DESIGN OPTIMISATION OF LAMINATED CYLINDERS FOR MAXIMUM BUCKLING STRENGTH

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SUMMARY: The design optimisation of simply supported laminated circular cylindrical shells subjected to a combination of axial and torsional buckling loads is considered in this paper. The objective is defined as the maximisation of a performance index specified by the sum of the non-dimensionalised weighted loadings, and the effect of cylinder radius, length and weighting are investigated.

KEYWORDS: design optimisation, laminated cylinders, maximum buckling strength

INTRODUCTION

Composite materials have seen increasing applications in various fields of engineering such as the marine and aerospace industries. This is mainly due to the high strength and stiffness to weight ratios that these materials can offer [I]. Another advantage of these materials over conventional materials is the possibility of tailoring their properties to the specific requirements of a given application. This is particularly useful when the composite structure is subjected to different combinations of loading conditions. In these cases, multiobjective design techniques can maximise the performance of such structures, and lead to efficient designs.

When thin cylindrical shells are subjected to axial or torsional loading conditions, they may fail by buckling. Since these structures have extensive practical applications, consideration of this type of failure during the design phase is important. In terms of design optimisation, with symmetrically laminated composite structures, the designer can select the layer fibre angle as his optimizing variable to maximize a given objective function. When these structures are subjected to multiple buckling loading conditions such as a combination of axial compression and torsion, a multiobjective design approach can be formulated in order to ensure an optimally designed structure. Various researchers have considered the design of thin laminated cylindrical shells. Tripathy and Rao [2], using the finite element method, considered optimal layups for laminated shells under axial buckling loads. Multiobjective design studies of composite structures include those by Adali et al [3], who studied the optimal design of laminated cylindrical shells under torsional, axial, external and internal pressure loadings. In this study, the multiobjective optimal design of simply supported symmetrically laminated cylindrical shells for a weighted combination of maximum axial and torsional buckling loads is investigated. The layer fibre angle is chosen as the optimizing variable, and the effects of cylinder length, radius, wall thickness and loading weighting are numerically investigated.

BASIC EQUATIONS

Consider a laminated circular cylindrical shell with length L, radius R, and total thickness H under an axial load N_x and torsional load T_{cr}, as shown in Figure 1. The shell has a symmetric layup consisting of K layers of equal thickness t. The structure is referenced in an orthogonal coordinate system (x,y,z), where x is the longitudinal, y the circumferential, and z the radial direction. The displacement components $u, v,$ and w are along the x, y and z directions, respectively.

The fiber angle is defined as the angle between the fibre direction and the longitudinal (x) axis. The fiber orientations are symmetric with respect to the mid-plane of the shell and are given by $\theta_k = (-1)^{k+1}\theta$ for $k \leq K/2$ and $\theta_k = (-1)^k\theta$ for $k \geq K/2 + 1$, where $k = 1, \ldots) K$ with k being the layer number.

The critical torsional buckling load for a simply supported cylindrical laminated shell of K layers is [4]:

$$T_{cr} = 21.75(D_{22}')^{5/8}\left(\frac{A_{11}A_{22} - A_{12}^2}{A_{22}}\right)^{3/8}\frac{R^{5/4}}{L^{1/2}} \tag{1}$$

A restriction on this equation is

$$\left(\frac{D_{22}}{D_{11}}\right)^{5/6}\left(\frac{A_{11}A_{22} - A_{12}^2}{12A_{22}D_{11}}\right)^{1/2}\frac{L^2}{R} \geq 500 \tag{2}$$

The extensional and bending stiffnesses A_{ij} and D_{ij} are given by

$$\mathbf{A,} = \sum_{k=1}^{K}\overline{Q}_{ij}^{(k)}(t_{k+1} - t_k) \tag{3}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{K}\overline{Q}_{ij}^{(k)}(t_{k+1}^3 + t_k^3) \tag{4}$$

where $\overline{Q}_{ij}^{(k)}$ are components of the transformed reduced stiffness matrix for the $k-th$ layer, with t_k denoting the coordinate of the interface between the layers k and $k+1$. For the same shell, the axial buckling load is [4]:

$$N_x(m,n,\theta) = \frac{1}{\lambda_m^2}\frac{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix}} \tag{5}$$

where m and n are the numbers of half waves in the buckle pattern in the axial and circumference direction, respectively, and

$$\begin{aligned} C_{11} &= A_{11}\lambda_m^2 + A_{66}\lambda_n^2 \\ C_{22} &= A_{22}\lambda_n^2 + A_{66}\lambda_m^2 \\ C_{33} &= D_{11}\lambda_m^4 + 2(D_{12} + 2D_{66})\lambda_m^2\lambda_n^2 + D_{22}\lambda_n^4 + A_{22}/R^2 \end{aligned} \tag{6}$$

$$C_{12} = C_{21} = (A_{12} + A_{66})\lambda_m\lambda_n$$
$$C_{13} = C_{31} = A_{12}\lambda_m/R$$
$$C_{23} = C_{32} = A_{22}\lambda_n/R$$

where $\lambda_m = m\pi/L$ and $\lambda_n = n/R$. *The* critical buckling load is N_{cr} computed by minimising N_x over the integers m and n, viz.

$$N_{cr} = \min_{m.n} N_x(m, n, \theta) \tag{7}$$

PROBLEM FORMULATION

The problem involves the objectives of maximising the torsional buckling load T_{cr}, and the axial buckling load N_{cr}, given in the previous section. Such problems can be handled by introducing a multiobjective design formulation in which both objectives are taken into account in the optimisation process. One method of achieving this is to seek Pareto optimal solutions of the design problem [5]. This can be achieved by introducing a performance index J(0) consisting of weighted sums of the single objectives, viz.

$$J(\theta) = \alpha T_{cr}/T_0 + (1-\alpha)N_{cr}/N_0 \tag{8}$$

where α is a given weighting factor $0 \leq \alpha \leq 1$, and N_0 and T_0 are the critical axial and torsional buckling loads at $\theta = 0$. For α = 0 and $\alpha = 1$, the solution of the design problem leads to single objective designs. The design optimisation is achieved by optimally determining the fiber orientations θ for a given α.

The design problem can be stated as

$$J_{\max} \equiv^{\Delta} \max_{\theta}[J(\theta) = \max_{\theta}[\alpha T_{cr}(\theta)/T_{cr}(0°)(1-\alpha)N_{cr}(\theta)/N_{cr}(0°)] \tag{9}$$

subject to

$$0° \leq \theta \leq 90° \tag{10}$$

where $T_{cr}(\theta)$ and $N_{cr}(\theta)$ are determined from equations (5) and (6). The optimisation procedure involves the stages of evaluating the performance index J(0) for a given θ and improving the fiber orientation to maximise *J,* at a given set of weightings. Thus, the computational solution consists of successive stages of analysis and optimisation until a convergence is obtained and the optimal angle θ_{opt} is determined within a specified accuracy. In the optimisation stage, the Golden *Section* method [6] is employed.

RESULTS

The results reported here are for cylinders with eight symmetric layers. In the first instance, the length of the shell was varied from 5 to $25m$, in 5m intervals, with the radius set at R = $1m$. The total wall thickness was specified as $0.05m$. The on-axis orthotropic material constants used for the analysis of these shells are for T300/5208 graphite epoxy and are given by Q_{xx} = 181.81 GPa, Q_{yy} = 10.34 Gpa, Q_{xy} = 2.90 Gpa and Q_{ss} = 7.17 Gpa. The dependence of the optimal fiber angle on the length is shown in Table 1 where α = 0.5. It is interesting to note that at L = $20m$, θ_{opt} decreases to 76.54" (from 74.26" at L = $15m$), and then increases to 90° at L = 25m.

$L(m)$	θ_{opt}	N_{cr}/N_o	T_{cr}/T_o
5	73.85°	1.70	1.92
10	75.88"	1.62	1.94
15	74.26"	1.66	1.92
20	76.54"	1.58	1.96
25	90	1.28	2.04

Table 1. Dependence of the optimal fibre angle on the cylinder length with R = $1m$,α = $0.5m$

Next the dependence of θ_{opt}on the cylinder radius R is investigated with the length specified as $L = 10m$ (Table 2) with α = 0.5. It is interesting to note that θ_{opt} increases as R increases.

$R(m)$	θ_{opt}	N_{cr}/N_o	T_{cr}/T_o
2	69.15"	1.62	1.84
3	71.77"	1.72	1.88
4	'12.09"	1.74	1.90
5	75.05"	1.66	1.94
7.5	75.32"	1.62	1.94
10	75.88"	1.50	1.94

Table 2. Dependence of the optimal fibre angle on the cylinder radius with L = $1m$,α = $0.5m$

Finally the dependence of the optimal fibre angle on the weighting factor α is given in Table 3. This relationship is monotonic, increasing from θ_{opt} = 34°at α = 0.0 (design for maximum N_{cr}) to 90°at α = 1.0 (design for maximum T_{cr}). It is observed that at single-objective designs, the other objective becomes quite low. This drawback can be overcome by choosing a compromise design obtained at around a = 0.5.

α	υ_{opt}	N_{cr}/N_o	T_{cr}/T_o
0.0	θ 34.23°	1.80	1.20
0.1	34.23"	1.80	1.20
0.2	34.23"	1.80	1.20
0.3	74.38"	1.63	1.94
0.4	75.17	1.63	1.94
0.5	75.88"	1.62	1.94
0.6	82.43"	1.53	2.03
0.7	87.41"	1.44	2.03
0.8	90"	1.43	2.05
0.9	90"	1.42	2.05
1.0	90"	1.42	2.05

Table 3. Dependence of the optimal fibre angle on the weighting factor α with L = 15m and R = lm.

CONCLUSION

The design optimisation of simply supported laminated circular cylindrical shells subjected to a combination of axial and torsional buckling loads is considered in this paper. The objective is defined as the maximjsation of a performance index specified by the sum of the non-dimensionalised weighted loadings, and the effect of cylinder radius, length and weighting are investigated.

REFERENCES

1. Walker, M., Reiss, T. & Adali, S., 'Multiobjective design of laminated cylindrical shells for maximum torsional and axial buckling loads', *Computers & Structures,* Vol. 62, 2, 1996, pp. 237 - 242.

2. Tripathy, B. & Rao, K. P., 'Stiffened composite axisymmetric shells - optimum layup for buckling by ranking', *Computers & Structures,* Vol. 46, 1993. pp. 299 - 309.

3. Adali, S., Richter, A. & Verijenko, V. E., 'Multiobjective design of laminated cylindrical shells for maximum pressure and buckling load', *Microcomputers in Civil Engineering,* (in press).

4. Vinson, J. R. & Sierakowski, R. L. *The Behaviour of Structures Composed of Composite Materials,* Martinus Nijhoff, Dordrecht, 1986.

5. Adali, S., 'Pareto Optimal Design of Beams Subjected to Support Motions', Computers *& Structures,* Vol. 16, 1983, pp. 297 - 303.

6. Haftka, R. 'I'. & Gürdal, Z. *Elements of Structural Optimisation,* 3rd Edition, Kluwer Academic Publishers, Dordrecht, *1992.*

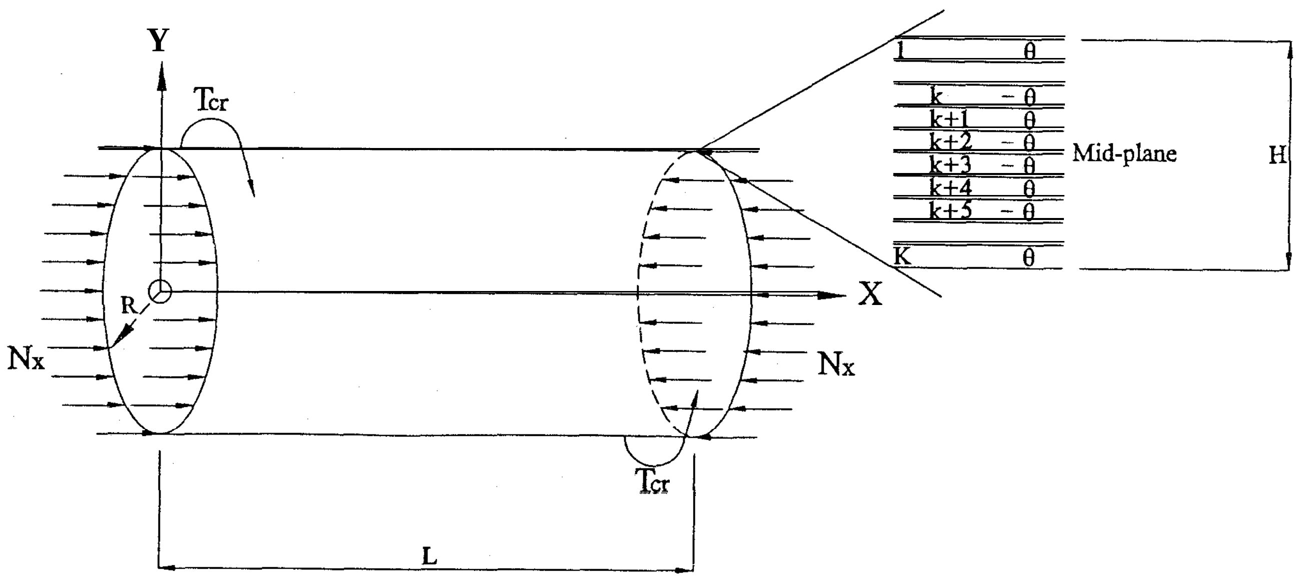

Figure 1. Geometry and loading of cylindrical shell.