations are derived by a variational method.

Omitting the details of this procedure, which is cited in [4], let us lay our emphasis on the analysis of the equations and the solution, separating a symmetric part from nonsymmetric one and their dependence on the parameters corresponding to a symmetric and asymmetric panel structure.

Symbolizing the components of the symmetric solution part by the index "*s*" and nonsymmetric one respectively by "*as*", the four displacement functions of face layer middle planes and the surface loads take the following form:

$$u_{s,as} = (u_2 \pm u_1)/2, \qquad w_{s,as} = (w_2 \mp w_1)/2, \qquad q_{s,as} = (q_2 \mp q_1)/2, \tag{1}$$

where the upper sign at the right part of Eqn (1) corresponds to the index "*s*" at the left-hand side and the lower sign respectively to "*as*". Variational equations of Euler- Ostrogradskii after that and having introduced the structure asymmetry parameters ϑ and θ (presented below) in them, assume the following dimensionless form:

$$\begin{cases} u''_{s,as} - \vartheta_{s,as}(k_1/4)\left[(1+\chi_0)w'_{as} - (1-\chi_0)w'_s + 4u_{as}\right] = 0 \\ w^{IV}_{s,as} - \theta_{s,as}(3k_1/2\mu^2)\left[(1+\chi_0)w''_{as} - (1-\chi_0)w''_s + 4u'_{as}\right] + 12k_2\theta_{as,s}w_s = q_{s,as} \end{cases} \tag{2}$$

The derivation in Eqns (2) is taken with respect to the dimensionless variable, all functions, physical and geometric parameters are expressed in a dimensionless form:

$$\xi = \frac{x}{h_1 + h_0}, \qquad u,w = \frac{\mathrm{u,w}}{h_1 + h_0}, \qquad \chi_0 = \frac{h_2 + h_1}{h_1 + h_0}, \qquad \chi = \frac{h_2}{h_1}$$

$$n_x = \frac{E_x^{(2)}(1-\nu_1^2)}{E_x^{(1)}(1-\nu_2^2)}, \quad n_z = \frac{E_z}{E_x^{(1)}}(1-\nu_1^2), \quad m_z = \frac{G_{xz}}{E_x^{(1)}}(1-\nu_1^2)$$

$$\gamma_1 = 1 + \frac{1}{n_x\chi}, \quad \gamma_2 = \frac{1}{n_x\chi^3}, \qquad \vartheta_{s,as} = \frac{1}{n_x\chi} \mp 1, \quad \theta_{s,as} = \frac{1}{n_x\chi^3} \mp 1 \tag{3}$$

$$\mu = \frac{h_1}{h_1 + h_0}, \qquad k_1 = \frac{m_z}{\mu(1-\mu)}, \quad k_2 = \frac{n_z}{\mu^3(1-\mu)}, \quad k_3 = \frac{3\chi_0 m_z}{\mu^3(1-\mu)}$$

$$q_0 = \frac{6P(1-\nu_1^2)}{E_x^{(1)}b(h_1+h_0)\mu^3}, \qquad q_1 = 2q_0\delta(\xi), \quad q_2 = -\gamma_2 q_0 \delta(|\xi| - \xi_1)$$

The last line of Eqn (3) including the Dirac function δ should be replaced by a linear combination of the unit functions η of "lagging" argument, when forces *P* and *P/2* are substituted for a uniform load $q = P/2ab$, applied respectively at lengths *2a* and *a* in the vicinity of points $x = 0$ and $x = \pm l_1$ of the upper and the lower face layers, see Fig.1 *b*. So that:

$$\delta(\xi) \Rightarrow (1/2\xi_a)\left[\eta|\xi| - \eta(|\xi| - \xi_a)\right], \qquad \delta(|\xi| - \xi_1) \Rightarrow (1/\xi_a)\left[\eta(|\xi| - \xi_1 + \xi_a) - \eta(|\xi| - \xi_1)\right]$$

$$\xi_1 = \frac{l_1}{h_1 + h_0}, \qquad \xi_l = \frac{l}{h_1 + h_0}, \quad \xi_a = \frac{a}{h_1 + h_0}, \quad 0 \le a \le l_1 \tag{4}$$

The inspection of the Eqns (2) shows that for the structure asymmetry when all $\vartheta_{s,as}$, $\theta_{s,as} \neq 0$ the equations of the symmetric and nonsymmetric parts of the displacements are not separated and in this situation these equations are devoid of advantages over the set of the equations in [4]. However, for the structure symmetry it follows that $\vartheta_s = \theta_s = 0$, $\chi_0 = 1$ and all equations can be separated into the sets each defining the symmetric and nonsymmetric component of the solution. Further still in the set for the symmetric part the equations of the functions u_s, w_s themselves can also be separated.

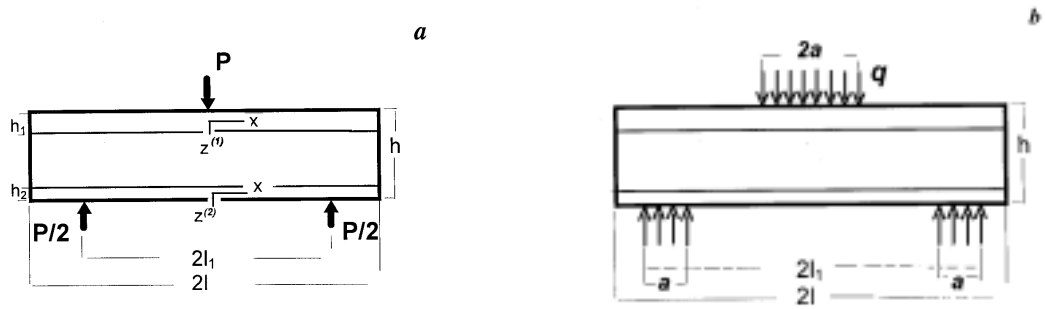

Fig. 1: Loading chart of a sandwich panel

Boundary conditions for the set of Eqns (2) of the twelfth order in the absence of the longitudinal force at the panel ends $\pm\xi_l$ are the natural homogeneous conditions derived from variation of an energy functional:

$$w'''_{s,as}(\pm\xi_l) - \bar{\theta}_{s,as}(3k_1/2\mu^2)[(1+\chi_0)w'_{as} - (1-\chi_0)w'_s + 4u_{as}]_{\pm\xi_l} = 0$$
$$w''_{s,as}(\pm\xi_l) = u'_{s,as}(\pm\xi_l) = 0, \qquad \left(\bar{\theta}_{s,as} = \chi_0/(n_x\chi^3) \mp 1\right) \tag{5}$$

By virtue of the fact that the membrane stresses of the face layers in the summarized axial load $N_x = 0$ make the couple $N_1 = -N_2$ the relation $u'_2(\xi) = -(1/n_x\chi)u'_1(\xi)$ is realized Thus the expression of the moment at the centre of a sandwich panel loaded by point forces (Fig.1a) in relative variables can be deduced as:

$$w''_1(0) + n_x\chi^3 w''_2(0) + (6/\mu^2)(1+\chi_0)u'_1(0) = -q_0\xi_1 \tag{6}$$

and with a piecewise distributed load (Fig.1b) as

$$w''_1(0) + n_x\chi^3 w''_2(0) + (6/\mu^2)(1+\chi_0)u'_1(0) = -q_0(\xi_1 - \xi_a) \tag{7}$$

The Laplace transform was used in solving the set of Eqns (2) in light of the discontinuous nature of the load upon the action of point forces and piecewise loads, that was specified by $q_i(\xi)$ in (3) and (4). All other attendant conditions for symmetric load distribution about the central section $\xi = 0$ as with the initial values at that section taken $w'_i(0) = u_i(0) = 0$, $i = 1,2$ match the applied method. On the other hand the fundamental functions available from this method give us a convenient way to deduce the kinematic characteristics at the central panel section in terms of the unknown constants in the Cauchy problem.

As a result the solution of bending problem treated for asymmetric structure panel would be expressible in a closed form in terms of the fundamental functions, which specific character is defined by the following properties:

1. Eleven linearly independent functions (with $N_x = 0$ the necessity for evaluation of the twelfth function is eliminated) are recursively deduced by derivation:

$$p_{n-5}(\xi) = \frac{d^n}{d\xi^n} p_{-5}(\xi), \qquad n = 0,...,10 \tag{8}$$

The expression $p_{-5}(\xi)$ can be written in the form of a sum of the terms which are the products of hyperbolic and trigonometric functions of a longitudinal coordinate. The arguments of these functions are rescaled by the factors, which are the real and the imaginary parts of complex characteristic numbers with the notation as α and β. The isolated terms of the sum being the hyperbolic functions dependent on real characteristic number denoted by a_1. The total notation of the set of functions is quoted in [4]. For the absolutely pliable in transverse shear material of the midlayer, that is $G_{xz} = 0$ (and $k_1 = 0$) the following expression of the primary fundamental function can be derived:

$$p_{-5}(\xi) = \frac{\xi^6}{6!\,b_1} - \frac{\xi^2}{2b_1^2} + \frac{1}{b_1^{5/2}} sh\,\alpha\xi\, sin\,\alpha\xi \tag{9}$$

where right now $\alpha = (\sqrt{2}/2) b_1^{1/4}$.

2. The functions $p_n(\xi)$ are normalized so that:

$$p_n(0) = 0, \text{ when } n \neq 5 \text{ and } p_5(0) = 1 \tag{10}$$

3. The following auxiliary linear relationship of the fundamental functions which identically equal to unity was deduced:

$$b_0\, p_{-1}(\xi) + b_1\, p_1(\xi) + b_2\, p_3(\xi) + p_5(\xi) \equiv 1 \tag{11}$$

where

$$\begin{aligned} b_0 &= -a_1^6 - b_2 a_1^4 - b_1 a_1^2 = -12 k_1 k_2 [\gamma_1(1+\gamma_2) + \gamma_2(3/\mu^2)(1+\chi_0)^2] \\ b_1 &= (\alpha^2+\beta^2)^2 + 2a_1^2(\alpha^2-\beta^2) = 12 k_2 (1+\gamma_2) \\ b_2 &= -a_1^2 - 2(\alpha^2-\beta^2) = -k_1[\gamma_1 + (3/\mu^2)(1+\gamma_2\chi_0^2)] \end{aligned} \tag{12}$$

The displacements and their derivatives at the central panel section are derived from the boundary relationships (5) considering (6) as well as from the fact that a zero value of the deflection of the lower face layer is defined at a point of the support $\xi = \xi_1$ i. e. $w_2(\xi_1) = 0$. Eliminating $w_i(0)$ and $u_1'(0)$ from these equations the set of two equations for $w_i''(0)$ $i = 1,2$ is obtainable, which solution provides a basis for the following analysis of the results given below. It should be noted as experience shows that the calculation procedure can be incorrect when a panel span $\xi_1 > 3$. This attendant upon necessity of regularization of the terms for the sum of products of hyperbolic and trigonometric functions with large arguments. The correcting calculation reception can be effected by the substitution of the mentioned products with the real and imaginary parts of these functions now of a complex argument, that is by isolation of their modulus as $\sqrt{sh^2\alpha\xi_l + cos^2\beta\xi_l}$ and argument as $arctg(tg\,\alpha\xi_l\, tg\,\beta\xi_l)$.

Determination of starting displacements and their derivatives at section $\xi = 0$ in the case of loading of the panel by a uniform piecewise distributed load over the sections of length *a* (Fig.1b) is carried out analogously to the case of point forces. For this purpose we should replace the functions $p_n(\xi)$ in the particular solution for the case of a point force by the finite differences of their primitives from the lagging argument divided by the displacement of the argument ξ_a. Also we should carry out the replacement $\xi_1 \to \xi_1 - \xi_a$ in the case when ξ_1 is a cofactor of q_0.

RESULTS AND DISCUSSION

The peculiarities of a discrete model kinematics can be revealed in the analysis of the higher derivatives of displacements. The third derivative of deflections by a discrete model is not step function of a longitudinal coordinate at point force loading, as it is in a classical global model along with the modified model considering transverse shear. The second derivative of deflections at the places of application of point forces defines the face layer curvature, which at the central section is related to a radius of curvature as $R_i = 1/\mathrm{w}''_{i,xx}(0) = (h_1 + h_0)/w''_{i,\xi\xi}(0),\ i = 1,2$.

The Central Curvatures of Face Layer

To estimate the curvature values $w''_i(0), i = 1,2$ let us assume a standard curvature value $w''_*(0)$ that is derived from the classical bending model that applied to asymmetric sandwich structure. For a three-point bending the classical curvature value at the panel centre can be expressed as:

$$\mathrm{w}''_{*,\mathrm{xx}}(0) = -\frac{6Pl_1(1-\nu_{xy}\nu_{yx})}{E_x^{(1)} n_{as}^* b h_1^3} \tag{13}$$

or for the greatest curvature radius R_* *this relationship in relative variables (3) takes the form* $w''_*(0) = -q_0\xi_1 / n_{as}^* = (h_1 + h_0)/R_*$. *The factor* n_{as}^* *in Eqn (13) corrects for structure asymmetry of a sandwich and equals in magnitude the ratio of its cylindrical rigidity to the one of the upper face layer. The distance counted from the outer surface of the lower face layer to the neutral plane of an asymmetric panel with the midlayer unloaded in bending is defined as*

$$h_e = \frac{2h + (n_x\chi^2 - 1)h_1}{2(n_x\chi + 1)} \tag{14}$$

The factor n_{as}^* can be readily available from (6) setting $w''_1(0) = w''_2(0) = w''_*(0)$ and under uniform normal turn $u'_1(0) = -[(h_2 + h_0 + h_1/2 - h_e)/(h_1 + h_0)]w''_*(0)$

$$n_{as}^* = 1 + n_x\chi^3 + \frac{3n_x\chi(1+\chi_0)[1+\chi+2(1-\mu)/\mu]}{\mu(1+n_x\chi)} \tag{15}$$

For the symmetry of a sandwich structure we have $n_{as}^* = 2 + 6/\mu^2$, which agrees to within designations with the formula from [5]. If. $h_0 = 0$ that is $\mu = 1$ and $\chi_0 = \chi$, then the factor for two layer asymmetric panel can be derived as $n_{as}^* = 1 + n_x\chi^3 + 3n_x\chi(1+\chi)^2/(1+n_x\chi)$. In the case of an ideal sliding of the layers along the surface of their contact we have $n_{as}^* = 1 + n_x\chi^3$.

Let us note another distinction of the factor n_{as}^{*} variation, which is unique to the classical model. With inversion of the ratio of face layer cylindrical rigidities $D_1 / D_2 = 1/(n_x \chi^3)$, bearing in mind therewith that $\mu \to h_2 /(h_2 + h_0)$ and $\chi_0 \to \chi_0^{-1}$ we derive from (15) $n_{as}^{*} \to n_{as}^{*} /(n_x \chi^3)$ and the cylindrical rigidity of a sandwich panel is $D = D_1 n_{as}^{*} = D_2 [n_{as}^{*} /(n_x \chi^3)] = const$.

Traditionally the classical method for the evaluation of maximum bending stress is based on the determination of the second derivative (13). For bending of the asymmetric panel with one degree of displacement freedom it will suffice to analyze the dependence of this derivative on physical and geometric parameters. The variation of these parameters was carried out in different directions from a symmetric structure, which is an intermediate variant with equal layer thickness $h_1 = h_2 = h_0 = 4\,mm$ and $E_x^{(i)} = E_x = 40\,GPa$ for a material of HEXCEL/Al type. The thickness of face layers was varied between the limits $(1/10)h \le h_i \le (2/3)h$ with the constant thickness of a panel h and the midlayer $h_0 = 0.24h$ and their elasticity modules were varied in the range $E_x \le E_x^{(i)} \le (20/3)E_x$. The variation of the structure parameters was inversely effected that is the initial values of parameters χ and n_x were respectively the same as the values $1/\chi$ and $1/n_x$ at the end of a variation interval $0.15 \le \chi, n_x \le 6.66$. To make the variation intervals symmetrical about the values $\chi = 1$ and $n_x = 1$ for these relative parameters a logarithmic scale was introduced.

Concave-convex surface, Fig. 2, resultant from the inversion of structure parameters of the sandwich panel of length $2l = 2l_1 = 6h$ and demonstrates the variation of the second derivative of a classical deflection at the central section of a panel that is its curvature with dimensionality $(1/m)$. As few as four section planes parallel to z-coordinate and passing through two diagonals of the range of varied parameters as well as through the axes $\chi, n_x = 1$ yield in intersection with the surface the symmetric curves of variation $w''_{*,xx}(0)$ with structure parameters. The principal surface curves demonstrate the monotonic variation of the panel curvature with its greatest value obtained at the maximum value of one of the parameters and the minimum - another one and the smallest curvature value occurs when both structure parameters are coincide among themselves being greatest or least.

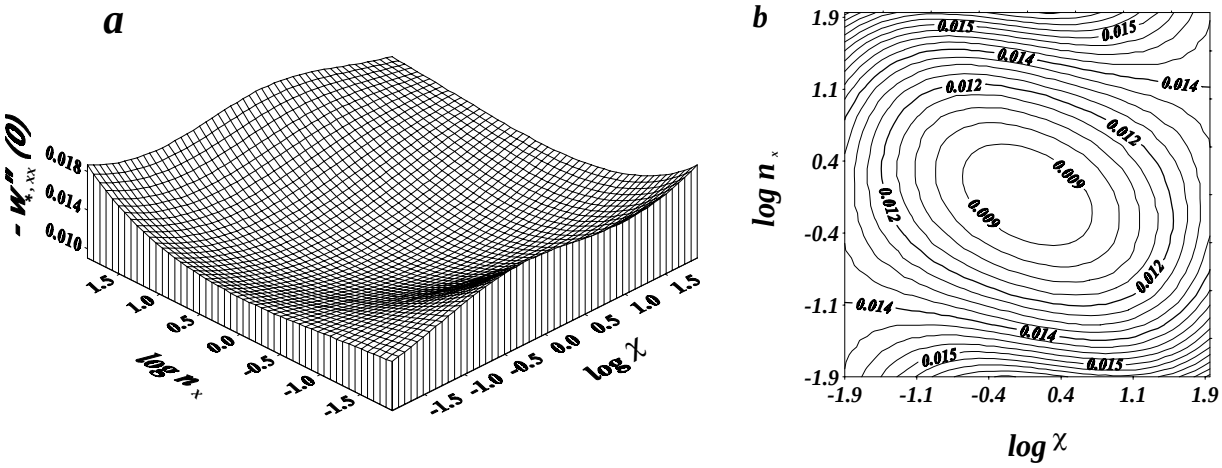

Fig. 2: Variation of the curvature (1/m) at a panel center by a classical model with the structural parameters of an asymmetric section. $P / bh = 1\,MPa$, the loading chart is in the Fig.1a, a - parametric surface of the central curvature; b - level lines on the surface

Substantial change in the classical curvature value follows from the discrete model. The symmetry and asymmetry elements of the surface plotted above in Fig. 2 *a, b* are violated due to

consideration of transverse deformation and a "drift" of the kinematics suitable to a broken line from the uniform turn of a panel section. As may be seen from the Fig. 3 *a, b* a curvature discordance with a classical model is most pronounced under variation of the geometric asymmetry parameter χ. The curves in Fig.3 *a* intersect the ordinate axis above the curves in Fig.3 *b*, as with the same index $n_x = 1$ the face layer modules in the first case are 3.83 times as large as the modules $E_x^{(i)} = E_x$, $i = 1,2$ and the values m_z and n_z in Fig.3 *a, b* are different. The curves 3,4 testify that with the boundary value $\mu_V = (h_1 + h_2)/h \approx 3/4$ the face layer curvature of a sandwich panel dependent on the transverse elastic characteristics of the midlayer $E_z = 0.310\ GPa,\ G_{xz} = 0.138\ GPa$ is great enough, that even classical bending of the two-layer panel of lower thickness $h_1 + h_2$ do not give rise to a greater curvature compared with the one of the upper face layer of three layer sandwich.

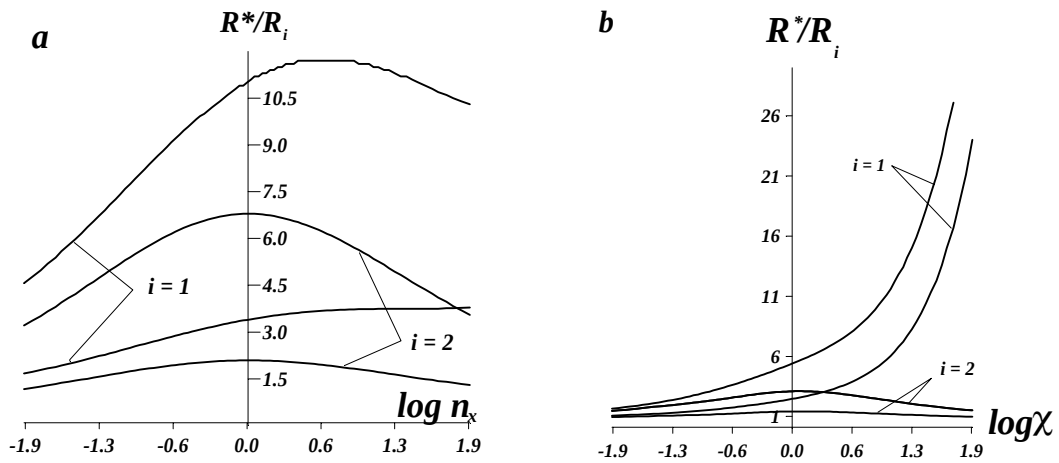

Fig. 3 The ratio of the largest curvature of the face layer $i = 1,2$ at a panel centre to the classical curvature (upper curves) as well as to the curvature of two rigidly coupled face layers (lower curves) versus the elastic ***a*** *($\chi = 1$) and geometric* ***b*** *($n_x = 1$) layer parameters.*

Relative Estimates of Maximal Bending Stresses

. The stresses at the upper and lower point of the central section of an asymmetrical structure panel by a classical theory are respectively equal to:

$$\sigma_x^*(0)\Big|_{z^{(1)}=-h_1/2} = \frac{E_x^{(1)}(h-h_e)}{(1-\nu_{xy}^{(1)}\nu_{yx}^{(1)})(h_1+h_0)} w_*''(0)$$

$$\sigma_x^*(0)\Big|_{z^{(2)}=h_2/2} = -\frac{E_x^{(2)}h_e}{(1-\nu_{xy}^{(2)}\nu_{yx}^{(2)})(h_1+h_0)} w_*''(0) \qquad (16)$$

The same stresses by a discrete theory may be written as:

$$\sigma_x^{(i)}(0)\Big|_{z^{(i)}=\mp h_i/2} = \frac{E_x^{(i)}}{1-\nu_{xy}^{(i)}\nu_{yx}^{(i)}}\left[u_i' \pm \frac{h_i}{2(h_1+h_0)} w_i''\right], \qquad i = 1,2 \qquad (17)$$

where the upper sign corresponds to the face layer with $i = 1$ and the lower sign - with $i = 2$

The ratio between the maximum bending stress by a discrete model and the corresponding stress by a classical theory can be derived from Eqns (16) and (17) considering Eqns (6), (13)-(15):

$$\left.\frac{\sigma_x^{(1)}(0)}{\sigma_x^*(0)}\right|_{z^{(1)}=-h_1/2} = \frac{h_1+h_0}{h-h_e}\left\{\left[n_{as}^* - \frac{w_1''(0)+n_x\chi^3 w_2''(0)}{w_*''(0)}\right]\frac{\mu^2}{6(1+\chi_0)} + \frac{\mu}{2}\cdot\frac{w_1''(0)}{w_*''(0)}\right\}$$

(18)

$$\left.\frac{\sigma_x^{(2)}(0)}{\sigma_x^*(0)}\right|_{z^{(2)}=h_2/2} = \frac{h_1+h_0}{h_e}\left\{\left[n_{as}^* - \frac{w_1''(0)+n_x\chi^3 w_2''(0)}{w_*''(0)}\right]\frac{\mu^2}{6n_x\chi(1+\chi_0)} + \frac{\mu\chi}{2}\cdot\frac{w_2''(0)}{w_*''(0)}\right\}$$

For ideal sliding with $G_{xz}=0$ taken as a result $u_i'(0)=0$ can de deduced and the expressions in square brackets of Eqns. (18) considering Eqn.6 are also zero. For this case instead of a classical stress σ_x^* of a sandwich panel it is worthwhile to take into consideration the bending stress σ_x^{*slip} of the two - layer panel of the layer thickness respectively h_1 and h_2. Then taking account of the location of these layers, let us denote $h_e^{(1)}=h_2+h_0+h_1/2$, $h_e^{(2)}=h_2/2$ and according to (18) the following relationship can be derived:

$$\left.\frac{\sigma_x^{(i)slip}}{\sigma_x^{*slip}}\right|_{z^{(i)}=\mp h_i/2} = \frac{w_i''}{w_*''}, \qquad i=1,2 \tag{19}$$

The ratio (19) defines the relative variation of bending stresses having regard to a filler transverse elasticity with the ideal sliding of face layers. The difference between the bold curves and the dashed ones given in Fig. 4 *a, b* actually shows the influence of transverse shear rigidity of a midlayer on the bending stresses.

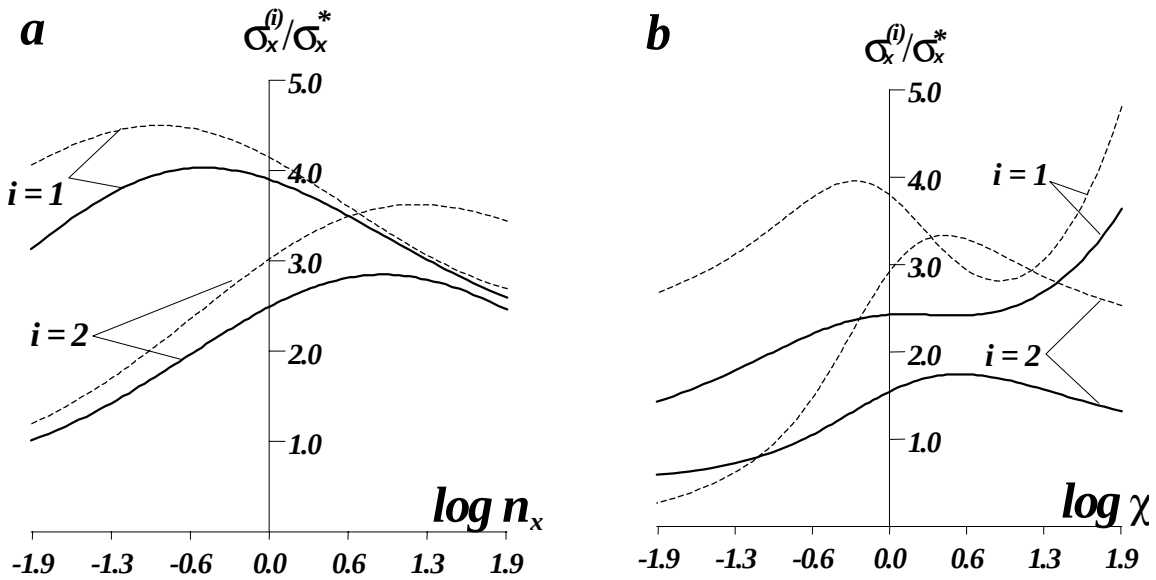

Fig. 4 Effect of the elastic (a) and geometric (b) asymmetry factors of a layered structure on the ratio of the bending stress (18) – the bold curves and (19) - the dashed curves

CONCLUSIONS

The analysis of the bending problem of a sandwich panel by a discrete model has revealed the distinctions of a local load effect on bending characteristics depending on the asymmetry of face layer elastic and geometric properties as well as the elasticity of a filler. It has been found that the face layer curvatures in the places of point force application are unlike classical values to a greater extent than bending stresses. The ratio of bending stresses to their classical values at a panel center is not monotonous function of the panel asymmetry parameters.
The results obtained as the curves, governing the effect of structural parameters at a panel local loading, supplement the work [6] in the engineering application of calculations for a sandwich structure asymmetry. They will be also available for the refinements concerning the selection of a panel structure [7] and comparing the project with its classical variant.

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