ANALYTICAL MODEL FOR DEBONDED INTERFACES ASSOCIATED WITH FIBRE FRACTURES OR MATRIX CRACKS

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SUMMARY : An analytical stress transfer model is described enabling an estimate to be made of the stress and displacement fields that are associated with fibre fractures or matrix cracks in unidirectional fibre reinforced composites. This model represents a great improvement on existing shear-lag based methodologies. The model takes account of thermal residual stresses, and is based on a single assumption, namely, that the axial stress in the fibre is independent of the radial coordinate, and similarly for the matrix. A representation for both the stress AND displacement fields is derived that satisfies exactly the equilibrium equations, the required interface continuity equations for displacement and tractions, and all stress-strain equations except for the one that relates to axial deformation. In addition, the representation is such that the Reissner energy functional has a stationary value provided that averaged axial stress-strain relations for the fibre and matrix are satisfied. The improved representation is fully consistent with variational mechanics and provides BOTH the stress and displacement distributions in the fibre and the matrix. For an isolated fibre fracture or matrix crack, interface debonding is considered where two types of condition are investigated. Firstly, it is assumed that the shear stress is uniform within the debonded region. Secondly, it is assumed that stress transfer in the debonded region is controlled by Coulomb friction. Preliminary predictions are made for carbon fibre reinforced epoxy composites.

KEYWORDS : Interface, debonding, stress, displacement, friction, variational mechanics.

INTRODUCTION

A prime technical objective in the composites field is to develop reliable methods of measuring the properties of fibre/matrix interfaces in unidirectionally reinforced continuous fibre composites. While several methods are currently used (see for example [1]), it is not clear that the techniques provide results which are indicative of the state of the interface rather than being a characteristic of the test method used. One reason for this uncertainty arises from the use of modelling techniques that are not adequate because too many approximations are made when developing the models. There is a great need therefore to develop models that are representative of the conditions expected when a fibre fractures or the matrix cracks leading to stress transfer at the fibre/matrix interface, and possibly to localised shear yielding at the interface, or to fibre/matrix debonding involving some kind of frictional contact. While such models will be able to improve the reliability of interpreting the results of some interfacial tests (e.g. fragmentation and pull-out tests), they will also be invaluable when attempting to predict some of the effects of interphase properties on the macroscopic behaviour of unidirectional

composites, e.g. by providing detailed localised stress and deformation predictions that can be used in Monte Carlo simulations modelling the loading and progressive failure of such composites.

The objective of this paper is to show analytically how stress transfer between two concentric cylinders can be estimated where the inner cylinder represents the fibre and the outer cylinder represents the matrix. Such stress transfer will arise when a fibre breaks or the matrix cracks in a unidirectional composite. An axisymmetric model is considered subject to applied axial load distributions that gives rise to axial stress transfer between the cylinders through the action of distributed shear stresses on the interface between the two cylinders. The model is constructed so that the cylinders can be made of different transversely isotropic materials. The model takes full account of the effects of the thermal residual stresses induced in the system during manufacture as a result of the difference in thermal expansion behaviour of the fibre and matrix.

The solution technique, described in much more detail in [2], is an improvement of an earlier stress transfer model [3]. The solution of the stress transfer problem involves the development of an ordinary differential equation that can be solved by analytical methods, and subsequent calculation of the stress and displacement distributions throughout the system. The axisymmetric model of stress transfer leads to stationary values of the Reissner energy functional [4] so that the stress and displacement distribution derived from the model would also result from carrying out a corresponding variational calculation. Thus the stress transfer model is the best that can be developed based upon the single fundamental assumption that axial stresses in each cylinder are independent of the radial coordinate. Nairn [5] has developed a solution for the stress field only for the case when a variational calculation is carried out based on the principle of minimising the complementary energy rather than seeking stationary values of the Reissner energy functional.

Stress transfer between fibre and matrix will be considered for the case when either a fibre is uniformly fragmented, or when the matrix has cracked uniformly so that the matrix cracks are normal to the fibre axis. Interaction effects between neighbouring fractures are ignored here, although the analysis can easily be generalised to take such interaction into account. A set of cylindrical polar coordinates (r, θ, z) is introduced such that the origin lies on the common axis of the cylinders (with the z-axis directed along the axis of the cylinders) at the mid-point between two neighbouring fibre fractures or two neighbouring matrix cracks. The inner cylinder represents the fibre which has radius R. The outer cylinder has radius a such that $a = R / \sqrt{V_f}$ where V_f is the volume fraction of the composite represented by the concentric cylinder model ensuring that the fibre volume fraction for the concentric cylinder model is the same as that of the composite being modelled.

Field equations

For axisymmetric problems the following equilibrium equations must be satisfied for the fibre and matrix,

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 , \qquad (1)$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r} = 0 . \qquad (2)$$

The fibre and matrix are regarded as transverse isotropic solids so that the stress-strain-temperature relations, in terms of the axial and transverse moduli E, Poisson's ratios ν, shear modulus μ and thermal expansion coefficients α are of the form

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} = \frac{1}{E_T}\sigma_{rr} - \frac{\nu_T}{E_T}\sigma_{\theta\theta} - \frac{\nu_A}{E_A}\sigma_{zz} + \alpha_T \Delta T , \tag{3}$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} = -\frac{\nu_T}{E_T}\sigma_{rr} + \frac{1}{E_T}\sigma_{\theta\theta} - \frac{\nu_A}{E_A}\sigma_{zz} + \alpha_T \Delta T , \tag{4}$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} = -\frac{\nu_A}{E_A}\sigma_{rr} - \frac{\nu_A}{E_A}\sigma_{\theta\theta} + \frac{1}{E_A}\sigma_{zz} + \alpha_A \Delta T , \tag{5}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) = \frac{\sigma_{rz}}{2\mu_A} , \quad \varepsilon_{\theta z} = \frac{\sigma_{\theta z}}{2\mu_A} , \quad \varepsilon_{r\theta} = \frac{\sigma_{r\theta}}{2\mu_T} , \tag{6}$$

$$\text{where} \quad E_T = 2\mu_T(1+\nu_T) \quad \text{but} \quad E_A \neq 2\mu_A(1+\nu_A) . \tag{7}$$

Following Nairn [5], when σ_{zz} is independent of r, the displacement u_r is compatible with the stress-strain relations (3) and (4) if the following compatibility equation for stresses is satisfied :

$$(1 + \nu_T)\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \frac{\partial}{\partial r}(\sigma_{\theta\theta} - \nu_T\sigma_{rr}) . \tag{8}$$

Interfacial and radial boundary conditions

At the interface $r = R$ between the cylinders, the following continuity conditions must be satisfied for all values of z :

Perfect interface bonding :

$$\begin{aligned} \sigma^f_{rr}(R, z) &= \sigma^m_{rr}(R, z), \qquad \sigma^f_{rz}(R, z) = \sigma^m_{rz}(R, z), \\ u^f_r(R, z) &= u^m_r(R, z), \qquad u^f_z(R, z) = u^m_z(R, z). \end{aligned} \tag{9}$$

Interface debonding :

$$\begin{aligned} \sigma^f_{rr}(R, z) &= \sigma^m_{rr}(R, z), \qquad \sigma^f_{rz}(R, z) = \sigma^m_{rz}(R, z), \\ u^f_r(R, z) &= u^m_r(R, z), \qquad \sigma^f_{rz}(R, z) = \lambda\,[\theta\tau + \phi\eta\sigma^f_{rr}(R, z)]. \end{aligned} \tag{10}$$

In (10), τ and η are regarded as material constants where θ, ϕ = 0 or 1, and where λ = -1 or 1. By selecting θ = 1 and ϕ = 0 the boundary conditions (10) correspond to the those for an

interface subject to a uniform interfacial shear stress $\pm\tau$. By selecting $\theta = 0$ and $\phi = 1$ the boundary conditions correspond to the those for an interface subject to the Coulomb law of friction where the parameter η is the friction coefficient. By selecting $\lambda = 1$ the boundary conditions (10) correspond to those for a matrix crack, and by selecting $\lambda = -1$ the conditions correspond to those for a fibre fracture. The boundary conditions (10) assume that on debonding at the interface the fibre and matrix remain in contact with one another and give rise to stress transfer between the cylinders where the fibre may slip relative to the matrix. If mechanical contact is lost then clearly the stress components σ_{rr} and σ_{rz} are both zero and the displacement component u_r is discontinuous across such an interface. As no stress transfer occurs at such an interface, this case will not be considered in this paper. On the external surface $r = a = R / \sqrt{V_f}$ of the outer cylinder the following boundary conditions are often imposed (especially when modelling for fibre fragmentation)

$$\sigma_{rr}(a, z) = \sigma_T , \quad \sigma_{rz}(a, z) = 0 , \tag{11}$$

where σ_T 1 is a uniform transverse stress applied to the external surface of the outer cylinder. In regions away from the loading mechanism, and any matrix crack or fibre fracture, it is assumed that

$$u_z^f \equiv u_z^m \equiv \varepsilon z , \tag{12}$$

where ε is the axial strain in such regions. The relation (12) is valid for all values of z when the system is undamaged, as shown in Appendix A of Ref. [2].

The boundary condition (11) for the radial stress could be replaced by a corresponding radial displacement condition which would be appropriate when modelling fibre fractures embedded with a unidirectional composite. The radial component could be selected to be the radial displacement that would arise in an undamaged composite (see [6] for a more detailed discussion).

Representation for the stress and displacement fields

The solution for the case when there is no damage present in the form of fibre fractures, matrix cracks or debonded interfaces is given in Appendix A of Ref. [2]. The approach to developing a solution for a damaged system is to express the solution as a sum of the undamaged solution and a perturbation solution arising from the damage. For the fibre region $0 \le r \le R$ the stress field when damage is present is assumed to be of the following form equivalent to that assumed by Nairn [5]

$$\sigma_{zz}^f = \sigma_f - C(z) , \tag{13}$$

$$\sigma_{rz}^f = \frac{1}{2} C'(z) r , \tag{14}$$

$$\sigma_{rr}^f = -\frac{1}{16}\left(3 + \nu_T^f\right) C''(z) r^2 + R_f(z) + \sigma_T - V_m \frac{\phi}{R^2} , \tag{15}$$

$$\sigma^f_{\theta\theta} = -\frac{1}{16}\left(1+3\nu^f_T\right)C''(z)r^2 + R_f(z) + \sigma_T - V_m\frac{\phi}{R^2} \; ,$$

(16)

where σ_f and ϕ relate to the undamaged solution, σ_f being the uniform axial fibre stress in an undamaged composite subject to the same loading conditions and temperature. The functions C(z) and $R_f(z)$ are regarded as being identically zero when no form of damage is present. For the matrix region $R \le r \le a$, the stress field is assumed to be of the following form, again equivalent to that used by Nairn [5]

$$\sigma^m_{zz} = \sigma_m + \frac{V_f}{V_m}C(z) \; , \tag{17}$$

$$\sigma^m_{rz} = \frac{C'(z)}{2V_m}\left[\frac{R^2}{r} - V_f r\right] ,$$

(18)

$$\sigma^m_{rr} = \left[\left(3+\nu^m_T\right)\frac{r^2}{a^2} - 4\left(1+\nu^m_T\right)\ln\frac{r}{a} - 2\left(1-\nu^m_T\right)\right]\frac{R^2}{16V_m}C''(z) + R_m(z) - \frac{S_m(z)}{r^2} + \sigma_T + \phi\left(\frac{1}{a^2}-\frac{1}{r^2}\right),$$

(19)

$$\sigma^m_{\theta\theta} = \left[\left(1+3\nu^m_T\right)\frac{r^2}{a^2} - 4\left(1+\nu^m_T\right)\ln\frac{r}{a} + 2\left(1-\nu^m_T\right)\right]\frac{R^2}{16V_m}C''(z) + R_m(z) + \frac{S_m(z)}{r^2} + \sigma_T + \phi\left(\frac{1}{a^2}+\frac{1}{r^2}\right),$$

(20)

where σ_m is the uniform axial matrix stress in an undamaged composite subject to the same loading conditions and temperature. The functions $R_m(z)$ and $S_m(z)$ are regarded as being identically zero when no form of damage is present.

On using (3) or (4), together with (13), (15) and (16), the corresponding representation for the displacement component u^f_r, for the fibre region $0 \le r \le R$, is given by

$$\frac{u^f_r}{r} = -\frac{1-\nu^f_T}{32\mu^f_T}C''(z)r^2 + \frac{\nu^f_A}{E^f_A}C(z) + \frac{1-\nu^f_T}{E^f_T}R_f(z) + A_f \; ,$$

(21)

where A_f is defined in [2]. On using (6), (14) and (21) it can be shown on integrating with respect to r that for $0 \le r \le R$,

$$u_z^f = -\frac{1}{2}\left[\left(\frac{\nu_A^f}{E_A^f} - \frac{1}{2\mu_A^f}\right)C'(z) + \frac{1-\nu_T^f}{E_T^f}R_f'(z)\right]\left(r^2 - R^2\right)$$
$$+ \frac{1-\nu_T^f}{128\mu_T^f}C'''(z)\left(r^4 - R^4\right) + H_f(z) + \varepsilon z\ , \tag{22}$$

where $H_f(z) + \varepsilon z \equiv u_z^f(R, z)$ arises from the integration representing the axial displacement distribution in the fibre along the interface. The function $H_f(z) \equiv 0$ when the system is in an undamaged state. Similarly, the corresponding displacement components for the matrix region $R \le r \le a$ are given by

$$\frac{u_r^m}{r} = \left[\frac{1}{2}\frac{r^2}{a^2} - 2\ln\frac{r}{a} + 1\right]\frac{1-\nu_T^m}{16\mu_T^m}\frac{R^2}{V_m}C''(z) - \frac{\nu_A^m}{E_A^m}\frac{V_f}{V_m}C(z)$$
$$+ \frac{1-\nu_T^m}{E_T^m}R_m(z) + \frac{S_m(z)}{2\mu_T^m r^2} + A_m + \frac{\phi}{2\mu_T^m r^2}\ , \tag{23}$$

$$u_z^m = \frac{1-\nu_T^m}{16\mu_T^m}\frac{R^2}{V_m}C'''(z)\,r^2\ln\frac{r}{R} + \left[\frac{R^2}{2\mu_A^m V_m}C'(z) - \frac{S_m'(z)}{2\mu_T^m}\right]\ln\frac{r}{R}$$
$$+ \frac{1}{2}\left[\left(\frac{\nu_A^m}{E_A^m} - \frac{1}{2\mu_A^m}\right)\frac{V_f}{V_m}C'(z) - \frac{1-\nu_T^m}{16\mu_T^m}\frac{2-\ln V_f}{V_m}R^2C'''(z) - \frac{1-\nu_T^m}{E_T^m}R_m'(z)\right]\left(r^2 - R^2\right)$$
$$- \frac{1-\nu_T^m}{128\mu_T^m}\frac{V_f}{V_m}C'''(z)\left(r^4 - R^4\right) + H_m(z) + \varepsilon z\ , \tag{24}$$

where $H_m(z) + \varepsilon z \equiv u_z^m(R, z)$ arises from the integration representing the axial displacement distribution in the matrix along the interface. The function $H_m(z) \equiv 0$ when the system is in an undamaged state.

The stress and displacement fields specified by (13-24) satisfy exactly the equilibrium equations, and the compatibility equations together with the stress-strain relations for <u>any</u> function C(z), and for any functions $R_f(z)$, $R_m(z)$, $S_m(z)$, $H_f(z)$ and $H_m(z)$ whose values are defined in [2].

Differential equation for a perfectly bonded interface

For the stress and displacement representations derived above it is not possible to satisfy exactly the axial stress-strain relations for the fibre and matrix having the form given by (5). However, it is possible to satisfy the corresponding averaged forms of these stress-strain relations, an approach that leads to the following homogeneous fourth order ordinary differential equation that must be satisfied by the stress transfer function C(z) in the perfectly bonded region where

$H_f(z) \equiv H_m(z)$

$$F\,R^4C''''(z) + G\,R^2C''(z) + H\,C(z) = 0\,, \tag{25}$$

where the coefficients F, G and H are defined in [2]. Using the algebraic programming language REDUCE, it has been shown that F, G and H correspond to formulae given by Nairn [5] derived using a variational technique. (N.B. the expression for C_{35} given by Nairn should include a minus sign before the ratio $(V_2A_1)/(V_1A_2)$ that appears in his result). It is concluded that the approach of this paper provides the displacement field corresponding to the stress-based variational calculation [5] that minimises the complementary energy.

Frictionally slipping interfaces

Consider now the situation where the fibre and matrix have debonded along the interface such that they remain in contact and are subject to frictional slip. The first case to be considered assumes that frictional slip is characterised by a uniform interfacial shear stress τ which is regarded as a material constant. From (14) the stress transfer function in the slip zone satisfies the first order differential equation

$$C'(z) = \frac{2\tau}{R} \qquad \text{so that} \qquad C(z) = \frac{2\tau}{R}z + k\,, \tag{26}$$

where k is a constant of integration. The assumption of a constant interfacial shear stress is frequently made in the literature, although the Coulomb friction law is more acceptable from a physical point of view as the interfacial shear stress can be expected to depend on the compressive normal stress. The Coulomb friction law is specified by

$$\sigma_{rz}(R, z) = \lambda\eta\sigma_{rr}(R, z)\,, \tag{27}$$

for all z lying in the interfacial slip zone, where η is the coefficient of friction and where $\lambda = -1$ for a fibre fracture and $\lambda = 1$ for a matrix crack. On substituting (14) and (15) into (27) it can be shown that in the slip zone the stress transfer function C(z) must satisfy the following second order ordinary differential equation

$$f\,R^2C''(z) - 2g\,RC'(z) - h\,C(z) = \rho\,, \tag{28}$$

where

$$f = 2\left(\frac{\alpha - V_m\gamma}{V_f} - \frac{3+\nu_T^f}{16}\right),\quad g = \frac{1}{2\lambda\eta},\quad h = 2\beta\frac{V_m}{V_f},\quad \rho = 2\left(V_m\frac{\phi}{R^2} - \sigma_T\right), \tag{29}$$

where α, β and γ are defined in [2].

Axial boundary conditions

A length 2L of fibre and matrix are now considered where the origin of the (r, z) coordinates is at the centre of the system on the axis of the fibre. On $z = \pm L$ there are either fibre fractures or matrix cracks, and the shear stress σ_{rz} is assumed to be everywhere zero, so that the solution to

the problem can be applied to the fibre fragmentation and matrix cracking problems for the special case where the crack distribution is uniform in either the fibre or the matrix. It then follows from (14) and (18) that this shear stress boundary condition is satisfied if

$$C'(\pm L) = 0 . \tag{30}$$

The boundary condition for the axial stress is written in the following generalised form that can be used for fibre fractures or matrix cracks

$$\sigma_{zz}^{f}(r, \pm L) = \frac{(1+\lambda)\sigma}{2V_f} , \quad 0 \le r \le R , \tag{31}$$

$$\sigma_{zz}^{m}(r, \pm L) = \frac{(1-\lambda)\sigma}{2V_m} , \quad R \le r \le a . \tag{32}$$

On setting λ = -1 the boundary conditions (31) and (32) are valid for fibre fractures, and on setting λ = 1 these boundary conditions are valid for matrix cracks. On using (13) and (17) the boundary conditions (31) and (32) lead to the condition

$$C(\pm L) = \sigma_f - \frac{(1+\lambda)\sigma}{2V_f} = \frac{1}{2V_f}\left[(1-\lambda)V_f\sigma_f - (1+\lambda)V_m\sigma_m\right] = F(\sigma,\sigma_T,\Delta T) , \tag{33}$$

where σ_f and σ_m , the uniform axial fibre and matrix stresses for an uncracked ply, depend linearly upon σ, σ_T and ΔT.

Preliminary predictions

The material to be considered is a carbon fibre reinforced epoxy composite where the radius of the fibres is 7 microns, the volume fraction of fibres is taken as 0.5, and the stress-free temperature is such that ΔT = -120°C. The properties assumed for the carbon fibre and the epoxy matrix are given by :

	Fibre	*Matrix*
E_A (GPa)	208.0	3.89
E_T (GPa)	16.7	3.89
μ_A (GPa)	18.0	1.41971
ν_A	0.25	0.37
ν_T	0.35	0.37
α_A(/^{o}C x 10^6)	-1.1	55.0
α_T(/^{o}C x 10^6)	22.1	55.0

For the carbon fibre reinforced epoxy composite in which a fibre has broken, and where the interface is subject to a constant interfacial shear stress, Fig.1 shows the axial stress distribution in both the fibre and matrix as a function of z/L, together with the interfacial shear and normal stresses. The axial stress applied to the composite has been taken to be 0.2 GPa.

It should be noted that the stresses are all continuous and that the interfacial shear stress has an imposed uniform value of 50 MPa in the debonded region. The location of the transition zone (discussed in [2] and [6]) in the range $0.9793 \leq z/L \leq 1$ is clearly seen. It should be noted that the normal stress is everywhere compressive. The axial fibre stress is seen to diminish to a zero value at the location z = L of the fibre fracture. The axial matrix stress increases to its maximum value on the plane of fibre fracture.

Consider now a carbon fibre reinforced epoxy composite having a low volume fraction $V_f = 0.02$ characteristic of a fragmentation test, in which a fibre has broken. The friction coefficient is selected to be $\eta = 0.3$ and the temperature difference has been taken to be $\Delta T = 20^oC$ corresponding to an elevated temperature fragmentation test. Fig.2 shows the axial stress distribution in both the fibre and matrix as a function of z/L, together with the interfacial shear and normal stresses. The axial stress applied to the composite has been taken to be 5 MPa. It should be noted that the stresses are all continuous and that the normal interfacial stress is compressive in the debond zone, as required by the model if frictional stress transfer is to occur. Because of the low volume fraction the axial fibre stress is much larger than the axial matrix stress. The axial fibre stress is seen to diminish to a zero value at the location z = L of the fibre fracture. The axial matrix stress increases to its maximum value on the plane of fibre fracture. The tip of the debond is located at the point z/L = 0.9892.

The average displacements corresponding to the stress distributions shown in Figs.1 and 2 can be calculated from (21-24), and it should be noted that they will be continuous at the location of the debond tips, and that they will become coincident well away from the fibre fracture in those regions of zero stress transfer where the axial strain in both the fibre and matrix has the same value. There will be discontinuities in the interfacial axial displacements of both the fibre and the matrix at the debond tip. This is a phenomenon that arises from the fact that it is not possible to achieve the continuity of $C'''(z)$ at the location of the debond.

Conclusion

A high quality analytical model of stress transfer has been described that can be used to predict the localised stress and displacement distributions associated with fibre fractures and matrix cracks in unidirectional composites whose fibres and matrix deform linear elastically. The approach to modelling interfacial debonding has been to ensure that, wherever possible, the expressions for all relevant physical parameters are given by analytical formulae. The stress transfer associated with debonding can be modelled using two different approaches. The first assumes that the interfacial shear stress is uniform in the debonded zone. Satisfactory solutions are possible only if a transition zone is included in the neighbourhood of the fibre fracture or matrix crack. The second assumes that stress transfer in the debonded region is governed by the Coulomb friction law; a situation that does not require the use of a transition zone. Preliminary predictions indicate that the stress transfer model works very well for a range of composite types and loading conditions.

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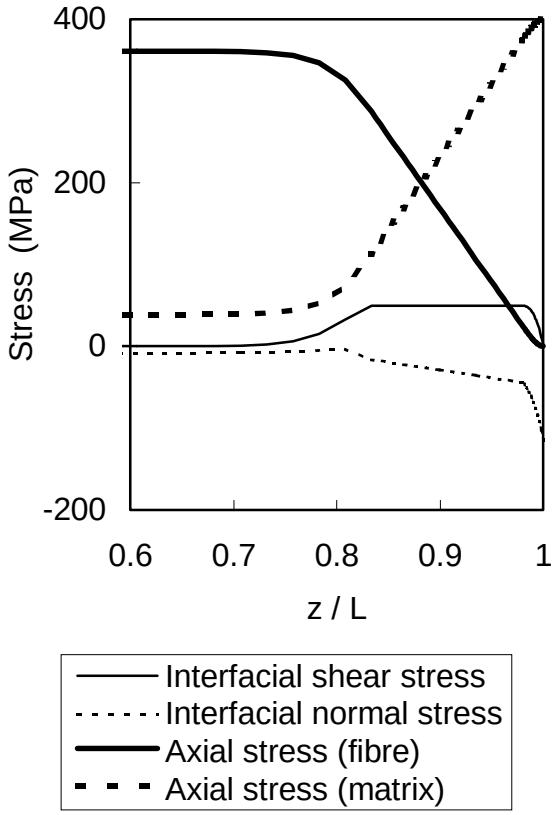

Fig.1 : Stress distribution for a carbon fibre reinforced epoxy composite having a volume fraction V_f = 0.5 where the interfacial shear stress is a constant τ =50 MPa.

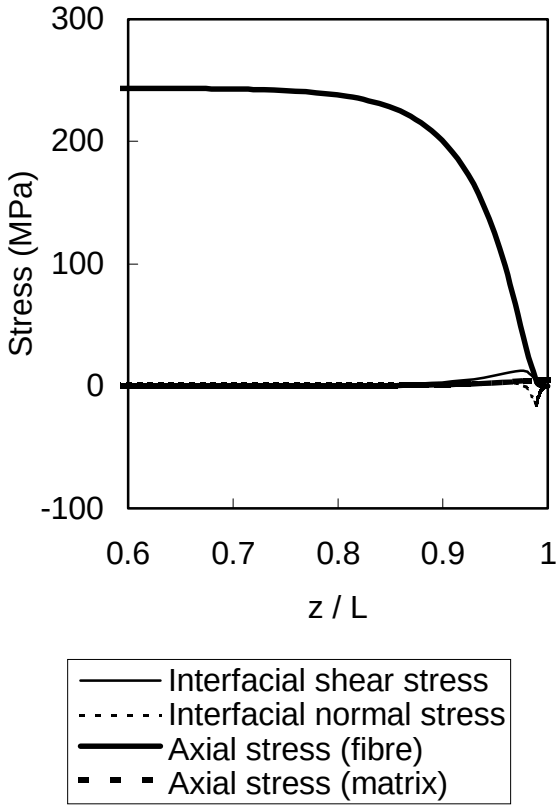

Fig.2 : Stress distribution for a carbon fibre reinforced epoxy composite having a volume fraction V_f= 0.02 where Coulomb friction occurs having coefficient η = 0.3.

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