

# DESIGN METHODOLOGY FOR BUCKLING OF THIN-WALLED LAMINATED COMPOSITE BEAMS

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**SUMMARY:** Thin-walled open section beams are extensively used in many structural engineering applications, and these structural elements are currently being made of composite materials. In addition to weight saving and relative freedom from corrosion and fatigue, composite materials permit designs with fewer parts and fasteners, which tends to reduce total costs and promotes maintainability and readiness. Column design curves are presented for composite beams with channel cross sections. The results show buckling by pure bending and by bending-torsion coupled modes can occur and that transition among modes are governed by structural parameters.

**KEYWORDS:** Composite Beams, Buckling, Design

## INTRODUCTION:

Thin-walled open section beams are extensively used in many structural engineering applications, and these structural elements are currently being made of composite materials. In addition to weight saving and relative freedom from corrosion and fatigue, composite materials permit designs with fewer parts and fasteners, which tends to reduce total costs and promotes maintainability and readiness.

The most significant difference between closed and open cross section beams is the analysis of torsion. Also, the beam kinematics must include the nonlinear geometric effects, which permit the study of buckling. While general purpose finite element codes[1]-[4] and laminated plate theory[5,6] are available, the literature pertaining to the buckling behavior of thin-walled composite open section beams remain somewhat limited. Bauld and Tzeng[7] have presented a Vlasov[8] type theory for thin-walled composite open section composite beams, which is restricted to symmetric layups and does not account for transverse shear effects. Lo and Johnson[9] reported finite element results based on the theory in Reference 7. Also, they compared the numerical results with their experimental results. There was poor agreement between the two.

More recent work has been reported by Rehfield and Atilgan[10] and Atilgan and Rehfield[11] pertaining to thin-walled open section composite beams. Better theoretical correlation with experimentally obtained buckling loads is achieved in Ref. 10 and 11. We do not, however, consider the correlation to be as good as desired. The quality of the test specimens is a possible issue.

The objective of this paper, in contrast to analysis, is to develop design information for buckling resistance. Two types of parameters are involved. One is the material system and ply layups. The second is the slenderness of the column. Emphasis will be given to resin matrix composite materials, so no plastic buckling zone is considered. This permits two outcomes --- linearly elastic buckling or compressive crushing failure.

A significant finding is that transverse shear deformation is not very pronounced. Earlier work[12] would suggest otherwise, at least for closed sections. However, in the present work, transverse shear will be ignored based on this finding.

The theoretical models are based upon those of Refs. 10 and 11. The difference involves the neglect of transverse shear deformations and torsion-related warping. The neglect of these effects has been justified by analytical studies.

## Applications

Emphasis will be given to channel sections. This is in keeping with earlier work. A thin-walled channel section is shown in Fig. 1. The cross section appears in Fig. 2.

Ply layups will be composed of [0] and [ $\pm 45$ ] plies. Configurations will have one axis of symmetry. Consequently, two distinct types of buckling modes can occur. The first is a pure bending mode about an axis parallel to the web of the channel section. The second is a coupled bending-twisting mode. Which mode appears corresponds to the one with the lowest buckling strain level.

An example of a buckling design chart is shown in Fig. 3. The strain levels  $\varepsilon_z$  and  $\varepsilon_1$  correspond to the pure bending mode and the coupled bending-twisting mode, respectively. The parameter  $L / \sqrt{C_{66} / C_{11}}$  is a slenderness parameter that combines geometry, ply layups, ply stiffness, and stacking sequence. The parameter  $C_{66}$  is the bending stiffness associated with the bending mode. The extensional stiffness is  $C_{11}$ . Note that a transition from one mode to the other occurs as the slenderness parameter varies, as shown in Figs. 3-8.

The material system is AS1/3501-6 carbon-epoxy with the properties

$$E_{11} = 19.3 \times 10^6 \text{ psi}$$

$$G_{12} = 0.77 \times 10^6 \text{ psi}$$

$$E_{22} = 1.10 \times 10^6 \text{ psi}$$

$$\nu_{12} = 0.35$$

The design parameters that are varied are the size of the flanges and the number of [0] plies in the flanges. The compressive strain level to failure is 0.01.

The sample calculations illustrate the transition of buckling modes and the identification of the critical modes.

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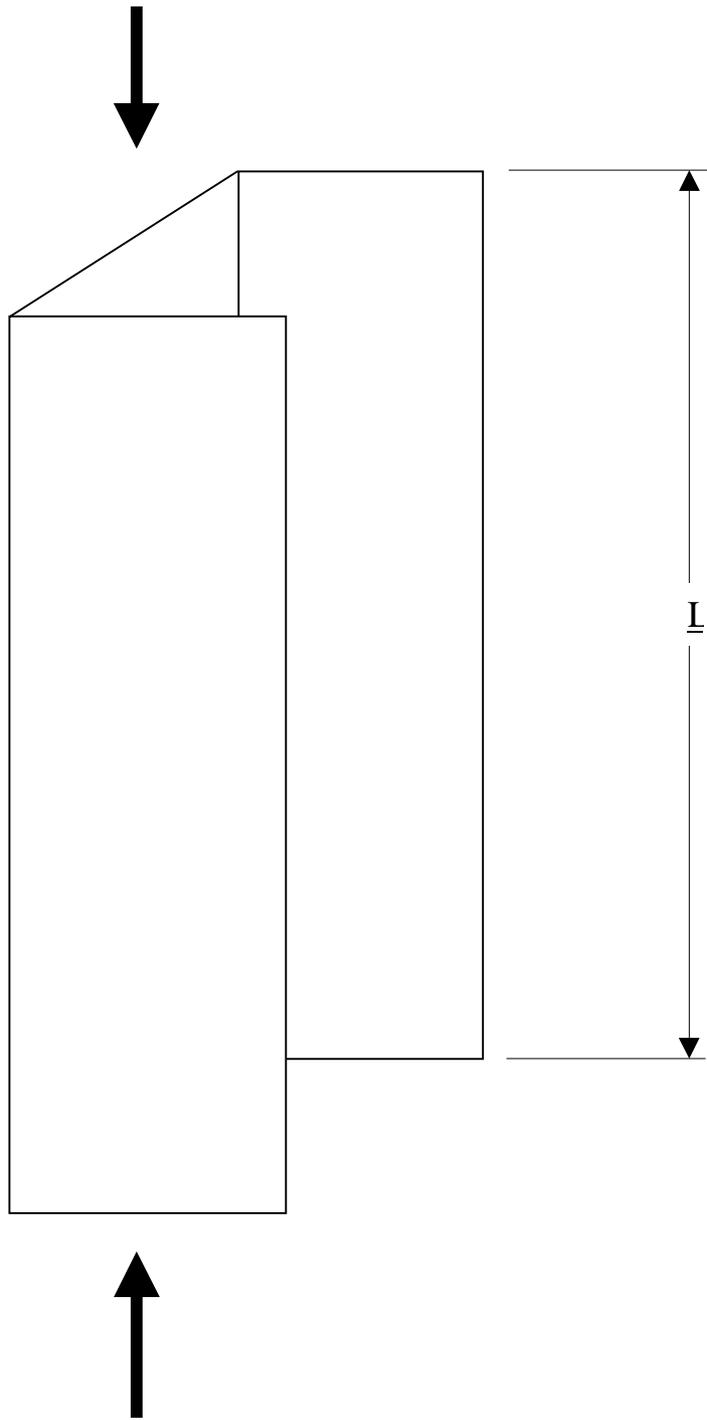


Fig. 1: Thin-walled composite column with C-Section

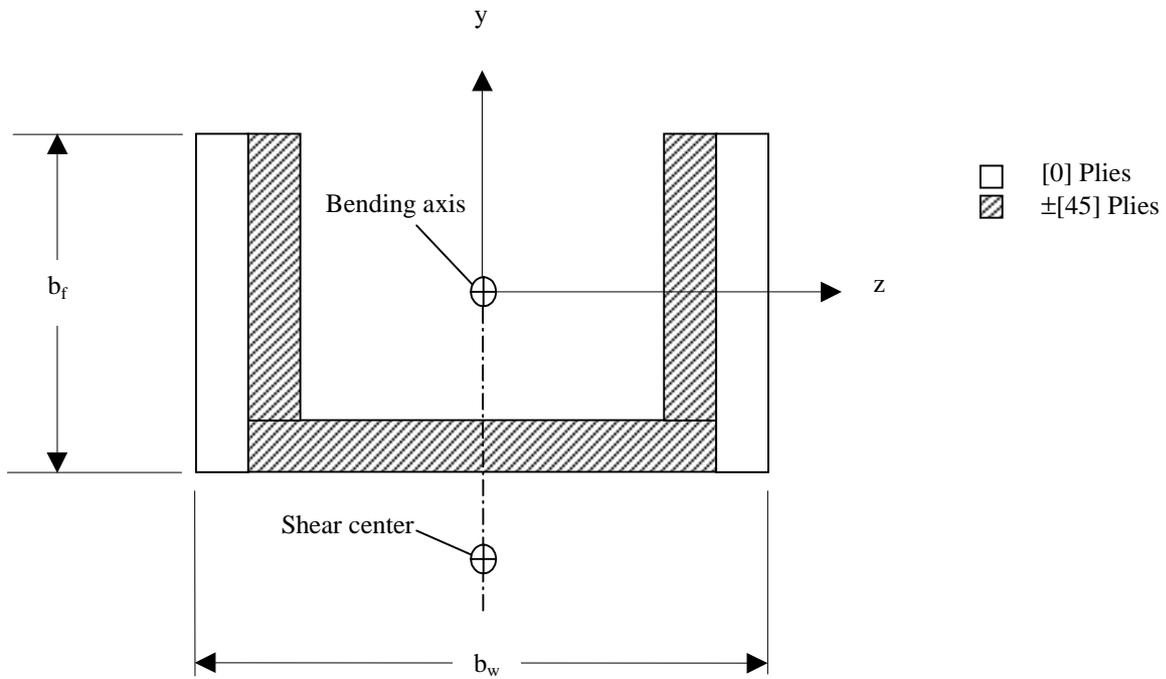


Fig. 2: Cross-section layout

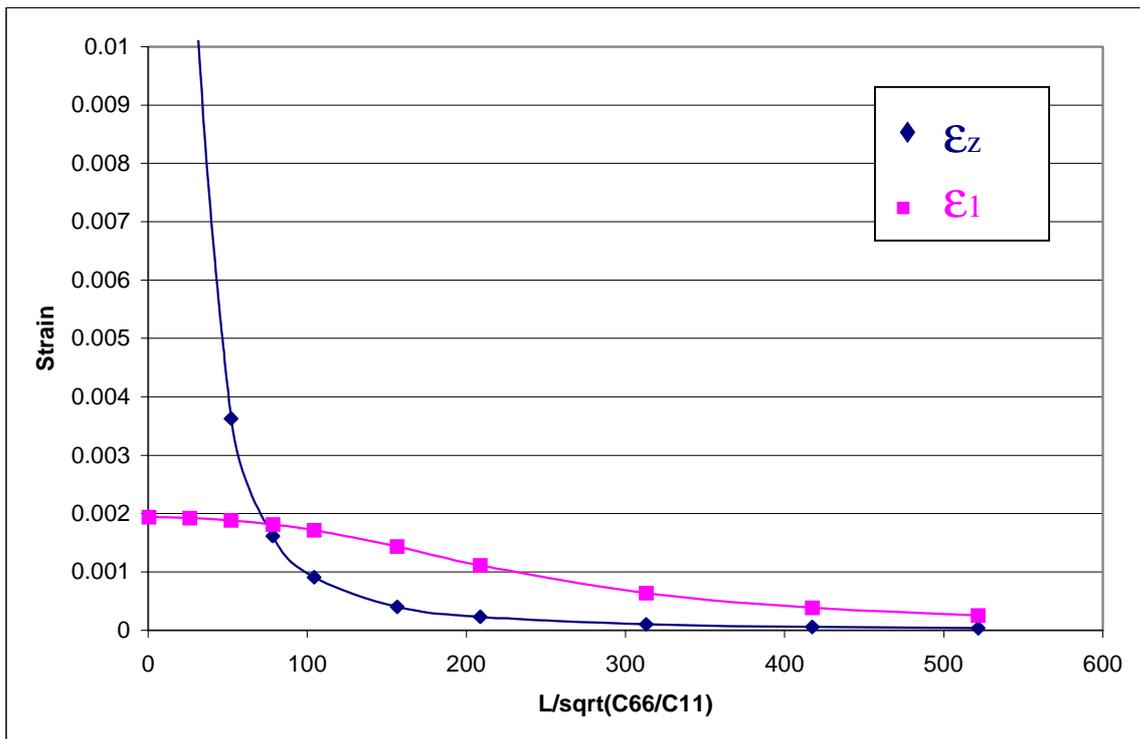


Fig. 3: Buckling strains with two 0-degree plies and 0.6" flange length

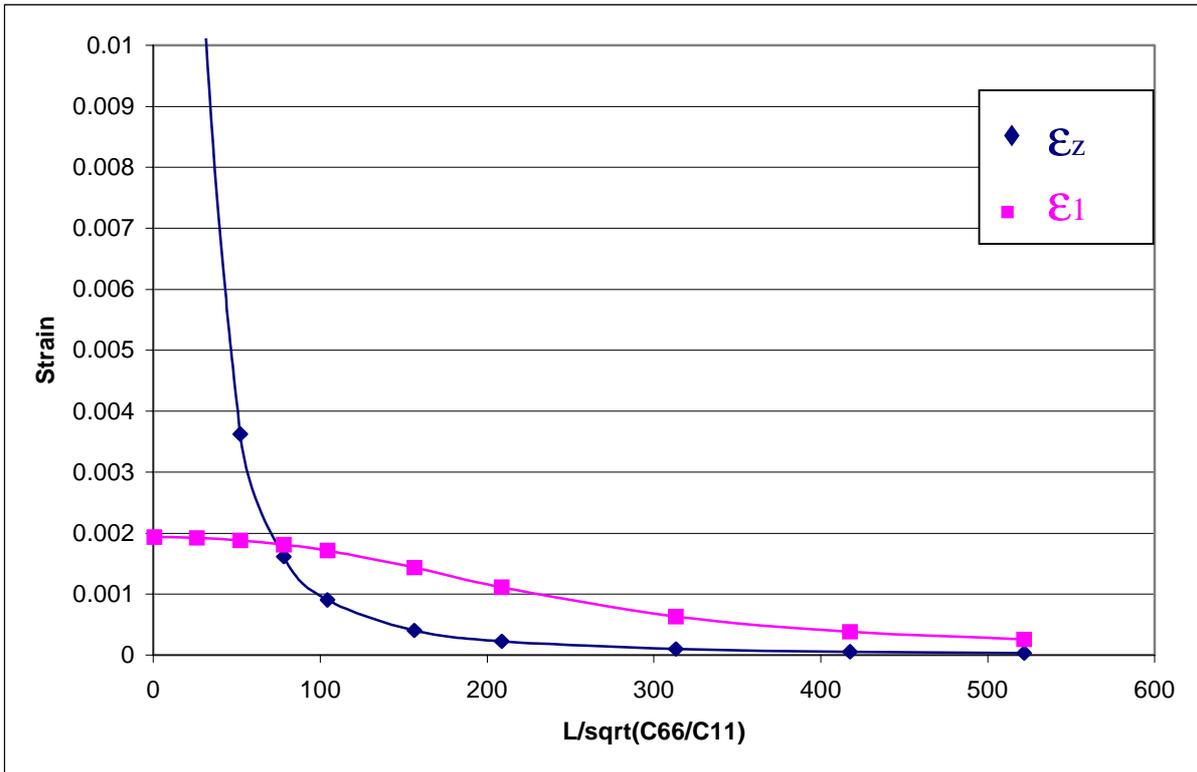


Fig. 4: Buckling strains with four 0-degree plies and 0.6" flange length

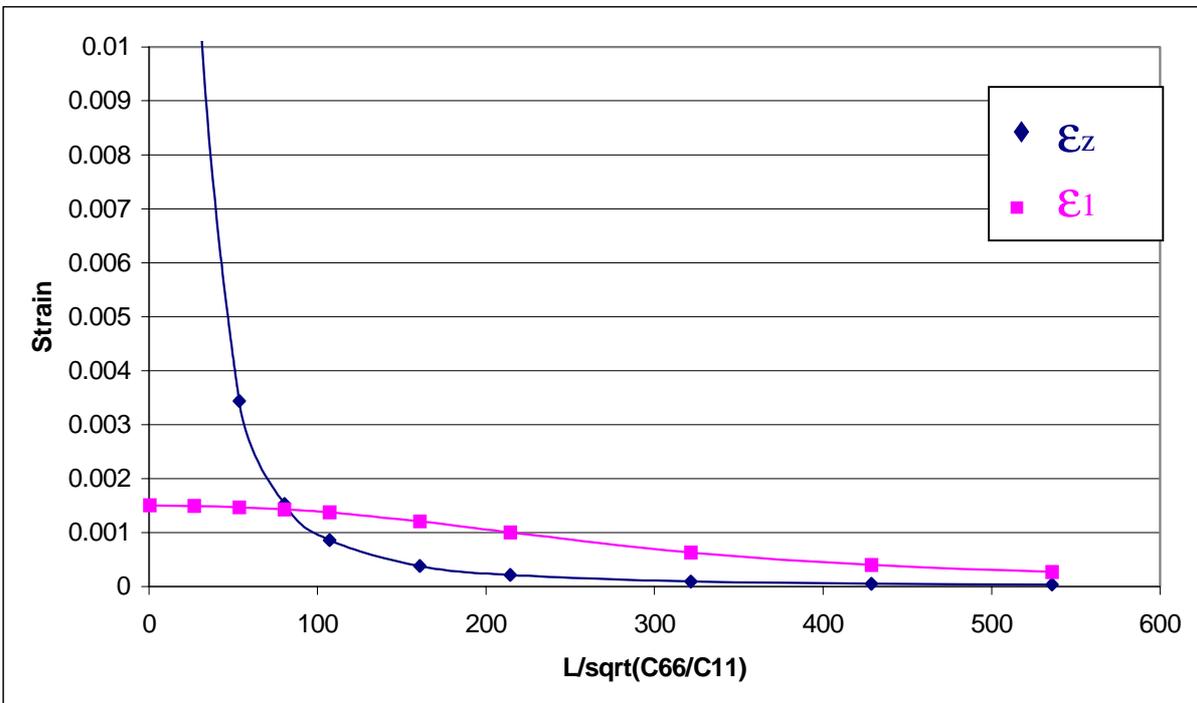


Fig. 5: Buckling strains with six 0-degree plies and 0.6" flange length

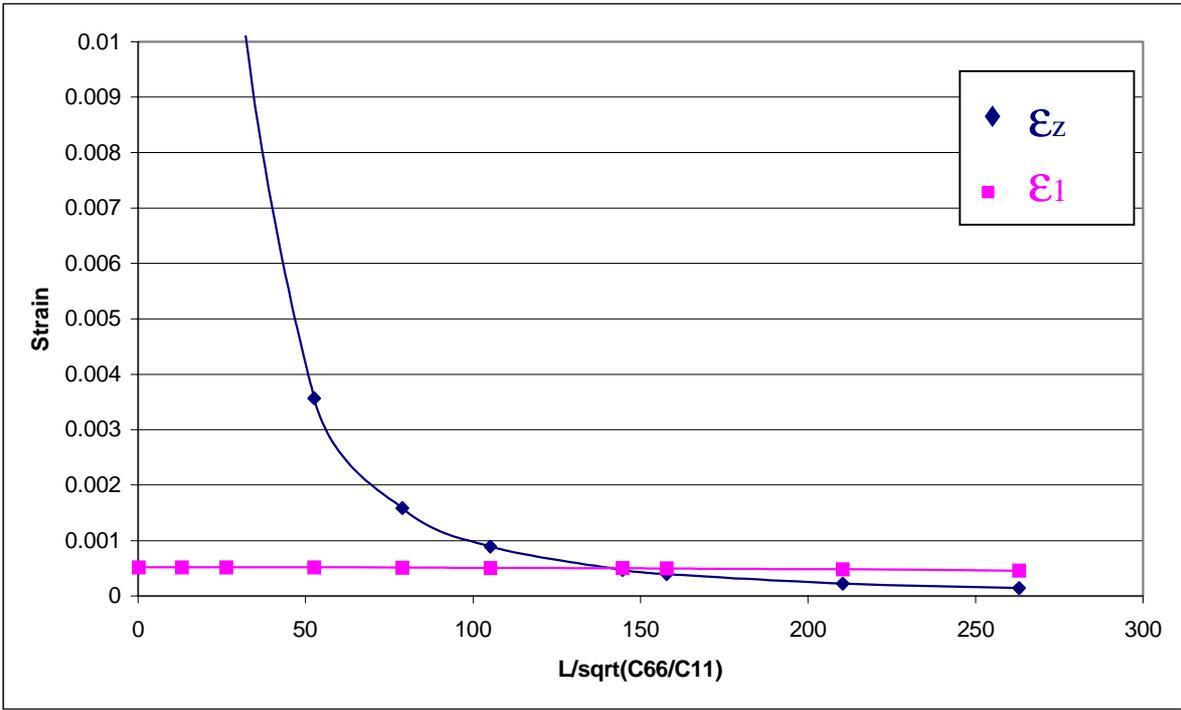


Fig. 6: Buckling strains with two 0-degree plies and 1.2" flange length

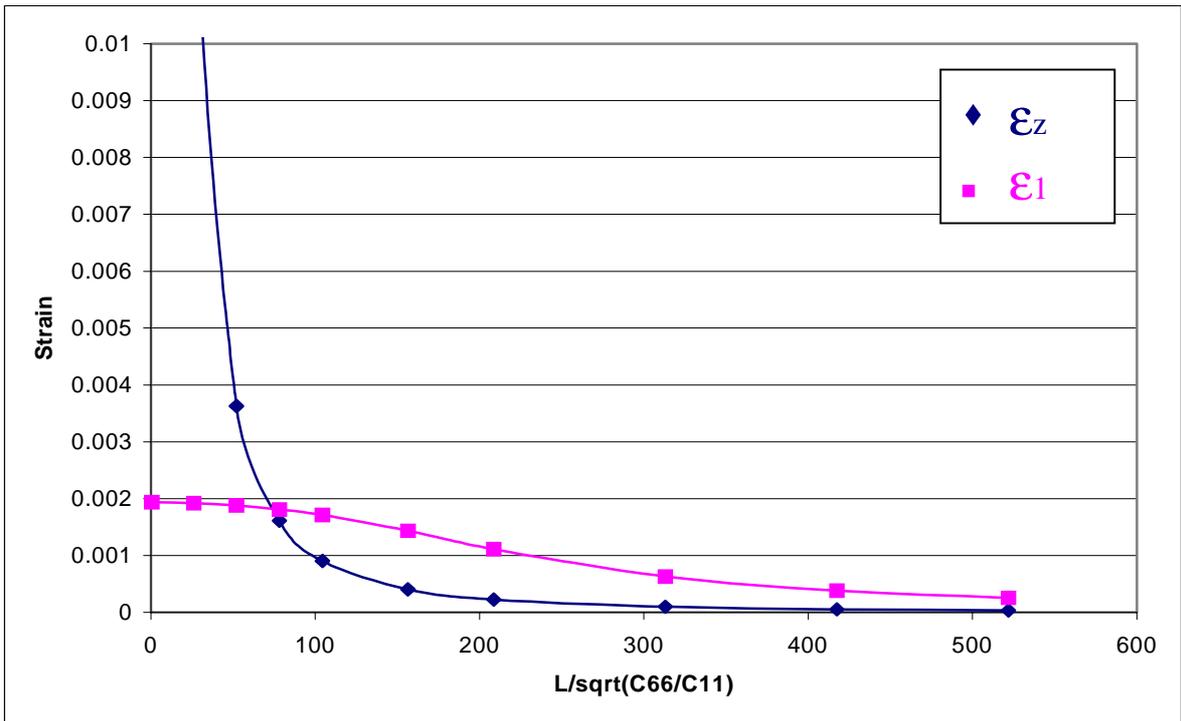


Fig. 7: Buckling strains with four 0-degree plies and 1.2" flange length

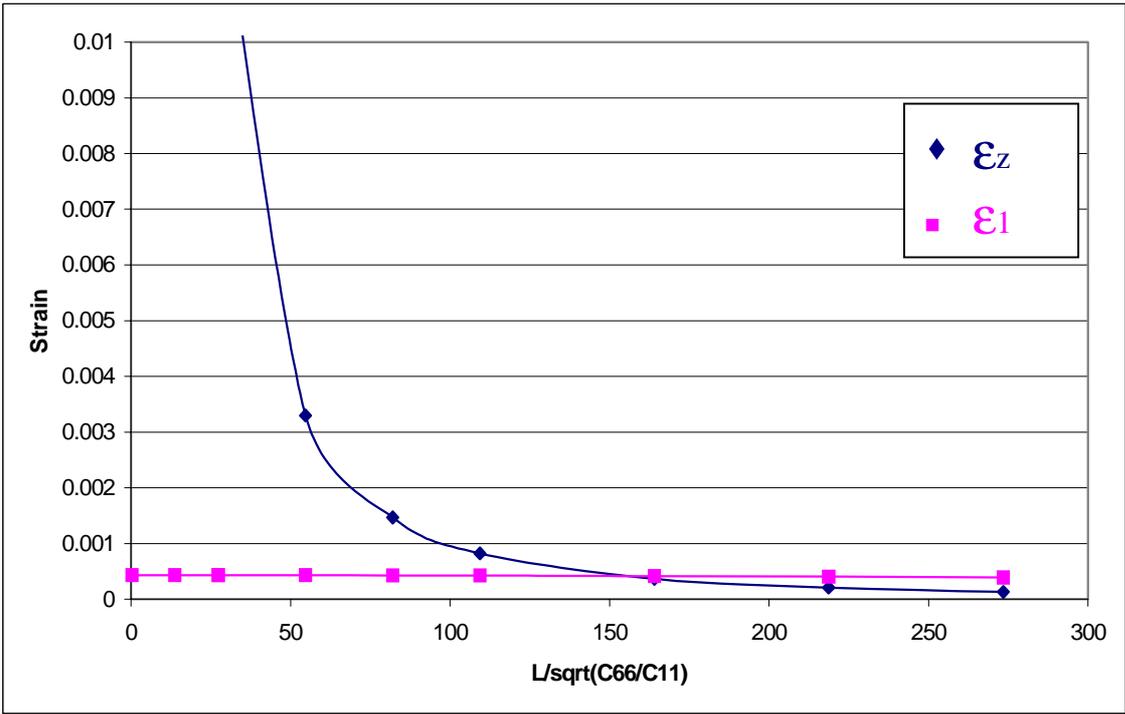


Fig. 8: Buckling strains with four 0-degree plies and 1.2" flange length