OPTIMIZATION OF A COMPOSITE MATERIAL BOX-BEAM FOR USE IN A HAPTIC DEVICE

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SUMMARY: The objective of this work was to develop an optimization method based on a finite-element procedure to find the optimal ply angles for a laminate used in a box-beam of specified dimensions and material. This box-beam can then become the base for the implementation of a linkage of the Freedom-7 Haptic Hand Controller [1], a small robotic device. The optimization algorithm is based on the Nedler-Mead Simplex Method and was used to determine the lay-ups providing the highest fundamental frequencies for a specific box-beam. For the box-beam studied, the most detrimental vibration mode was wall squinting and the optimal lay-up involved high ply angles at the top and bottom of the laminate. Changing the dimensions of the box-beam had some impact on the optimal lay-up, but more importantly on the highest achievable fundamental frequency. A cross-section with a height-to-width ratio of 2 yielded the highest fundamental frequency.

KEYWORDS: Optimization, Finite-element method (FEM), Vibration, Fundamental mode, Box-beam.

INTRODUCTION

The quality of a haptic-robotic device is highly related to its design. High-performance devices are built using specific design objectives. Mass minimization of the device is desired to properly simulate free motion. Stiffness must also be maximized to accurately simulate the restraint caused by an immovable wall [2]. Furthermore, high-frequency and low-amplitude vibrations resulting from high-stiffness and low-weight components along with high structural damping ease controller stability [3]. Composite materials, with their high stiffness-to-weight ratios and good damping properties, are therefore very appealing for use in the construction of a high-performance haptic device. However, the number of parameters involved in the definition of a composite laminate can be large and the effect that the modification of those parameters has on the structure properties is often non-intuitive. Previous work was performed by Sung and Thompson [4] and Sung and Shyl [5]. Their approach was based on the optimization of the desired structure property using Biggs' recursive-programming algorithm. A simpler optimization technique not involving Lagrangians or derivatives was desired for this work, as the function to optimize originates from a finite-element model and cannot be expressed in closed form.

FINITE-ELEMENT MODEL

The use of finite-element analysis was necessary in order to perform dynamic analysis on a structure such as a box-beam, rather than just a plate. The stress interactions at the corners make it indeed very difficult to solve analytically for a general laminate.

Element Definition

Three types of 4-node rectangular elements were used to model structures. The definition of those elements is described in Table 1 where u, v, w represents the displacements in the x, y, z directions respectively. The subscript 0 indicates that the values are those at the mid-plane of the laminate. The adaptive integration used in element 3 is described in [6] and has for effect of compensating for the excessive transverse shear stiffness (shear locking) occurring in thin shells. An in-plane characteristic dimension-to-thickness ratio of 50 is a good cut-off for determining the use of an element [7]. A ratio below that cut-off indicates transverse shears will not be negligible and element 3 should be preferred. However, structures made of very thin shells should not be modeled with this element as shear locking may occur, particularly in the case of a box-beam.

Table 1: Definition of Elements

Element Type	1	2	3
Degrees of Freedom	$u_0, v_0, w_0, \frac{\partial w_0}{\partial y}, -\frac{\partial w_0}{\partial x}$	$u_0, v_0, w_0, \frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, -\frac{\partial u_0}{\partial y}, \frac{\partial v_0}{\partial x}, \frac{\partial w_0}{\partial y}, -\frac{\partial w_0}{\partial x}$	$u_0, v_0, w_0, \frac{\partial v_0}{\partial z}, -\frac{\partial u_0}{\partial z}$
Corner/Coupling Compatibility	No	Yes	Yes
Displacement Field	$u = u_0 - z\frac{\partial w_0}{\partial x}$ $v = v_0 - z\frac{\partial w_0}{\partial y}$ $w = w_0$	$u = u_0 - z\frac{\partial w_0}{\partial x}$ $v = v_0 - z\frac{\partial w_0}{\partial y}$ $w = w_0$	$u = u_0 + z\frac{\partial u_0}{\partial z}$ $v = v_0 + z\frac{\partial v_0}{\partial z}$ $w = w_0$
Rotational Inertia	No	No	Yes
Stress-Strain Relationship	Classical Laminate Theory	Classical Laminate Theory	First-Order Laminate Theory with Adaptive Integration

Element Effectiveness

To verify the effectiveness of the elements defined above, the first vibration mode of plates and box-beams made of isotropic and composite materials were obtained for each element type and compared to reference solutions.

Isotropic Plate

An 8 by 8 mesh was used to obtain the first mode of vibration for an isotropic plate under various boundary conditions. The plate had a side length of 0.6 m and a thickness of 0.002 m. The material used had a density of 4540 kg/m^3, a Young's modulus of 114 GPa and a Poisson's ratio of 0.33. The results appear in Table 2.

It can be seen that element type 1 and 2 yield the same results for an isotropic plate. This is also the case for any symmetric laminated plate as there is no bending/in-plane deformation coupling for these cases.

Table 2: Comparison between predicted fundamental frequencies for an thin isotropic square plate

Boundary Conditions	Analytical [8] Hz	Element 1 Hz (Error)	Element 2 Hz (Error)	Element 3 Hz (Error)
SSSS	26.754	26.523 (-0.86%)	26.523 (-0.86%)	27.209 (+1.70%)
CCCC	48.814	48.021 (-1.62%)	48.021 (-1.62%)	50.809 (+4.09%)
CFFF	4.741	4.689 (-1.10%)	4.689 (-1.10%)	4.700 (-0.86%)
CCFF	9.434	9.289 (-1.54%)	9.289 (-1.54%)	9.328 (-1.12%)
CSCS	39.286	38.751 (-1.36%)	38.751 (-1.36%)	40.657 (+3.49%)
CSSS	32.057	31.707 (-1.09%)	31.707 (-1.09%)	32.816 (+2.37%)

S: Simply supported edge
C: Clamped edge
F: Free edge

Isotropic Box-Beam

Comparison was performed on a slender cantilever box-beam as described in [6]. A 20 (lengthwise) by 1 by 1 element-mesh was used as shown in Fig. 1. The box beam had a 20m length, a 1m by 1m cross-section and a 0.005m wall thickness. The material used had a density of 7861 kg/m^3, a Young's modulus of 207 GPa and a Poisson's ratio of 0.3. The results appear in Table 3.

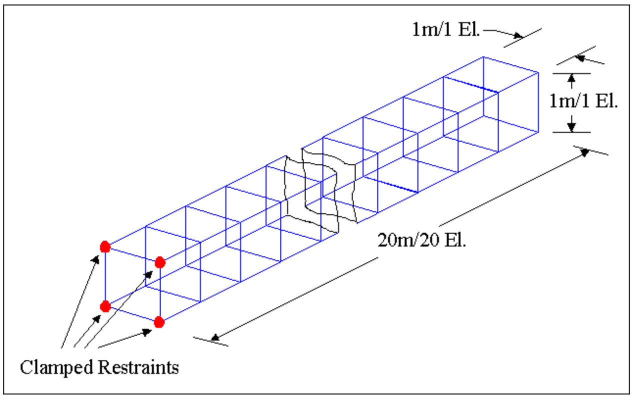

Fig. 1: Mesh used to model a slender isotropic box-beam

Table 3: Comparison between predicted modes for an isotropic box-beam

Mode #	Analytical [6] Hz	Element 1 Hz (Error)	Element 2 Hz (Error)	Element 3 Hz (Error)
1	2.912	3.080 (+5.77%)	2.942 (+1.03%)	3.090 (+6.11%)
2	17.565	7.325 (+58.3%)	7.234 (+58.8%)	18.751 (+6.75%)

Although very effective for isotropic plates, elements 1 and 2 yield poor predictions for all but the first mode in this case. It thus appears that more elements must be used to analyze a box-beam than it is required for a plate to obtain the same accuracy.

Composite Plate

A reference composite plate was modeled through a 10 by 10 thin shell quadrilateral element-mesh using I-Deas [9] finite-element software. An 8 by 8 element-mesh was used for the elements to validate. The plate side length was set to 0.2m. Each ply was 0.125mm thick and made of HMS-DX-210 as described in Table 4. The plate was simply supported. Results appear in Table 5.

Table 4: Properties of composite material used in this work [10]

Material	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	ν_{LT}	ψ_L	ψ_T	ψ_{LT}	VF	ρ (kg/m^3)
HMS-DX-210	172.7	7.20	3.76	0.3	0.0045	0.0422	0.0705	0.5	1550

E_L: Longitudinal modulus
E_T: Transverse modulus
G_{LT}: Shear modulus
ν_{LT}: Poisson's ratio
ψ_L: Longitudinal specific damping ratio
ψ_T: Transverse specific damping ratio
ψ_{LT}: Shear specific damping ratio
VF: Volume fraction
ρ: Density

Table 5: Comparison between predicted modes for a composite plate

Lay-up	Mode	Reference [9] Hz	Element 1 Hz (Difference)	Element 2 Hz (Difference)	Element 3 Hz (Difference)
$[0_4]$	1	64.092	63.519 (-0.89%)	63.519 (-0.89%)	65.488 (+2.18%)
$[0_4]$	2	85.939	84.630 (-1.52%)	84.630 (-1.52%)	90.885 (+5.76%)
$[0_4]$	3	139.503	135.203 (-3.08%)	135.203 (-3.08%)	159.465 (+14.3%)
$[(+/-30)_2]$	1	77.561	77.444 (-0.15%)	77.248 (-0.40%)	79.303 (+2.25%)
$[(+/-30)_2]$	2	147.368	147.188 (-0.12%)	146.648 (-0.49%)	156.194 (+5.99%)
$[(+/-30)_2]$	3	209.946	207.776 (-1.03%)	207.039 (-1.38%)	223.814 (+6.61%)

Composite Box-Beam

A cross-ply laminate $[0/90]_S$ was used for a cantilever box-beam 0.15m long with a 0.06m by 0.03m cross-section. A comparison test with a 10 by 3 by 2 thin shell element-mesh (respective to the previous dimensions) was performed with I-Deas [9]. The elements to validate were analyzed in a coarse mesh of 6 by 3 by 2 elements and a finer 10 by 3 by 2 mesh. Results are shown in Table 6.

Table 6: Comparison between predicted modes for a composite box-beam

Mode #	Reference [9] Hz	Element 1 Mesh: Hz (Difference)	Element 2 Mesh: Hz (Difference)	Element 3 Mesh: Hz (Difference)
1	430.465	Coarse: 392.152 (-8.90%) Fine: 392.807 (-8.75%)	Coarse: 392.145 (-8.90%) Fine: 393.410 (-8.61%)	Coarse: 602.767 (+40.0%) Fine: 602.768 (+40.0%)
2	487.577	Coarse: 453.116 (-7.07%) Fine: 458.038 (-7.07%)	Coarse: 453.109 (-7.07%) Fine: 458.703 (-5.92%)	Coarse: 653.747 (+34.1%) Fine: 650.425 (+34.1%)
3	578.006	Coarse: 469.926 (-18.7%) Fine: 471.119 (-18.5%)	Coarse: 469.898 (-18.7%) Fine: 471.409 (-18.4%)	Coarse: 855.092 (+47.9%) Fine: 854.044 (+47.8%)

This is clearly an example in which element type 3 should not be used because of the very small thickness of the laminate. Furthermore, increasing the number of elements did not have a significant effect on the frequency prediction, showing that convergence can be obtained with few elements.

In general, the elements used in this work can be trusted to yield results of a respectable accuracy, provided they are used in cases for which their definition assumptions apply.

OPTIMIZATION

While a designer usually has an idea of the material and the number of layers to use in a composite structure, each ply angle remains a parameter defining the laminate and finding the optimal configuration is not obvious. Several efficient approaches exist to optimize a function of several variables as described in [11]. In all cases, the difficulty and the number of computations required to optimize a function increases exponentially with the number of independent parameters. When the function is evaluated through finite-element analysis, each function call requires a substantial amount of computation time. In an effort to minimize the number of function evaluations, each test is recorded and for a case deemed “close enough”; a previously found result is outputted. This has some effect on the accuracy of convergence that will be explained soon.

Optimization Routine

The user must first define a structure, a suitable finite-element-mesh and boundary conditions. The user also sets the number of plies, the materials and the angular ranges. Some additional constraints can be added to make the laminate symmetric or anti-symmetric or to constrain a single ply. Finally the angular precision of the overall process and the optimization objective are entered.

Three optimization methods are successively used in this work to obtain a final result. Let n be the number of parameters (independent ply angles). A random search is done within the specified range of the various angles during which the function to optimize is evaluated on 20n points. From these tests, the points suspected to be near different optimums are isolated. These points are then used as starting points in a modified version of the Nedler-Mead Simplex Method [12]. The original convergence test, putting emphasis on the function to optimize, was changed to observe convergence of the ply angles. As a result, unnecessary accuracy is avoided. However, when approaching convergence, previous test results are often outputted instead of the actual evaluation. The grid effect due to the test-recording feature translates into a convergence point that can be close but off the actual optimum. For this reason, a third method is necessary. Very simply, each parameter is modified independently by units of angular precision to find the optimum from the neighboring points and to insure that modifying only one parameter from that optimum will always lead to a worse result. This three-phase optimization method ensures good, consistent convergence. It was observed that the degree of precision involved in evaluating the function had a definite influence on the location of the optimum found. This last optimization phase is therefore solved with a higher accuracy and previously found results are not called unless they were of the same accuracy.

When several starting points are found by the random search, a supplemental convergence test predicts if an optimization round will converge towards a previously evaluated optimum and the search is stopped when appropriate. When other optimums are found within an acceptable range of the maximum of the function, they are outputted as alternatives.

OPTIMIZATION RESULTS

Repetitiveness

An optimization test for a simply supported square plate (0.2m by 0.2m) composed of four 0.125mm thick plies of HMS-DX-210 was performed using different element types and mesh size. The objective was to achieve the highest natural frequency. No constraint on the layers was used. The angular precision was set to 1 degree. The results shown in Table 7 clearly show that anti-symmetric and symmetric laminates were favored for this square plate having symmetric boundary conditions. These results are consistent with those found by Jones, Morgan and Whitney [13] as to the location of the optimum for the anti-symmetric case.

Table 7: Optimal lay-up for a specific simply supported rectangular plate

Mesh	El. #	Solution (Hz)	Alternative (Hz)
4 by 4	1	$[(+/-45)_2]$ (81.477)	$[+/-45]_S$ (77.038)
4 by 4	3	$[(+/-45)_2]$ (89.010)	$[+/-45]_S$ (84.451)
6 by 6	1	$[(+/-45)_2]$ (82.040)	$[+/-45]_S$ (76.818)
6 by 6	3	$[(+/-45)_2]$ (85.354)	$[+/-45]_S$ (80.138)

These encouraging results show the consistency of the convergence routine and that small but sufficient element-meshes can be used to find the optimum efficiently. This, of course, means a lot of computational time can be saved by the use of a coarser mesh.

Box-Beam Optimization

A box-beam made of 0.125mm thick layers of HMS-DX-210 had to be optimized. The dimensions chosen were those of a box-beam under consideration of being part of a haptic-robotic structure: a 0.15m length and a 0.06m by 0.03m cross-section. The optimization objective was to obtain the highest first mode frequency and reference to the optimal lay-up for the remainder of this work will refer to that objective unless otherwise stated. The box-beam was assumed cantilevered. Since reducing the mass of the part is also an objective, the test was done using different numbers of plies. One could then judge if the added weight of an extra layer is worth the possible increase in fundamental frequency. A 6 by 3 by 2 element-mesh was used. The angular precision was set to 1 degree. The optimized lay-ups are presented in Table 8. A fiber along the length of the box-beam would be at 0 degree and layer 1 is on the interior of the box-beam.

Table 8: Optimal lay-ups for a box-beam of specified dimensions

Test #	Plies	El. #	Constraint	Solution Lay-up (Freq. Hz)	Second Mode Freq. Hz (From 1st mode freq.)	Alternative (Freq. Hz)
1	2	1	None	[78/78] (394.907)	395.285 (+0.0957%)	none found
2	3	1	None	[78/-6/79] (636.228)	664.125 (+4.38%)	none found
3	3	1	Anti-Symm.	[90/0/-90] (630.115)	657.207 (+4.30%)	none found
4	3	3	None	[82/2/83] (1061.925)	1062.380 (+0.0428%)	none found
5	4	1	None	[89/-17/-36/85] (863.302)	864.297 (+0.115%)	[82/-26/25/-86] (852.153)
6	4	1	Symm.	[88/-14/-14/88] (862.378)	864.837 (+0.285%)	none found
7	4	1	Anti-Symm.	[84/-25/25/-84] (852.137)	852.137 (+3.16%)	none found
8	5	1	None	[88/-47/5/-49/87]* (1081.542)	1082.335 (+0.0733%)	[88/-38/2/-53/86] (1079.870)
9	5	1	Symm.	[87/-46/3/-46/87] (1080.952)	1081.004 (+0.00481%)	[-89/30/-48/30/-89] (1067.433)
10	5	1	Anti-Symm.	[79/-55/0/55/-79] (1062.721)	1065.112 (+0.225%)	[84/-36/-44/36/-84] (1045.600)
12	6	1	Symm.	[86/-46/13/13/-46/86] (1267.039)	1267.044 (+0.00039%)	none found
13	6	1	Anti-Symm.	[84/-49/10/-10/49/-84] (1262.075)	1262.084 (+0.00071%)	[88/10/-59/59/-10/-88] (1217.386)

* The optimization process was repeated three times in order to obtain this solution

The very high angles for the bottom and top plies are an indication that the most detrimental vibrations come from wall squinting as shown in Fig. 2. Putting fibers at a high angle adds the stiffness necessary to reduce this phenomenon. Test 4 done with element type 3 resulted in an exaggerated frequency as observed in the previous composite box-beam comparison. Nonetheless, each ply of this optimal lay-up prediction is within 8 degrees of the prediction obtained using element type 1, proving the possibility of optimization using a low accuracy model.

Observation of the second mode frequency for the optimal lay-ups revealed that in most cases it is almost identical to the first mode frequency. This unexpected behavior is a reassuring result. It shows that while adding stiffness to resist the first mode and increasing its frequency, the other mode frequencies are being reduced. This improvement of the first mode frequency can only be done until another mode of vibration becomes as critical and has an equivalent frequency.

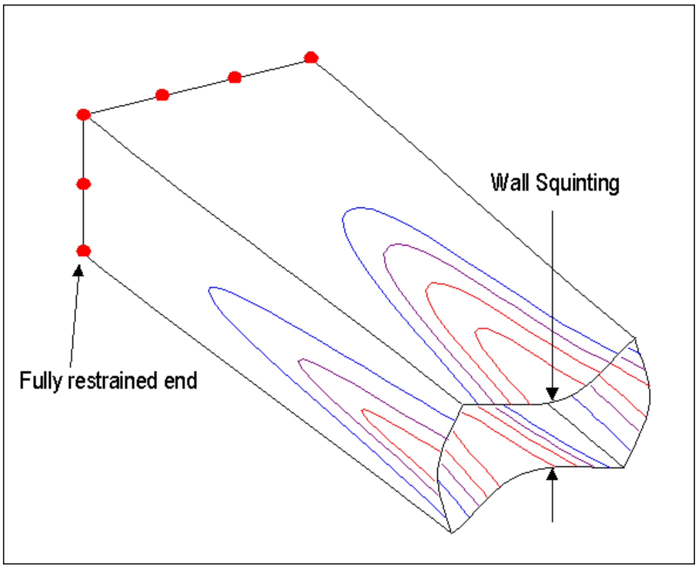

Fig. 2: Illustration of wall squinting

Adding Constraints

The test results shown in Table 8 indicate that the highest first mode frequency is obtained from a general laminate. However, there exists an almost equally good symmetric solution and there also exists an anti-symmetric alternative which, in most cases, is quite acceptable. This anti-symmetric solution tends to be as good as the symmetric solution as the number of layers increases and the coupling modulus of the laminate decreases. This behavior is consistent with the plate example studied in [13].

This quality of the symmetric and anti-symmetric solution compared to that of the actual optimum laminate might not be present for every function to optimize. This should therefore be verified before making any assumption. Nonetheless, symmetric and anti-symmetric laminates have twice as few parameters to optimize as a general laminate and the computational time required to obtain convergence is reduced as well as the risk of only finding a local optimum. It is also possible to constrain a specific angle to a range or a unique value (for strength for example). This reduces the computational effort as well.

Parametric Study

It would be advantageous to have beforehand an idea of the range of all layer angles for the optimized laminate, thus reducing the search effort. The sensitivity of the fundamental frequency to changes in ply angle is first studied. The effects of changing dimensions on the optimal lay-up are then demonstrated.

Angular Sensitivity

The optimal lay-up for the unconstrained four-ply box-beam as found in Table 8 was modified in different ways to determine the sensitivity of fundamental frequency to ply angle variation. The results are shown in Table 9.

Table 9: Sensitivity of fundamental frequency to ply angle variation

Layer #	+2° Change: Frequency Hz (Difference)	-2° Change: Frequency Hz (Difference)	+5° Change: Frequency Hz (Difference)	-5° Change: Frequency Hz (Difference)
4	862.856 (-0.052%)	862.091 (-0.140%)	859.395 (-0.453%)	856.986 (-0.732%)
3	861.856 (-0.167%)	862.804 (-0.058%)	858.227 (-0.588%)	861.747 (-0.180%)
2	862.389 (-0.106%)	852.833 (-1.213%)	860.890 (-0.279%)	835.571 (-3.212%)
1	862.446 (-0.010%)	862.889 (-0.048%)	858.885 (-0.512%)	859.788 (-0.407%)

From these results, it can be seen that for this optimal lay-up, the second layer is the critical one and the fundamental frequency reacts promptly when it is given a negative angle variation.

Effects of Modifying Dimensions

The effects of the modification of only one or all dimensions of the box-beam on the optimal laminate to be used have been studied. Table 10 shows the modifications that were done to the original 4-ply box-beam. The numbers in the dimension columns represent the factor by which each dimension was multiplied. Element type 1 was used throughout these tests with a mesh of 6 by 3 by 2 and an angular precision of 1 degree.

Table 10: Effects of multiplying dimensions of a box-beam on its optimal lay-up for maximum fundamental frequency

Case	Length	Height	Width	Solution (Freq. Hz)	Alternative (Freq. Hz)	Remark
1	1	1	1	[89/-17/-36/85] (863.302)	None found	Original solution (2 to 1 cross-section)
2	1	1	1.5	[88/-68/-69/88] (456.672)	[-84/75/-73/86] (456.486)	Flatter box-beam (4 to 1 cross-section)
3	1	2	1	[90/54/55/90] (633.577)	[80/-68/62/-86] (632.343)	Square cross-section (1 to 1 cross-section)
4	2	1	1	[-88/-3/32/-29] (470.886)	[-27/34/-2/-89] (468.379)	Slender box-beam (2 to 1 cross-section)
5	1.5	1.5	1.5	[88/-32/-32/88] (385.846)	None found	Full scaling (2 to 1 cross-section)

These tests seem to indicate that changes to the cross-section will result in some changes of the optimal lay-ups, mainly in the inner layers. However, it must be pointed out that the maximum achievable fundamental frequency is very strongly influenced by the cross-section dimensions. Case 1 resulted in the highest fundamental frequency with a height-to-width ratio of 2.

Increasing the length of the box-beam as in case 4 resulted in the axial vibrations to be no longer negligible. To resist those modes requires the presence of more fibers along the length of the box-beam. This is indeed what is observed for that case in layer 4 or in layer 1. Also, as one could expect, the maximum achievable fundamental frequency decreases with the increasing length of the box-beam.

Sensitivity to Boundary Conditions

Wall squinting is definitely at its worse at the free end of the box-beam. Clamping the other end as well would reduce the phenomenon and the optimal laminate could be expected to be different. The optimal lay-up obtained for the original 4-layer box-beam with both ends clamped was [80/-56/-55/81] with a fundamental frequency of 907.986 Hz. An alternative lay-up was [90/48/-69/74] with a fundamental frequency of 904.810 Hz. Clamping the other end of the box-beam reduced the squinting effect, but also reduced the vibration from other modes, leaving wall squinting the critical vibration mode, this time in the middle of the box-beam.

Inserting a Core

Another way to reduce wall squinting is to add a core to the laminate, increasing its flexural modulus. A 2mm core of density 25 kg/m^3 was inserted in the middle of the four-ply laminate used in the original cantilevered box-beam. The optimal lay-up was [-1/0/Core/47/-49] with a first mode frequency of 1934.060 Hz. Two other local maximums were found at [69/0/Core/26/-38] (1854.736 Hz) and [0/71/Core/27/-38] (1852.897 Hz). The solution to this problem shows that adding a core is an efficient way to reduce the effects of wall squinting.

CONCLUSIONS

A method to calculate optimal composite laminate ply angles has been presented in this work. Numerical examples have been shown for the case of a box-beam destined to be part of a haptic-robotic structure where the frequency of the first mode had to be maximized. It was shown that wall squinting is the critical vibration factor for a cantilevered box-beam of that size and high ply angles are required at the top and the bottom of the laminate to obtain the highest possible first mode frequency.

Modifications to the cross-section of the beam did not have a large influence on the optimal lay-up configuration, but the fundamental frequency obtained with this optimal lay-up was affected a great deal. A height-to-width ratio of 2 yielded the highest natural frequency. Increasing the length of the box-beam had for result an optimal lay-up with smaller ply angles and a lower possible fundamental mode. The parametric study performed in this work yielded results that could be explained, but was insufficient to draw a pattern allowing prediction of an optimal lay-up without the use of the computer routine. Further study should reveal whether it is possible to obtain a table of optimal laminates for non-dimensional cases.

The type of element used to model the box-beam had little influence on the finding of the optimal lay-up. Nonetheless, it would be a good idea to use an element yielding better vibration frequency predictions provided the size-order of the problem to be optimized is not increased.

As the number of free parameters increases in the lay-up to optimize, the risk of finding only a local optimum increases as well. Further improvements to the optimization routine could be to use the present results to guess the location of the optimum and start the iterative process from there. Alternatively, the optimum symmetric lay-up could be used as a starting point if the symmetric case represents a good alternative for the function to optimize.

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