ANALYSIS OF FRACTURE MECHANICS ON SKEW CRACK NEAR INTERFACE OF BIMATERIALS

Chen Haoran[1], Wang Limin[1,2], Xu Shilang[1], Wang Jindong[1]

1 Dalian University of Technology, Dalian 116024, Liaoning, P. R. China.
(E-mail ChenHR@GingKO.DLUT.Edu.CN)
2 Shandong Institute of Technology, Zibo 255012, Shandong, P. R. China

SUMMARY: Bimaterials are composed of two kinds of media. The effects of the mechanical state of the interfaces of different media on the material's performance are important. In the fields of the linear elastic fracture mechanics, J-integral is equal to Strain Energy Release Rate(SERR), and Stress Intensity Factor (SIF) is another character in the discipline. The scope of application of the J-integral is firstly discussed in this paper. A schematic model of a skew crack near the bimaterial interface is established, and the corresponding numerical methods of calculating the two fracture mechanics characters are put forward. When the crack is very close to the interface, the values of the SERR and SIF of the crack-tip near the interface mainly depend on the bimaterial parameters and the angle between the crack face and the interface. If the crack occurs in the harder medium, the SERR is sensitive to the distance between the crack tip and the bimaterial interface.

KEYWORDS: crack, interface, bimaterials, strain energy release rate, J-integral, fracture mechanics

INTRODUCTION

The composite materials are composed of many kinds of media. The functions and mechanical behavior of the materials are closely related to their components, and the effect of damages and cracks near the interfaces of the materials on the materials' performance is very important. When a crack occurs on an interface between two different media, the fields of stress and displacement are different from those of a crack in the homogenous medium, in which the crack-tip stress is singular with oscillation[1-2] and the surfaces of the crack terminal parts are mutually inserted or contact with each other[3]. In Ref.4-8, the mechanics characters of the crack are respectively analyzed through different schematic model of interphase. As to multi-media materials, the characters of the stress field for a crack paralleling to the interface is calculated and analyzed in Ref. 9-11. The stress singular problem of the crack terminating at a bimaterial interface is researched by Bogy[12]. When a crack is vertical to the interface, a lot of analysis results are presented by in Ref 13-14. In the material system including two media, Dundurs indicated that the system depends on two material parameters[15]:

$$\alpha=\frac{\mu_2(1-v_1)-\mu_1(1-v_2)}{\mu_2(1-v_1)+\mu_1(1-v_2)}\ ,\qquad \beta=\frac{1}{2}\cdot\frac{\mu_2(1-2v_1)-\mu_1(1-2v_2)}{\mu_2(1-v_1)+\mu_1(1-v_2)} \tag{1}$$

Where, μ, ν are shear elastic module and Poisson's ratio, and the subscripts 1 and 2 refer to medium 1 and medium 2, respectively.

Bimaterials are composted of two kinds of media. For composite materials or bimaterials, because of the difference of their mechanical performances, the investigation on their stiffness and strength are very complex; and requires a lot of hard-work in several respects. In this paper, the study of the bimaterial mechanical behavior is done through the fracture mechanics. The applicable scope of J-integral for bimaterials is firstly analyzed, and the corresponding numerical method of calculating SERR and SIF is presented. When the position of a crack near the interface varies, the entire figure of SERR is roughly given by the author's method. A lot of numerical results and discussions are also included in the paper.

APPLICABLE SCOPE OF J-INTEGRAL FOR BIMATERIALS

As is known to all, the value of J-integral along the path of enclosing a crack terminal is conservative for the homogeneous medium in which the derivative of the strain energy density with respect to some strain is equal to the associated stress and there is no body force[16]. The definition of the J-integral is expressed as follows:

Where, w is strain energy density; n, ds are normal vector and differential length of a shorter curve respectively.

$$J = \int w dy - \int \sigma_{ij} n_j \left(\partial u_i / \partial x\right) ds \tag{2}$$

As shown in Fig.1, assume the X-axes of the coordinate and a crack lying on the bimaterial interface. The marking numbers of the upper medium and the lower medium are indicated as 1# and 2#. For the close path P1=*abcdefgha* & P2=*abcmgha*, Considering the direction of the integral and putting the integral-area to be the left side of the integral path, we obtain the following relationship: P2=P1-*gmcdefg*=P1-*gmefg* -*mcdem*-*me*-*em*. Because the value of J-integral is vanished when the region closed by the integral path has not singular point, so there are:

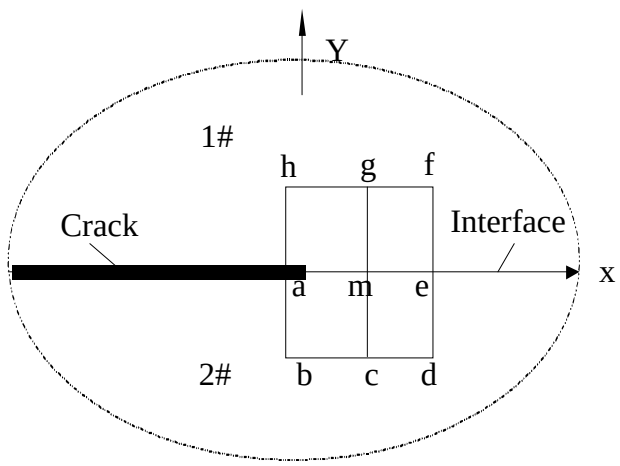

Fig.1 A crack along the interface of bimaterials

$$\begin{aligned} J_{P2} &= J_{P1} - J_{gmefg} - J_{mcdem} - J_{me} - J_{em} \\ &= J_{P1} - 0 - 0 - J_{me} - J_{em} \end{aligned} \tag{3a}$$

If '+' and '-' denote medium 1# and medium 2#, respectively; and there are $dy = 0, T_i^+ = \sigma_{ij}^+ n_j^+ = T_i^- = \sigma_{ij}^- n_j^-, u_i^+ = u_i^-$, along the interface. So Eq.3 can be rewritten as follows:

$$J_{P2} = J_{P1} - J_{me} + J_{me} = J_{P1} \tag{3b}$$

If a crack is parallel to the interface of bimaterials, the conservative of the J-integral's value is also certified in the above way. On the other hand, when a crack is vertical to the interface, because of $dy \neq 0$ alone the interface and, the strain energy densities of two sides for bimaterial interface are not equal to each other, the value of the J-integral along the path closing one medium is not the same as that of the path closing two different media. **In a word, if only a crack is parallel to or lying on the bimaterial's interface, the value of J-intergral along the curve closing the crack-tip will be constant.**

THREE KINDS OF VALUES OF J-INTEGRAL

According to the conservative law of J-integral, as a crack is parallel to the interface, it is supposed that the value of J-integral of the crack on the interface is approximately equal to either of the values of the crack near and parallel to the interface. It is expressed as follows:

$$J^0 \cong J^+ \cong J^- \tag{4}$$

Where the superscripts '0', '+' and '-' denote the crack lying on, over, under and parallel to the interface of the bimaterials; respectively. Under the conditions of the different bimaterial parameters (α, β), the comparison of the values among the three kinds of J-integral is listed in Table 1. These results are calculated by Finite Element Method (FEM). For the same bimaterial parameter, the differences of the nondimensional values of three kinds of J-integral are tiny, so the above expression may be regarded as an equality.

Table 1 The values of J^0, J^+ and J^- in different bimaterial parameters

J^0	0.1274	0.8500	0.7390	0.7164	0.7031	0.6925
J^+	0.1275	0.8439	0.7281	0.7044	0.6905	0.6794
J^-	0.1272	0.8386	0.7298	0.7037	0.6868	0.6733
α	0.1111	0.6364	0.8518	0.9048	0.9355	0.9608
β	0.03175	0.1818	0.2424	0.2585	0.2673	0.2745

FRACTURE MECHANICS CHARACTERS FOR A SKEW CRACK NEAR INTERFACE OF BIMATERIALS

Characters For Crack End Near Interface

As shown in Fig 2, assume the X-axes of coordinate lying on the bimaterial interface, the Young's module of the upper medium 1# being larger than that of the lower medium 2# and a crack being in either medium. The elastic module's proportion of two media is expressed to be E1/E2=6/1, and the remote load is the single-direction pull stress vertical to the crack. The numerical calculation of J-integral is limited in one medium and the integral path is close to the crack-tip near the interface. If E1>E2, and when the crack is placed in medium 1#, let the angle θ between the crack axes and the interface positive, otherwise negative. Because the values of J-integral for three kinds of J-integral are approximately equal to each other, the variation of the J-integral value with the crack in one medium to another can be expressed by one formula. By numerical analysis, the variation law of J-integral value with θ can be written as:

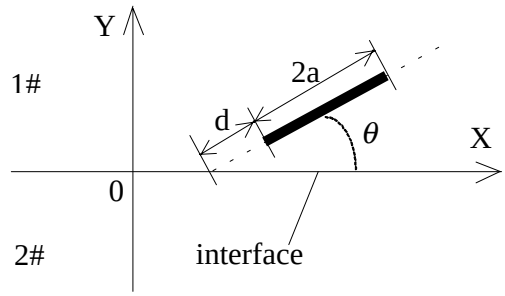

Fig. 2 *Schematic model of a skew crack near bimaterial interface*

$$J = J_0 e^{\xi \theta^3} \tag{5}$$

Where ξ is the function of the crack position relative to the interface, and, the corresponding values of ξ are 0.99, 0.55, 0.45 and 0.31, when *d/a* are equal to 0.01, 0.05, 0.10 and 0.30 respectively. J_o is the value of SERR (G_I or J-integral) for the crack being on or parallel to the interface, which can be calculated by SIF (K) obtained from Ref. 17 and the following formulation:

$$G_I + G_{II} = \frac{(1-\nu_1)/\mu_1 + (1-\nu_2)/\mu_2}{4\cosh^2 \pi\varepsilon}[K_1^2 + K_2^2] \tag{6}$$

Where, $\varepsilon = \ln[(1-\beta)/(1+\beta)]/(2\pi)$ is a bimaterial parameter.

When the distance between the interface of the bimaterials and the crack end near the interface is changed from small to large, the value variation law of J-integral for the crack tip near the interface is shown in Fig. 3. According to the figure, the following conclusion can be drawn. The value of J-integral or SERR is increasing with the value decreasing of *d/a* when the crack is in the harder medium, especially the crack being very near the interface and the crack being towards the position vertical to the interface. While, if the crack is in the softer medium, the value of SERR decreases with the decreasing *d/a,* and the SERR approaches to a larger value as the angle θ between 30~40°.

When a skew crack is in one medium of the bimaterials, the SIF value of the crack tip can be calculated by the relation-ship between SIF and SERR for the homogeneous medium. Although the values of SERRs are approximately equal to each other for a crack parallel to the interface in two media, because of different elastic modules for two media, the difference of SIF values is not very small. For example, if E1/E2=6, the proportion of SIFs can be more than 2.

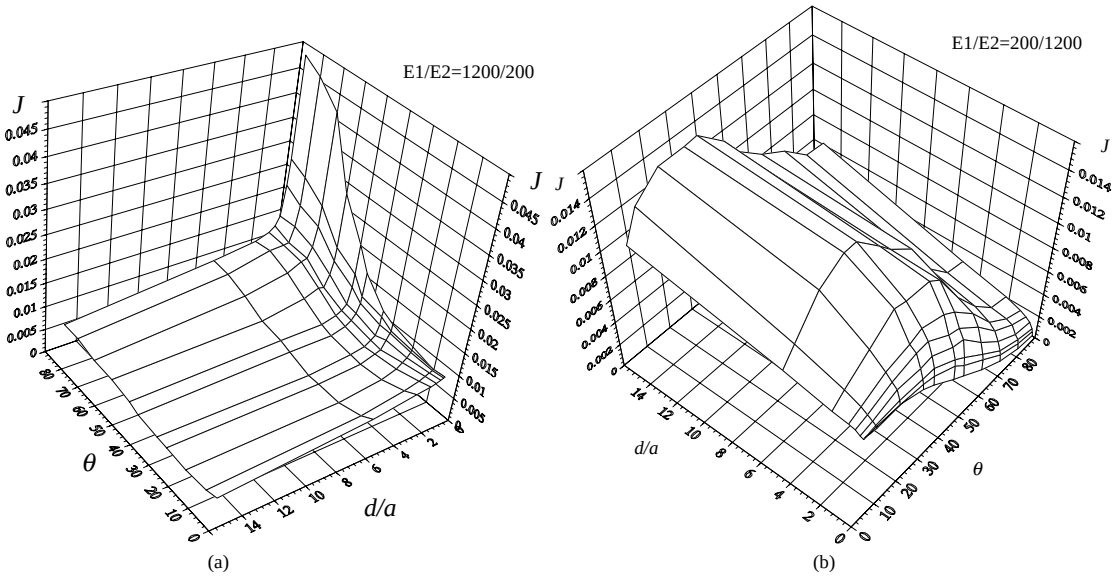

Fig. 3 The variation laws of J-integral of the skew crack near the interface with d/a and θ in the following cases: (a) the crack is located in the harder medium; (b) the crack is in the softer medium.

CHARACTERS FOR CRACK-TIP FARTHER FROM INTERFACE

If the nondimensional crack length in Fig.2 is 2, the SERR for the crack-tip farther from the interface can be calculated by FEM, and its variations with θ and *d/a* are presented in Fig. 4, where *d* is the distance from the crack-tip near the interface to the interface. When the crack is in the harder medium, the variation law of SERR value can be known as follows. The value is increasing at the beging, and then decreasing with the increasing of θ value. The value of SERR approaches a largest point at $\theta \approx 30^{\circ}$. This law is different from that of the crack-tip near the interface. >From the comparison of Fig.3 and Fig. 4, it is known that the value of SERR for the crack-tip near the interface is much larger than that for the crack-tip farther from the interface when the crack tends to be vertical to the interface of the bimaterials. It is implicated that the expanding of the crack is towards the position of the crack-tip near the interface. In the softer medium, the SERR value of the crack end far from the interface is decreasing firstly, then is increasing with the increasing of the θ. Comparing the SERR values of two ends of the crack in Fig.3 and 4, it can be known that the value of SERR for the crack-tip near the interface is less than that for the crack-tip far from the interface as the crack tends to be perpendicular to the interface.

CONCLUSIONS

For the problem of the crack close to the bimaterial interface, following conclusions can be reached by the above analyses and discussions:

(a) If a crack is parallel to or along the interface of the bimaterials, the J-integral value along the path closing one end of the crack is independent on the integral path.

(b) When a crack in one of the two media is parallel to and close to the interface, the values of J-integral are approximately equal to those of the crack on the interface.

(c) The fracture mechanics characters for the crack near the bimaterial interface are dependent on the bimaterial parameter and the angle between the crack and the interface, and the

variation law of the SERR can be expressed by a formula when the remote single direction pull load is exerted on.

(d) When a crack is in the harder medium, the SERR value is increasing sharply with the crack towards the position vertical to the interface and with the decreasing of the distance between the interface and the crack-tip.

(e) For a crack vertical to the interface, if the crack is in the harder medium, the crack-tip near the interface is easy to expand; while the crack-tip farther from the interface is easy to extend in the softer medium.

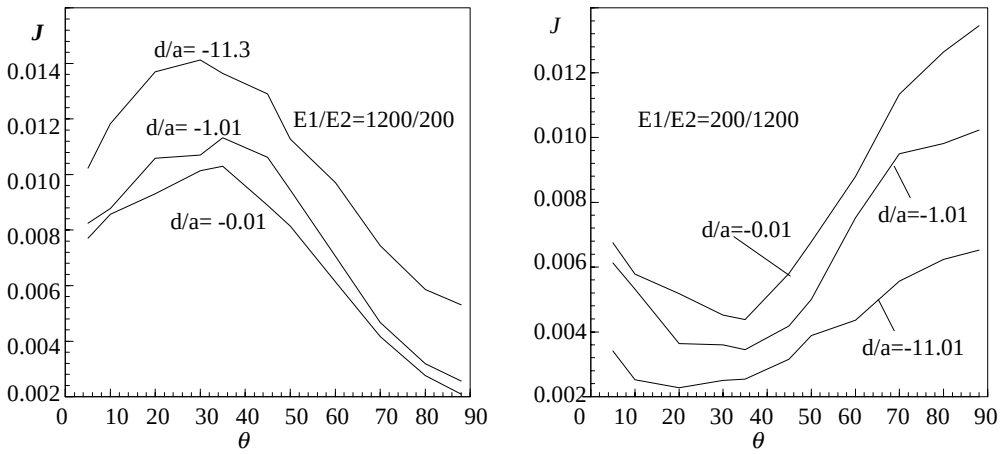

Fig. 4 The SERR value for the end of a crack far from the interface while the another end of the crack near the interface of the bimaterials in the following cases: (a). The crack is located in the harder medium; (b). The crack is in the softer medium.

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REFERENCES

1. Rice, J. R., «Elastic fracture mechanics concepts for interfacial cracks», *J.Appl.Mech*., Vol. 55, 1988, pp.98-103

2. Williams, M. L., «The stress around a fault or crack in dissimilar media», *Bull. Seismol. Soc. Am.*, Vol.49,1959, pp.199-204

3. Comnimou, M., Schmueser D, «The interface crack in a combined tension-compression and shear field», *J. Appl. Mech.* Vol. 46, 1979, pp.345-348

4. Delale, F. and Erdogan, F., « On the mechanical modeling of the interfacial region in bonded half-planes», *J. Appl. Mech*., Vol. 55**,** 1988, pp.317-323

5. Akisanya, A. R. and Fleck, N. A., «Brittle fracture of adhesive joints»**,** *Int.J.Fracture,* Vol. 58**,** 1992, pp.93-114

6. Yang, W. and Shih, C. F., « Fracture along an interlayer», *Int. J. Solids and structures,* Vol.31, 1994, pp.985-1002

7. Shih, C. F. and Asaro, R., «Elastic-plastic analysis of cracks on bimaterial interfaces»;Part 1:Small scale yielding, *J.Appl. Mech*., Vol.55. 1988, pp.299-316

8. Ozturk, M. and Erdogan, F., «An axisymmetric crack in bonded materials with a nonhomogeneous interfacial zone under torsion», *J. Appl. Mech*., Vol.62, 1995, pp.116-125

9. Jha, M. and Charalambides, P. G., «Crack-tip micro mechanical fields in layered elastic composites: Crack parallel to the interfaces», *Int. J. Solids structures,*Vol.35, No.1-2 1998, pp.149-179

10. Hutchinson, J. W. and Suo, Z., «Mixed mode cracking in layered materials», *Advances in applied mechanics,*Hutchinson J W, Wu T Y Eds, Boston, Vol.29, 1992. pp.63-191

11. Chen, H., Wang, L., Karihaloo, B. L., Williams, F. W., «Fracture Analysis for multi-materials with interface crack», *Comput. Mater. Science* , No.12, 1998, pp.1-8

12. Bogy, D. B., «On the plane elastostic problem of a loaded crack terminating at a material interface», *J. Appl. Mech*., 1971, pp.911-918

13. Romeo, A. and Ballarini, R., «A crack very close to a bimaterial interface», *J. Appl. Mech*., Vol.62, 1995, pp.614-619

14. Hilton, P. D. and Sih, G. C., «A laminate composite with a crack normal to the interfaces», *Int. Solides Structures*, Vol.7,1971, pp.913-930

15. Dundurs, J. , «Edge-bonded dissimilar orthogonal elastic wedges under normal and shear loading», *J. Appl. Mech*., Vol. 36, 1969, pp.650-652

16. Rice, J.R., «A path indenpendent integral and the approximate analysis of strain concentration by notches and cracks», *J.Appl.Mech*. 1968, pp.379-386

17. Hutchinson, J. W., Mear, M. E. and Rice, J. R., «Crack paralleling an interface between dissimilar materials», *J. Appl. Mech.*, Vol.54, 1987, pp.828-832