NEW FEM ANALYSIS OF CRACK GROWTH BEHAVIOR IN COMPOSITE MATERIALS

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SUMMARY: The phenomena of crack growth and interfacial debonding can be regarded as the formation of a new surface. Thus, it is quite natural to model these problems by introducing the mechanism of surface formation. In this study, a new and simple method is developed in order to simulate the fracture phenomena that can be considered as the formation of new surface as a crack growth. Based on the fact that surface energy must be supplied for the formation of new surface, a potential function representing the surface energy density is introduced in the finite element method by an interface element. The general idea of the interface element and its application to a peeling test of film, dynamic crack propagation, and three-point bending test of heterogeneous material are presented.

KEYWORDS: Finite element method, interface element, interface potential, peeling test, dynamic crack propagation

INTRODUCTION

Fiber reinforced composite materials and composites with a thin film coating are applied in various fields as structural materials because of their high specific strength and stiffness which contribute to weight savings. The conventional materials, such as metals and ceramics, are also used under severe conditions due to the recent improvement of their performances. From the point of view of safe design of the structures, it is very important to estimate the fracture strength of materials with a reasonable accuracy. Many methods to evaluate the failure strength of materials have been proposed. There are basically two approaches. One is the macroscopic approach in which the concepts of stress intensity factor, energy release rate and J-integral are employed. The other is the microscopic approach such as the simulation of crack propagation using molecular dynamics [1]. To evaluate the strength of a structural component, both the macroscopic and the microscopic nature of the phenomena must be taken into account.

In this study, a new and simple method is developed in order to simulate the fracture phenomena that can be considered as the formation of new surface as a result of crack growth.
Based on the fact that surface energy must be supplied for the formation of new surface, a potential function representing the surface energy density is introduced in the finite element method by an interface element. The proposed method is applied to various types of crack propagation problems and its capability for static and dynamic analyses is demonstrated.

**METHOD OF ANALYSIS**

**Surface Potential**

Figure 1 shows an illustration of the crack propagation modeled using interface elements. The interface element consists of two surfaces and it has no thickness when the load is not acting. When the load is applied, the two surfaces separate from each other. The distance between the surfaces, or the crack opening, is denoted by $d$. The mechanical characteristics of the interface element are defined through a potential function $\phi(\delta)$. Since the function $\phi(\delta)$ can be chosen rather arbitrarily, the Lennard-Jones type potential energy [2] described by the following equation is employed in this study.

$$
\phi(\delta) = 2\gamma \cdot \left( \frac{r_0}{r_0 + \delta} \right)^{2n} - 2 \cdot \left( \frac{r_0}{r_0 + \delta} \right)^n
$$  \hspace{1cm} (1)

Where $\delta$ is the crack opening displacement and $\gamma$, $n$, and $r_0$ are material constants. In particular, $2\gamma$ is the surface energy per unit area. As shown in Fig. 2(a), $n$ controls the shape of the potential energy curve. The derivative of $\phi(\delta)$ with respect to $\delta$ gives the bonding force per unit area of the surface. As shown in Fig. 2(b), the bonding stress rapidly decreases with increasing $\delta$. Through this phenomenon, the formation of new surface within the interface element can be described [3]. Further, the crack growth can be modeled by arranging such interface elements along the crack propagation path.

![Representation of crack growth using interface element](image_url)

**Fig. 1: Representation of crack growth using interface element**
Equilibrium Equation of System

For simplicity, the outline of the mathematical formulation is presented using the crack propagation problem in an elastic solid. When the material is elastic, the equilibrium equation can be derived based on the principle of minimum potential energy.

The total energy $\Pi$ of an elastic body with a propagating crack can be described as the sum of the strain energy $U$, and the potential of the external load $W$ and the interface energy for the newly formed surface during crack propagation $U_S$, i.e.

$$\Pi = U + U_S + W$$  \hspace{1cm} (2)

In case of the finite element method, the elastic body to be analyzed is subdivided into small elements and the displacements in each element are interpolated by nodal displacement $u_0$. Noting this, the total energy is described as,

$$\Pi = \Pi(u_0) = U(u_0) + U_S(u_0) + W(u_0)$$ \hspace{1cm} (3)

Further, $U(u_0)$, $U_S(u_0)$ and $W(u_0)$ can be represented as the sum of the contributions from each element $U^e(u_0^e)$, $U_S^e(u_0^e)$ and $W^e(u_0^e)$, i.e.

$$\Pi(u_0) = \sum \{ U^e(u_0^e) + U_S^e(u_0^e) + W^e(u_0^e) \}$$ \hspace{1cm} (4)

where, $u_0^e$ is the nodal displacement vector for each element extracted from the nodal displacement vector of the whole system $u_0$.

Once the total energy $\Pi$ is given as in Eqn 4, the equilibrium equation in incremental form can be derived in the following manner. Denoting the nodal displacement at the present step and its increment to the next step as $u_0$ and $\Delta u_0$, the total energy $\Pi$ can be described as a function of $u_0 + \Delta u_0$ and it can be expanded into Taylor’s series, i.e.
\[ \Pi(u_0 + \Delta u_0) \approx \Pi(u_0) + \Delta^1\Pi(\Delta u_0) + \Delta^2\Pi(\Delta u_0) \]
\[ = \Pi(u_0) - \{\Delta u_0\}^T \{f\} + \frac{1}{2} \cdot \{\Delta u_0\}^T [k]\{\Delta u_0\} \] (5)

where, \( \Delta^1\Pi \) and \( \Delta^2\Pi \) are the first and the second terms in \( \Delta u_0 \), i.e.

\[ \Delta^1\Pi(\Delta u_0) = -\{\Delta u_0\}^T \{f\} \] (6)
\[ \Delta^2\Pi(\Delta u_0) = \frac{1}{2} \cdot \{\Delta u_0\}^T [k]\{\Delta u_0\} \] (7)

Further, the equilibrium equation can be derived as the stationarity condition of \( \Pi(u_0 + \Delta u_0) \) with respect to \( \Delta u_0 \), i.e.

\[ \frac{\partial \Pi(u_0 + \Delta u_0)}{\partial \Delta u_0} = -\{f\} + [k]\{\Delta u_0\} = 0 \] (8)

or,

\[ [k]\{\Delta u_0\} = \{f\} \] (9)

where, \([k]\) and \(\{f\}\) are the tangent stiffness matrix and the load vector, respectively.

**Stiffness Matrix and Load Vector of Interface Element**

The stiffness matrix and the load vector of the interface element can be derived in a manner basically similar to that for the whole system. Since the FEM code developed in this research is a three-dimensional code using a solid element, the same 8-node solid element is used for the interface element. As shown in Fig. 3, the interface element consists of two surfaces containing four nodes, namely nodes 1-4 for the bottom surface and 5-8 for the top surface, and it has no thickness when load is not applied. The two surfaces separate when the load is applied and the distance or the opening is denoted by \( d \). When the surface area of the interface element is \( S^e \), the interface energy for an element, \( U^e_s(u^e_0) \), is given by the following equation.

\[ U^e_s(u^e_0) = \int \phi(\delta) \cdot dS^e \] (10)

where, \( \delta \) is the opening displacement at an arbitrary point on the surface and it can be interpolated using the interpolation function \( N_i(\xi,\eta) \), i.e.

\[ \delta(\xi,\eta) = \sum N_i(\xi,\eta) \cdot (w_i + w_i) \] (11)

where \( w_i \) is the nodal displacement normal to the surface.

Finally, the stiffness matrix \([k^e]\) and the load vector \(\{f^e\}\) of the interface element can be derived by expanding \( U^e_s(u^e_0 + \Delta u^e_0) \) with respect to \( \Delta u_0 \) in the following manner.
\[
U^e (u^e_0 + \Delta u^e_0) = \int \phi (\delta + \Delta \delta) \cdot dS^e
\]

\[
= \int \phi (\delta) \cdot dS^e + \int \frac{d\phi (\delta)}{d\delta} \cdot \frac{\partial \delta}{\partial u^e_0} \cdot \Delta u^e_0 \cdot dS^e + \frac{1}{2} \int \frac{d^2\phi (\delta)}{d\delta^2} \left( \frac{\partial \delta}{\partial u^e_0} \cdot \Delta u^e_0 \right)^2 \cdot dS^e + H.O.T. \quad (12)
\]

where,

\[
\int \frac{d\phi (\delta)}{d\delta} \cdot \frac{\partial \delta}{\partial u^e_0} \cdot \Delta u^e_0 \cdot dS^e = -\left\{ f^e \right\} \left\{ \Delta u^e_0 \right\} \quad (13)
\]

\[
\frac{1}{2} \int \frac{d^2\phi (\delta)}{d\delta^2} \left( \frac{\partial \delta}{\partial u^e_0} \cdot \Delta u^e_0 \right)^2 \cdot dS^e = \frac{1}{2} \left\{ \Delta u^e_0 \right\} \left[ k^e \right] \left\{ \Delta u^e_0 \right\} \quad (14)
\]

Since the interface element has no volume or mass, the same formulation can be applied to both the static and the dynamic problems.

**Fig. 3: Interface element for mode I and interpolation of crack opening displacement**

**COMBINED MODEL CONSISTS OF SURFACE AND VOLUME**

The important feature of the proposed method is the fact that it is a combined model consisting of the surface and the volume. When the failure of materials involving crack formation and extension is discussed, the phenomena must be viewed from both mechanical properties of the surface and the volume. In other words, the failure process of the material or the structure changes with the combination of these properties [4]. Figure 4 illustrates a simple example of combined models consisting of two elements or springs. In these models, element-1 or spring-1 represents the properties of the surface. Element-2 or spring-2 represents those for the volume. The mechanical properties of the surface and the volume alone are shown in Fig. 5. In this example, three types of surface properties with the same bonding force \( F_{\text{max}} \) but with different surface energies are assumed. The volume part of the model is assumed to be elastic-plastic material with \( F_y \) as yield load. When \( F_{\text{max}} < F_y \), the phenomenon becomes elastic. While, it becomes elastic-plastic when \( F_{\text{max}} > F_y \). As shown in Fig. 6, the mode of failure changes from brittle (or unstable) to ductile (or stable) with the increase of surface energy \( \gamma \) in both the elastic and the plastic cases.
**SIMULATION OF PEELING TEST**

The surface potential given by Eqn 1 involves, \( \gamma \), \( r_0 \) and \( n \) as parameters. The parameters \( r_0 \) and \( n \) influence only the start of the crack growth but have no effect on the crack propagation process when the problem is elastic [3]. The crack propagation is mainly influenced by the
surface energy $\gamma$. Further, it has been shown that the mesh size of the FEM model does not show significant influence on the computed crack growth process.

In this section, the applicability of the proposed method to the peeling test is examined. Figure 7 shows the peeling test of metal plating on the ABS resin to be analyzed. The computed load-displacement curves are compared with the measured ones [5] in Fig. 8. The computed results show that the crack starts to grow at the maximum load. Then, the load gradually decreases with increase of peeling length. Excellent agreement between the computation and the experiment demonstrates the applicability of the proposed method.

![Fig. 7: Schematic illustration of peeling test](image)

Fig. 7: Schematic illustration of peeling test

![Fig. 8: Measured and computed load-displacement curves](image)

Fig. 8: Measured and computed load-displacement curves

**DYNAMIC CRACK PROPAGATION**

In many cases, fracture problems are dynamic phenomena. To demonstrate the potential capability of the proposed method, the dynamic crack propagation in an elastic plate as shown in Fig. 9 is analyzed. The plate has an initial crack with a length of 400 mm, and it is prestressed by the forced displacement at the top and the bottom edges. The crack is initiated by a pulse load applied at the tip of the initial crack. The pulse load increases and decreases linearly in 10 $\mu$s. The time histories of crack extension lengths $\Delta a$ for different values of pre-stress displacement $u_0$ are plotted in Fig. 10. As theoretically predicted, the speed of crack propagation increases with the value of pre-stress and it converges to the elastic Rayleigh surface wave speed, $v_R$ [6]. In the case of the plane strain problem, $v_R$ is given by the following equation:
By substituting the values of Young’s modulus, $E$, Poisson’s ratio, $\nu$, and density, $\rho$, into Eqn 15, $2.97 \times 10^3$ m/s is obtained as the value of $v_R$. While the converged value of crack propagation speed observed in Fig. 10 is approximately $2.5 \times 10^3$ m/s. Considering that the present case is a three-dimensional problem which is close to the plane stress state, this value seems reasonable.

**Fig. 9: Model for dynamic crack propagation problem**

**Fig. 10: Influence of pre-stress on crack propagation length**

**THREE POINT BENDING OF HETEROGENEOUS MATERIAL**

Composite material is heterogeneous in mechanical properties. The heterogeneity influences the ductility of the material. Three-point bending test of heterogeneous material as shown in Fig. 11 is analyzed. The material is elastic-plastic material with partial hard zone near the
initial crack. The yield stress of the hard zone is assumed to be 1.5, 2.0 and 3.0 times of the base material in each case. The width between the hard zone is assumed to be 3 mm. The load displacement curves are compared with those of the homogeneous material in Fig. 12. The load increases with the displacement until the maximum load and it decreases with the extension of the crack. When the hard zone exists near the crack, the maximum load increases. However, the drop rate of the load becomes large due to the faster crack growth in case of large yield stress of the hard zone. As a consequence, the energy absorption capacity decreases.

Fig. 11: Model of three-point bending specimen with hard zones and its dimension used for analysis

Fig. 12: Influence of yield strength of hard zone on load-displacement curve

CONCLUSIONS

In order to analyze crack propagation and peeling, a computer simulation method using an interface element is proposed. It is applied to the peeling test of film, the dynamic crack in pre-stressed plate and the three-point bending test of the heterogeneous material. Through these examples, the applicability of the proposed method is demonstrated.

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REFERENCES


