IMPERFECTION SENSITIVITY OF MODERATELY THICK COMPOSITE CYLINDRICAL SHELLS

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SUMMARY: The problem of instability of imperfect, moderately thick, circular cylindrical shells under the action of uniform lateral pressure is investigated. Two approaches are followed: First, an analysis is done based on nonlinear kinematic relations, where the effect of transverse shear is taken into account and an imperfection function is assumed. The Galerkin procedure is employed to solve the resulting partial differential equations. The second method is based on applying Koiter's general postbuckling theory. To this extent, the objective is the calculation of imperfection sensitivity by relating to the initial post-buckling behavior of the perfect structure. Again, a shear deformation theory which accounts for transverse shear strains and rotations about the normal to the shell midsurface is employed to formulate the shell equations. The initial postbuckling analysis indicates that the range of imperfection sensitivity depends strongly on the material anisotropy, and also on the shell thickness and whether the end pressure loading is included or not.

KEYWORDS: buckling, compression, transverse shear, composite structures, postbuckling, imperfection sensitivity, external pressure.

INTRODUCTION

Recent studies on the buckling of moderately thick orthotropic shells under external pressure have pointed to the importance of the effects of orthotropy and thickness in lowering the critical load and rendering classical shell theory estimates in some cases quite non-conservative, in comparison to isotropic thin shell construction [1-3]. It is natural to consider next the extent to which these effects influence the imperfection-sensitivity of the shell.

This can be achieved in an efficient manner by applying Koiter's [4,5] general post-buckling theory, according to which, the slope of the secondary curve and the degradation of the critical loads with imperfections are described by means of the value and sign of the coefficient of the post-buckled state, *b*. A comprehensive survey by Hutchinson and Koiter [6] provides a very useful bibliography, together with an overview of the achievements and goals of this theory. In addition to Koiter's original work, several papers have produced variations of the theory with a bias towards virtual work [7,8].

The other approach to calculating imperfection sensitivity is the formulation and solution of the full nonlinear imperfect shell problem. To this extent, a solution methodology for the analysis of an isotropic, geometrically imperfect, thin, circular cylindrical shell loaded by uniform axial compression, based on the Galerkin procedure, was described by Sheinman and Simitses [9]. In this research, both approaches are applied to the study of an orthotropic cylindrical shell under external pressure.

Nonlinear Imperfect Shell Analysis

Let u, v and w denote the displacement components along x (shell axis), θ (circumferential) and z (through-thickness) coordinates and let w^0 denote the initial geometric imperfection. The first order shear deformation theory (FOSD) is employed:

$$\bar{u}(x,\theta,z) = u(x,\theta) + z\psi(x,y),$$
$$\bar{v}(x,\theta,z) = v(x,\theta) + z\phi(x,y),$$
$$\bar{w}(x,\theta,z) = w(x,\theta).$$

where $u(x,\theta)$, $v(x,\theta)$ and $w(x,\theta)$ are the reference surface displacements, measured from the imperfect undeformed geometry and ϕ, ψ are the rotations of a normal to the middle surface in the θz- and xz- planes respectively. The initial geometric imperfection is also assumed to be independent of the z coordinate.

Use of the principle of the stationary value of the total potential energy yields the equilibrium equations and the associated boundary conditions. The five equilibrium equations and the associated boundary conditions for the circular cylindrical shell are then expressed in terms of u, v, w, ψ and ϕ. The details can be found in Simitses and Hsiung [10]. This is accomplished by first expressing the stress resultants in terms of the reference strains and changes in curvature by employing the laminate constitutive equations that include transverse shear. Then, the nonlinear kinematic relations are employed to express these in terms of the displacement and rotation functions. The solution procedure is an extension of the one used for thin shells [11] and consists of the following steps:

The five reference surface displacement and rotation functions are represented in a sine and cosine (Fourier) series in the hoop direction, multiplied by undetermined functions of the axial (x) coordinate, e.g.:

$$u(x,y) = \sum_{j=0}^{k}\left[A_j^1(x)\sin(jn\theta) + B_j^1(x)\cos(jn\theta)\right]$$

the superscripts 1,2,3,4,5 are used for u, v, w, ψ and ϕ. A similar expression is used for the initial geometric imperfection:

$$w^0(x,y) = \sum_{j=0}^{k}\left[A_j^0(x)\sin(jn\theta) + B_j^0(x)\cos(jn\theta)\right]$$

These expressions are then substituted into the equilibrium equations and boundary conditions. Because the convergence is very rapid, only j = 0 and 1 terms are kept; thus, each function is approximated by three terms. Note that there is a free parameter, n, which denotes the wave number at the limit point. In order to determine the (lowest) critical load, all the steps outlined herein are applied to several sequential values of the wave number.

The Galerkin procedure (in the hoop direction) is then employed and the partial differential equations are converted to a set of ordinary nonlinear differential equations. Subsequently, by use of Newton's method, these are reduced to a set of linear differential equations. These, in turn, are cast into a finite difference form which are solved through iterations at each load level. The load is step-increased until a limit point is reached. This occurs when the solution goes from convergence to divergence during step-increased loading or the sign of the determinant of the coefficients of the unknowns reverses [12].

Results from the Nonlinear Analysis

The material considered is graphite/epoxy with moduli in Gpa: E_{11} = 149.617, $E_{22} = E_{33}$ = 9.928, $G_{12} = G_{13}$ = 4.481, G_{23} = 2.551, and Poisson's ratios: $\nu_{12} = \nu_{13}$ = 0.28 and ν_{23} = 0.45. The geometry studied is: R = 19.05 cm, R/h = 60, 30 and 15 and $\ell/R = 1,2,$ and 5 where R, h and ℓ are the radius, thickness and length of the shell. The imperfection shape considered is virtually axisymmetric and has the form:

$$w^0(x,\theta) = \xi h[-\cos(2\pi x/\ell) + 0.1\sin(\pi x/\ell)\cos n\theta]$$

where (is a measure of the imperfection amplitude given by

$$\xi = w^0_{\max}/h$$

Critical loads are calculated for axial compression and uniform external pressure (modeled as lateral deadweight). Here we show only the lateral pressure; for more results regarding axial compression see Simitses and Hsiung [10]. Fig. 1 shows the imperfection sensitivity of the configuration for R/h = 15 (relatively thick shell). As a comparison, the critical pressure for the perfect shell (ξ = 0) is 22 MPa. Furthermore, a similar study with axial compression instead of lateral pressure has shown that this configuration is more imperfection sensitive for pressure than for compression.

Koiter-based Solution

Asymptotic expansions for the post-buckling behavior of the perfect shell

In the post-critical regime, the structure suffers deviations in the displacement profile from the buckling mode $\vec{V}_1 = \{u^{(1)}, v^{(1)}, w^{(1)}\}$, and simultaneously, p will deviate from p_c, the critical pressure. Define $\eta = p/p_c$. Then, the displacements of the structure in the initial post-buckling phase can be written as

$$\vec{V} = \eta\vec{V}_0 + \xi\vec{V}_1 + \xi^2\vec{V}_2 + \xi^3\vec{V}_3 + \xi^4\vec{V}_4 + \ldots,$$

where η depends on ξ. In this expansion $\vec{V}_0$ is associated with the prebuckling state, $\vec{V}_1$ describe a normalized buckling mode, and the remaining terms are orthogonal to the buckling mode. For example,

$$u = \eta u_0 + \xi u^{(1)} + \xi^2 u^{(2)} + \xi^3 u^{(3)} + \xi^4 u^{(4)} + \ldots$$

Also, similar expansions are assumed for the resultant forces and moments. For example,

$$N_x = \eta N_{x0} + \xi N_x^{(1)} + \xi^2 N_x^{(2)} + \xi^3 N_x^{(3)} + \xi^4 N_x^{(4)} + \ldots$$

Moreover, substituting into the nonlinear strain displacement eqs. gives the strains in the form

$$\varepsilon_{ij} = \eta\varepsilon_{ij}^0 + \xi\varepsilon_{ij}^{(1)} + \xi^2\varepsilon_{ij}^{(2)} + \xi^3\varepsilon_{ij}^{(3)} + \xi^4\varepsilon_{ij}^{(4)} + \ldots$$

Stress Resultants

In the first order shear deformation shell theory considered, the generalized stress σ consists of the five force resultants N_x, N_θ, $N_{x\theta}$, Q_{xz} and $Q_{\theta z}$ and the three moment resultants M_x, M_θ and $M_{x\theta}$. The generalized strain ε represents the five membrane strains ε_x, ε_θ, $\gamma_{x\theta}$, γ_{xz}, $\gamma_{\theta z}$ and the three bending strains κ_x, κ_θ, and $\kappa_{x\theta}$. The generalized displacement consists of the five mid-point linear and angular displacements u, v, w, ϕ and ψ.

For orthotropy, the stress resultants are related to the strain components by:

$$\sigma = \begin{bmatrix} N_x \\ N_\theta \\ Q_{\theta z} \\ Q_{xz} \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_2^2 c_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_1^2 c_{55} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} \\ \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{bmatrix} = C\ .\varepsilon\ .$$

where c_{ij} are the stiffness constants (we have used the notation $1 \equiv x$ (axial), $2 \equiv \theta$, $3 \equiv z$ (radial), $4 \equiv \theta z$, $5 \equiv xz$ and $6 \equiv x\theta$) and k_1^2, k_2^2 are the shear correction factors. Results will be presented for the usual values of $k_1^2 = k_2^2 = 5/6$.

The relationship $\eta(\xi)$

In this subsection, we shall use the abbreviation 1/2 $\sigma\varepsilon$ to denote the strain energy of the shell, which can be written in the form of an integral over the volume, V:

$$\frac{1}{2}\sigma\varepsilon = \frac{1}{2}\int_0^\ell \int_0^{2\pi} \begin{pmatrix} N_x\varepsilon_x + N_\theta\varepsilon_\theta + N_{x\theta}\gamma_{x\theta} + Q_{xz}\gamma_{xz} + Q_{\theta z}\gamma_{\theta z} + \\ + M_x\kappa_x + M_\theta\kappa_\theta + M_{x\theta}\kappa_{x\theta} \end{pmatrix} R d\theta\, dx.$$

Also, we shall denote by Ω, the work done by the uniform fluid pressure (which remains always normal to the surface as the shell deforms); this is the product of the pressure and the change in the volume enclosed by the shell. An expression in terms of the displacements can be found in Brush and Almroth [13]:

$$\Omega = p\int_0^\ell \int_0^{2\pi} \left[w + \frac{1}{2R}\left(v^2 - vw_{,\theta} + v_{,\theta}w + w^2\right) \right] R d\theta\, dx.$$

A complete and extensive presentation of the $\eta(\xi)$ relationship is given by Budiansky [8], in which use is made of Frechet derivatives. The formal definition given for Frechet derivatives of any order is entirely equivalent to the familiar process of "taking variations" in the calculus of variations. We shall use prime to denote Frechet derivatives and the subscript c means evaluation at the critical state.

If we set now

$$\eta = 1 + a\xi + b\xi^2,$$

$$a = -\frac{3}{2}\frac{\sigma^{(1)}(\varepsilon_c^{'''}V_1^2)}{\sigma_c^0(\varepsilon_c^{''}V_1^2) - (\Omega_c^{''}V_1^2)},$$

(1*a*)

we can find the coefficients a, b for the case of linear strain-strain relations, quadratic strain-displacement relations and quadratic shortening-displacement relations, as follows [8]

(1*b*)

$$b = -\frac{2\sigma^{(1)}(\varepsilon_c^{''}V_1V_2) + \sigma^{(2)}(\varepsilon_c^{''}V_1^2)}{\sigma_c^0(\varepsilon_c^{''}V_1^2) - (\Omega_c^{''}V_1^2)},$$

(1*c*)

Notice that the second term in the denominator is due to the hydrostatic loading and it would not exist in a dead-loading situation. Also, the variable V is identified as the set of functions u, v, w. The Frechet derivatives of the generalized strains are found to be

$$\varepsilon_x^{''}V_1V_2 = w_{,x}^{(1)}w_{,x}^{(2)}; \quad \varepsilon_x^{''}V_1^2 = w_{,x}^{(1)2},$$

(2*a*)

$$\varepsilon_\theta^{''}V_1V_2 = w_{,\theta}^{(1)}w_{,\theta}^{(2)}/R^2; \quad \varepsilon_\theta^{''}V_1^2 = w_{,\theta}^{(1)2} \quad ,$$

(2*b*)

$$\gamma_{x\theta}^{''}V_1V_2 = \frac{1}{R}\left(w_{,x}^{(1)}w_{,\theta}^{(2)} + w_{,x}^{(2)}w_{,\theta}^{(1)}\right); \gamma_{x\theta}^{''}V_1^2 = 2w_{,w}^{(1)}w_{,\theta}^{(1)}/R,$$

(2*c*)

All the other components of the generalized strain result in zero Frechet second derivatives.

Also, the second Frechet derivative of the external work is,

$$\Omega_c^{''}V_1^2 = p_c\int\int\left(v^{(1)2} - v^{(1)}w_{,\theta}^{(1)} + v_{,\theta}^{(1)}w^{(1)} + w^{(1)2}\right)d\theta\, dx.$$

(2*d*)

It is also to be noted that the derivation of the relationship $\eta(\xi)$ makes use of the orthogonality conditions

$$s^{(2)}e^{(1)} = s^{(1)}e^{(2)} = s^0(\varepsilon_c^{''}V_1V_2) = 0$$

(2*e*)

The first and second order displacement fields are now needed to calculate the post-buckling coefficient.

The First Order Displacement Field

The governing equations of equilibrium and boundary conditions for the shear deformable orthotropic shell can be derived from the principle of virtual work, namely $\delta I = \delta\Omega$, by integrating by parts and setting the coefficients of δu, δv, δw, $\delta\phi$, and $\delta\psi$ to zero separately. We shall consider in the present paper a shell loaded by external pressure in a simply supported configuration and in which there are no prescribed end forces or moments.

In the pre-buckling state, the axially symmetric distribution of external forces produces stresses identical at all cross sections. For external pressure,

$$N_{\theta 0} = -pR; \quad N_{x0} = -\alpha pR/2; \quad N_{x\theta 0} = 0.$$

The parameter α is used to conveniently allow for end pressure loading; if the pressure contributes to axial stress through end plates, $\alpha = 1$, whereas if the pressure only acts laterally, $\alpha = 0$. We shall also use the superscript c to refer to the critical state, i.e. $N_{\theta 0}^{c} = -p_c R$.

Substituting the asymptotic expansions into the equilibrium equations thus derived, retaining the first order terms and then using the constitutive relations to express the first order resultant forces and moments in terms of the first order displacements gives five partial differential equations for the displacements and corresponding boundary conditions.

The first order displacement field is set in the form:

$$u^{(1)}(x,\theta) = U_1 \sin n\theta \cos \lambda x; \quad v^{(1)}(x,\theta) = V_1 \cos n\theta \sin \lambda x,$$

$$w^{(1)}(x,\theta) = W_1 \sin n\theta \sin \lambda x,$$

$$\psi^{(1)}(x,\theta) = \Psi_1 \sin n\theta \cos \lambda x; \quad \phi^{(1)}(x,\theta) = \Phi_1 \cos n\theta \sin \lambda x,$$

where

$$\lambda = \frac{m\pi}{\ell}$$

and a resulting set of five linear algebraic homogeneous equations is obtained, which gives the critical pressure, p_c. The buckling modes, i.e. the constants U_1, V_1, W_1, Ψ_1 and Φ_1, are subsequently obtained by choosing the normalization $W_1 = h$, where h is the shell thickness.

The Second Order Displacements

Substituting next the asymptotic expansions into the equilibrium equations, retaining the ξ^2 terms and using the anticipated result that $p = p_c + O(\xi^2)$, gives the second order equilibrium equations (for details see [14]). Then, substituting the resultant force-displacement relations gives five differential equations for the second order displacement field with the associated boundary conditions.

The second order displacement is sought in the separated form:

$$u^{(2)}(x,\theta) = U_{20}(x) + U_{22}(x)\cos 2n\theta; \quad v^{(2)}(x,\theta) = V_{20}(x) + V_{22}(x)\sin 2n\theta,$$

$$w^{(2)}(x,\theta) = W_{20}(x) + W_{22}(x)\cos 2n\theta; \quad \psi^{(2)}(x,\theta) = \Psi_{20}(x) + \Psi_{22}(x)\cos 2n\theta,$$

$$\Phi^{(2)}(x,\theta) = \Phi_{20}(x) + \Phi_{22}(x)\sin 2n\theta.$$

Two sets of ordinary differential equations, one set for the functions in the θ-independent terms and another set for the functions in the θ- dependent terms are obtained. The first set consists of the five ordinary differential equations for U_{20} (x), $V_{20}(x)$, $W_{20}(x)$, $\Psi_{20}(x)$ and $\Phi_{20}(x)$, and the corresponding boundary conditions. Likewise, the second set consists of five ordinary differential equations with the corresponding boundary conditions for U_{22} (x), $V_{22}(x)$, $W_{22}(x)$, $\Psi_{22}(x)$ and $\Phi_{22}(x)$. Also, due to symmetry only half of the shell need be considered in the solution procedure.

The two point boundary value problem for half of the shell is solved separately for the θ-independent and the θ-dependent functions by the relaxation method [15], in which the five coupled ordinary differential equations are replaced by finite difference equations on a mesh of points that spans half the length of the shell.

Once the second order displacement field is determined from the solution of the foregoing two-point boundary value problem, the post-buckling coefficient, b, can be determined from (1) and (2).

The initial variation of pressure after buckling is:

$$\eta = p / p_c = 1 + b(\delta / h)^2, \tag{3}$$

where, since the buckling modes were normalized so that W_1 is equal to the shell thickness, h, the general perturbation variable ξ has been replaced by δ/h. the maximum amplitude of the buckling mode over the shell thickness.

Results from the Koiter-based Solution

Let us consider a shell being made of unidirectional graphite/epoxy with the following typical properties for the material (1 is along the fibers, 2 is the in-plane transverse and 3 is the out-of-plane transverse direction): moduli in GN/m^2 E_1= 140.0, E_2= 9.1, $G_{12} = G_{23} = G_{31}$ = 4.3, and Poisson's ratio ν_{12} = 0.300. The shear correction factors are assumed to be $k_1^2 = k_2^2 = 5/6$. The shell has a mean radius of R = 1m. Two cases of shell thickness were studied: one corresponding to radii ratio R_2/R_1 = 1.05 and another thicker construction with R_2/R_1 = 1.10. Also, two cases of loading are considered: lateral only external pressure loading, i.e. no axial load and α = 0 and external pressure with hydrostatic end loading, i.e. with axial compressive loads determined with α = 1. Here we shall present only results for the thicker shell and lateral external pressure only; for additional results refer to Kardomateas [14].

The Batdorf parameter has been used as a convenient nondimensional parameter to present results for shell buckling and postbuckling. For isotropic cylindrical shells, this is only a function of the thickness of the shell. Analogues for anisotropy have been derived by Nemeth [16] by performing a nondimensionalization of the shell buckling equations and it was shown that, for anisotropic shells, the Batdorf parameter depends not only on the geometry but also on the stiffness constants. In particular, for an orthotropic cylindrical shell, the Batdorf parameter becomes:

$$\Xi = \frac{\ell^2 (c_{11}c_{22} - c_{12}^2)^{1/2}}{R\sqrt{12}(c_{11}c_{22}D_{11}D_{22})^{1/4}}.$$

The postbuckling coefficient b is calculated following the solution of the two-point boundary value problem for the second order displacements, as has already been outlined. If b is negative, the shell is imperfection-sensitive and the load carrying capacity diminishes following buckling. Also, the degree of imperfection-sensitivity is governed by the magnitude of b. If, on the other hand, b is positive, the structure retains some ability to support increased loads once bifurcation has taken place.

In all cases considered the structure buckles at m =1, whereas n depends on the shell length, ℓ, becoming equal to 2 for very long shells. Table 1 gives b, calculated from the shear deformable theory, for the cases of circumferentially reinforced, axially reinforced graphite/epoxy, and isotropic material in a shell under lateral external pressure only, and for radii ratio, R_2/R_1 = 1.10. The main conclusion is that the regions of imperfection sensitivity are strongly dependent on the anisotropy of the material. For the circumferentially reinforced case, it can be concluded that the critical pressure (at the bifurcation point) ought to be reliable above $\Xi \simeq 270$, whereas for the axially reinforced case the structure is imperfection-sensitive even at the high range of length values, and therefore reduction in the buckling pressure from the bifurcation values should be anticipated; for the isotropic case, the bifurcation pressure ought to be reliable above $\Xi \simeq 1{,}000$. For the rather short shells, i.e. small values of Ξ, the circumferentially reinforced case shows a large amount of imperfection-sensitivity above $\Xi \simeq 2.3$ (and up to $\simeq 270$), whereas the axially reinforced case exhibits no imperfection-sensitivity below $\Xi \simeq 27$. One interesting observation is that the absolute maximum of the negative range of the postbuckling coefficient, *b*, is reached in the isotropic case.

Table 1. *Postbuckling coefficient, b*
Lateral External Pressure Only, $R = 1$m, $R_2/R_1 = 1.10$, $k_1^2 = k_2^2 = 5/6$

Ξ* (Batdorf Parameter)	*b* Circum† Reinf	*b* Axial‡ Reinf	*b* Isotropic**
2	0.0847	1.6421	0.3483
5	-0.2170	0.9241	-0.0949
10	-0.2681	0.5118	-0.2075
23	-0.0839	0.0152	-0.2052
53	-0.0934	-0.1460	-0.1940
120	-0.0232	-0.2119	-0.0474
271	0.0001	-0.0916	-0.0618
615	0.0064	-0.1102	-0.0101
1395	0.0078	-0.0339	0.0043
3162	0.0081	-0.0036	0.0075

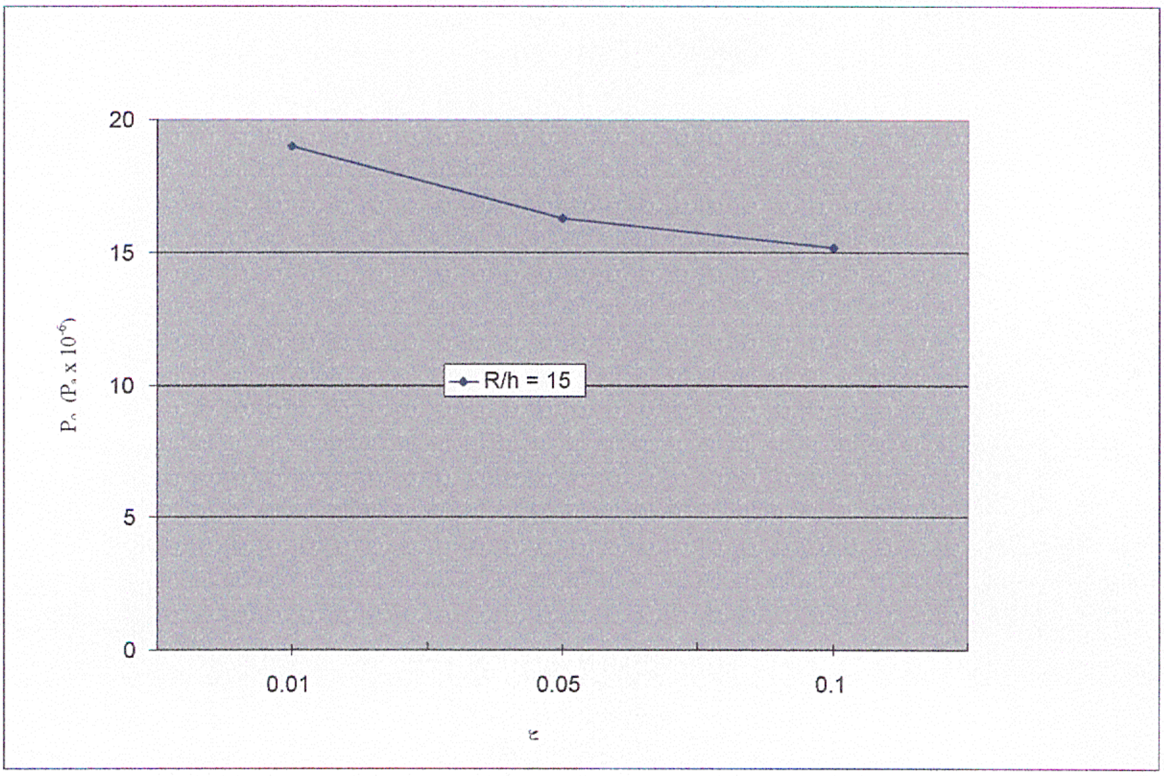

Fig.1. *Imperfection Sensitivity – Lateral Pressure. Graphite/Epoxy,* $\left(45^0_2/-45^0_2\right)$, $\ell/R = 1$.

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