THE NEW ACTION PLANE RELATED STRENGTH CRITERION IN COMPARISON WITH COMMON STRENGTH CRITERIA

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SUMMARY: Inter fibre fracture (IFF) of UD-laminae with polymer matrix is governed by the stresses σ_n, τ_{nt}, τ_{n1} on the fracture plane, which can be found as the plane with the highest stress exposure factor calculated from a (σ_n, τ_{nt}, τ_{n1})-strength criterion by *Puck*. For visualisation a given (σ_2, σ_3, τ_{23}, τ_{31}, τ_{21})-fracture condition is rewritten as a (σ_{II}, σ_{III}, $\tau_{\omega 1}$, δ)-fracture condition, wherein σ_{II} and σ_{III} are the 'transverse principal stresses' and $\tau_{\omega 1}$ is the longitudinal shear stress resulting from τ_{31} and τ_{21} while δ is the deviation angle between the action planes of σ_{II} and $\tau_{\omega 1}$. The fracture body of *Puck's* criterion shows a distinct influence of δ in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$)-space, whereas classical IFF-criteria as the *Tsai,Wu*- or the *Hashin*-criterion do not exhibit any influence of this parameter. In order to avoid the search of the fracture plane several attempts have been made to develop approximations of the (σ_n, τ_{nt}, τ_{n1})-strength criterion by direct invariant formulations in (σ_2, σ_3, τ_{23}, τ_{31}, τ_{21}), which are investigated in this paper too. However the saving of some calculation time seems unjustified in consideration of the loss of information about the fracture angle and mode.

KEYWORDS: strength criteria, fracture body, visualisation technique, inter fibre fracture, *Coulomb,Mohr* hypothesis, action plane, fracture angle, new testing technologies

INTRODUCTION

In some UD-layers of laminated tubular torsional springs harmless inter fibre fracture (IFF) and in others catastrophic IFF occurred. This depends on the inclination angle θ_{fp} of the fracture plane (fp) against the thickness direction of the layer. Fortunately a new generation of IFF-criteria has been developed, which does not only calculate the combined stresses causing IFF realistically but also predicts the expected fracture angle θ_{fp} theoretically. The new IFF-criteria are based on brittle fracture observed in most composites with polymer matrix for structural applications and therefore are attributable to the hypotheses of *Coulomb* and *Mohr*. Consequently it is assumed that only the normal stress and the shear stress acting on a fibre parallel plane are responsible for the risk of IFF in that plane. As the additional information about the fracture angle is very helpful for laminate design the new method has been improved and verified by a great number of experiments within the last years and implementation is on the way now.

In the 'ESDU Data Sheet – Failure Criteria' [1] innumerable strength criteria are listed, but the user does not get any reasonable suggestion for his choice. As the circumstances are quite the same in other surveys it is nearly impossible for him to find out, in which way even well-known strength criteria delimit from each other. Today the designer more or less has to trust the models implemented in the design software blindly.

Up to now the only comparison of strength criteria available in literature is limited to fracture bodies that capture at best three of the five IFF causing stresses σ_2, σ_3, τ_{23}, τ_{31}, τ_{21}; usually only selected two dimensional fracture envelopes are examined. This shortcoming is overcome by an appropriate 3D-visualisation technique presented in the next chapter, which is based on elementary relations known from mechanics. The resulting display is somehow a little unusual but one is enabled to compare the complete fracture bodies of the highest developed strength criteria for unidirectionally reinforced plastics to the fracture body of the new action plane related IFF criterion.

DERIVATION OF THE (s_{II}, s_{III}, t_{w1})-STRESS STATE

A general three-dimensional state of stress in a UD-lamina consists of three normal stresses σ and three shear stresses τ. It is advisable to relate them to a (x_1, x_2, x_3)-coordinate system with the x_1-direction parallel to the fibres (compare Fig. 5). However, to characterise a certain kind of stress the use of subscripts $\parallel$ and $\perp$ is more appropriate, because they emphasise the transverse isotropic nature of the material. If we use the symbols ($\parallel$) and ($\perp$) the expression 'stressing' instead of stress is more appropriate. This leads to the distinction of a longitudinal normal stressing $\sigma_\parallel$ and transverse normal stressing $\sigma_\perp$, a transverse shear stressing $\tau_{\perp\perp}$ and a longitudinal shear stressing $\tau_{\perp\parallel}$. Furthermore the $\sigma_\parallel$- and $\sigma_\perp$-stressings have to be distinguished by using $^{(+)}$ for tension and $^{(-)}$ for compression. The sign of shear stresses has no effect on the strengths. Knowing this one can define the basic strength parameters $R_\parallel^{(+)}$, $R_\parallel^{(-)}$, $R_\perp^{(+)}$, $R_\perp^{(-)}$, $R_{\perp\perp}$, $R_{\perp\parallel}$, which are used as positive values in fracture criteria. Correspondingly a strength criterion for UD-laminae is in principle described as:

$$F(\sigma_1,\sigma_2,\sigma_3,\tau_{23},\tau_{31},\tau_{21},R_\parallel^{(+)},R_\parallel^{(-)},R_\perp^{(+)},R_\perp^{(-)},R_{\perp\perp},R_{\perp\parallel},...)\overset{<}{\underset{>}{=}}1 \qquad (1)$$

In most cases the so called 'fracture function' F on the left hand side of Eqn (1) is a polynomial of the existing stresses and the strength parameters appear in the coefficients. The points in Eqn (1) mean that occasionally in the material coefficients some strength values from tests with combined stresses may occur. In most cases the strength criterion is formulated for the limiting stress state with $F(\sigma_1,...) = 1$. Then Eqn (1) is called a fracture condition.

The visualisation of fracture conditions for isotropic material is not difficult at all. In this case the 6 stress components σ_1, σ_2, σ_3, τ_{23}, τ_{31}, τ_{21} are substituted by 3 principle normal stresses σ_{IP}, σ_{IIP}, σ_{IIIP}. For fibrous transversal-isotropic materials such a change of the coordinate systems is not possible, as a rotation is only allowed in the so called transversal-isotropic (x_2, x_3)-plane. The x_1-axis must remain parallel to the fibre direction, because only for these 'natural directions' the basic strength parameters of the UD-layer are known.

The following investigations will only consider stresses having an direct influence on IFF. The fibre parallel stress σ_1 is disregarded, since IFF occurs in a fibre parallel plane [2, 3] and with that the fracture plane is parallel to σ_1. The corresponding situation for an isotropic material is that $\sigma_1 = \sigma_{IP}$ is neither the minimal nor the maximal principle normal stress. According to *Mohr* only the extreme values of the principle stresses σ_{IIP} and σ_{IIIP} do have an influence on the fracture process; for this reason fracture occurs parallel to the direction of σ_{IP} similar to the IFF in an UD-lamina.

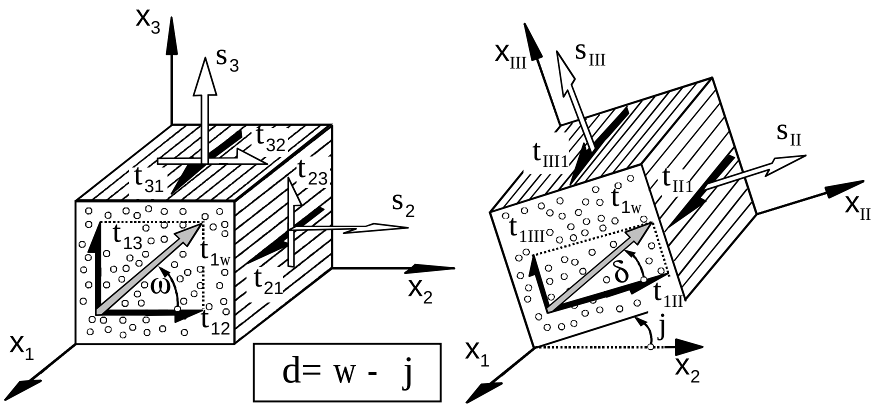

Fig. 1: Principle stresses s_{II}, s_{III} *of the transversal-isotropic plane*

For human imagination it is necessary to restrict fracture bodies to three dimensions. Therefore it seems reasonable in a first step to transform the (σ_2, σ_3, τ_{23}, τ_{31}, τ_{21})-stress state in consideration of the transversal isotropy into a (σ_{II}, σ_{III}, 0, τ_{III1}, τ_{II1})-stress state (Fig. 1). By no means σ_{II} and σ_{III} are 'real' principle stresses σ_{IIP} and σ_{IIIP}, because also the shear stresses τ_{III1}, τ_{II1} occur on their action planes (Fig. 1). Using the following transformation rule one is enabled to substitute any stress component appearing in a fracture condition in the way wanted:

$$\begin{Bmatrix} \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix} = \begin{bmatrix} \cos^2\varphi & \sin^2\varphi & -2\cdot\sin\varphi\cdot\cos\varphi & 0 & 0 \\ \sin^2\varphi & \cos^2\varphi & 2\cdot\sin\varphi\cdot\cos\varphi & 0 & 0 \\ \sin\varphi\cdot\cos\varphi & -\sin\varphi\cdot\cos\varphi & \cos^2\varphi-\sin^2\varphi & 0 & 0 \\ 0 & 0 & 0 & \cos\varphi & \sin\varphi \\ 0 & 0 & 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{Bmatrix} \sigma_{II} \\ \sigma_{III} \\ 0 \\ \tau_{III1} \\ \tau_{II1} \end{Bmatrix} \tag{2}$$

with $\varphi = \dfrac{1}{2}\arctan\dfrac{2\cdot\tau_{23}}{\sigma_2-\sigma_3}$

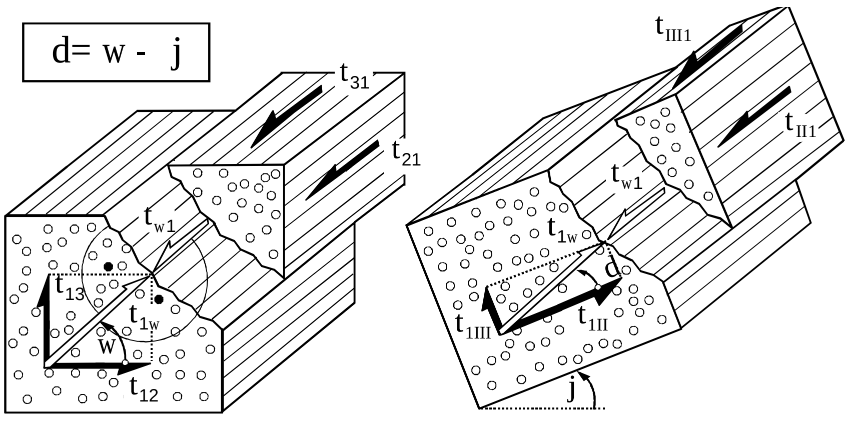

Fig. 2: Resulting shear stress $t_{\omega 1}$ *and deviation angle d*

The next step is to replace the shear stresses τ_{III1} and τ_{II1} by the resulting longitudinal shear stress $\tau_{\omega 1}$ as shown in Fig. 2. The Figure clarifies that τ_{1III} and τ_{1II} are nothing else but the components of a longitudinal shear stress $\tau_{1\omega}$ acting on a plane perpendicular to the x_1-direction [4]:

$$\tau_{1\omega} = \sqrt{\tau_{1III}^2 + \tau_{1II}^2} = \sqrt{\tau_{III1}^2 + \tau_{II1}^2} = \tau_{\omega 1} \tag{3}$$

The direction of $\tau_{1\omega}$ is defined by the angle δ:

$$\delta = \arctan\frac{\tau_{1III}}{\tau_{1II}} = \arctan\frac{\tau_{III1}}{\tau_{II1}} \tag{4}$$

We call δ the difference angle or deviation angle because its value is a measure for the deviation of the action planes of $\tau_{\omega 1}$ and σ_{II}. Stress states without transversal principal stresses

σ_{II} and σ_{III} will lead to an IFF in the fibre parallel plane the resultant shear stress $\tau_{\omega 1}$ is acting in. In this special case the fracture angle is $\theta_{fp} = \omega = \arctan \tau_{31}/\tau_{21}$ and fracture takes place after $\tau_{\omega 1}$ reached the longitudinal shear strength $R_{\perp\parallel}$. One can imagine that in general the deviation angle δ between the action planes of $\tau_{\omega 1}$ and σ_{II} is of main importance for the fracture process. It is obvious that in dependence of δ the interaction between the transverse principle stresses σ_{II}, σ_{III} and the resulting shear stress $\tau_{\omega 1}$ is more or less distinct. As an example one can regard a ($\sigma_{II} > 0$, 0, $\tau_{\omega 1}$, δ)-stress state with $\delta = 0°$ on the one hand and $\delta = 90°$ on the other hand. In contrast to the complete interaction of σ_{II} and $\tau_{\omega 1}$ for $\delta = 0°$ there is no interaction at all for $\delta = 90°$ and failure does not take place as long as either σ_{II} reaches the transverse tension strength $R_{\perp}^{(+)}$ or $\tau_{\omega 1}$ reaches $R_{\perp\parallel}$. It is inevitable to consider δ as a parameter for the visualisation of fracture conditions. From an academic point of view this leads to an infinite number of fracture bodies in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$)-stress space, as any small change of δ might modify the fracture body slightly. For the comparison of different fracture conditions it is absolutely sufficient to make a $\Delta\delta = 22{,}5°$ grading.

VISUALISATION OF FRACTURE BODIES

All fracture conditions visualised in this chapter were calculated with the same set of basic strength values published in [5]. If a criterion is complemented by additional material coefficients the suggestions of the accompanying developer were fulfilled.

Fracture conditions independent of d

Tsai,Wu-criterion

A classical representative of a global strength criterion is the well-known criterion of *Tsai* and *Wu* [6]. It is not based on a fracture hypothesis, that means a plausible assumption about the physical reason of fracture, instead it is a pure interpolation polynomial deduced from the basic strength values of the UD-layer, not regarding the fact that some strength values are dominated by fibre strength and others by matrix or interface strength.

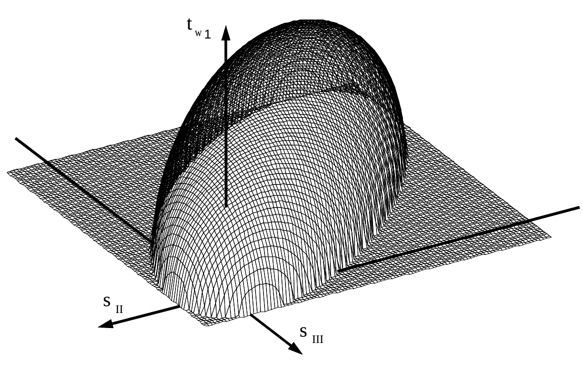

Fig. 3: Fracture body of the Tsai,Wu-criterion

The result is a smooth fracture body without any edges. Most certainly the fracture body does not change its shape if the influence of σ_1 is neglected. According to expectations the *Tsai,Wu*-criterion takes on the shape of an ellipsoid in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$)-stress space, too (Fig. 3). There is no dependence on the deviation angle δ. As σ_{II} and σ_{III} are treated absolutely equally in the invariant formulation of the criterion the fracture body is moreover symmetrical with regard to the ($\sigma_{II} = \sigma_{III}$)-plane. Therefore stress states having a transverse normal stress and a longitudinal shear stress acting on the same plane are treated equally to those having their action planes perpendicular to each other. Another remarkable aspect of this fracture body is that it is closed for biaxial compressive stresses $\sigma_{II} \approx \sigma_{III}$. From a makromechanical point of view –which is the basis of all strength criteria investigated in this paper – the shear fracture due to transverse compression is not possible for $\sigma_{II} = \sigma_{III}$ as the maximum transverse shear stress $\tau_{\perp\perp,max} = ½\cdot(\sigma_{II} - \sigma_{III})$ always remains zero. However – because of micromechanical inhomogeneity – a 'damage limit' for biaxial transverse compressive stresses must be expected [7]. After exceeding this 'damage limit' the micro-cracks in the matrix will reduce the original mechanical properties noticeably but a real material separation does not happen.

Hashin-criterion

Usually the term *Hashin*-criterion for IFF is related to two invariant fracture conditions published in [2], although in the same publication an action plane related approach is discussed too, which gave *Puck* the impulse to develop his action plane related IFF-criterion [8]. As *Hashin* was aware of the importance of the sign of the normal stress σ_n on the fracture plane, his invariant formulations differentiate between the cases $\sigma_n \geq 0$ and $\sigma_n < 0$. Unfortunately the fracture plane is not known a priori and can not be found by using invariant formulations. Therefore *Hashin* had to define a more practical condition to delimit his fracture conditions from each other. He fixed the boundary of the sections for $\sigma_n \geq 0$ and for $\sigma_n < 0$ in the plane $\sigma_{III} = -\sigma_{II}$. However he simultaneously pointed out the severe physical contradictions, which result from this rather arbitrary boundary.

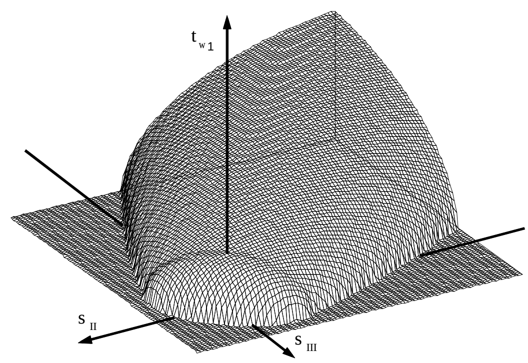

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Fig. 4: Fracture body of the Hashin-criterion

The corresponding fracture body is composed of a paraboloid with elliptical cross sections for $\sigma_n < 0$ and a crosswise oriented ellipsoid for $\sigma_n \geq 0$. Both are symmetric to the ($\sigma_{II} = \sigma_{III}$)-plane and intersect each other in the ($\sigma_{III} = -\sigma_{II}$)-plane in an elliptical borderline. The fracture curve for $\tau_{\omega 1} = 0$ and $\sigma_n \geq 0$ contradicts to the theory of *Paul* [9], which assumes no interactions of the principle stresses in this area, but *Hashin* already takes into account that a real fracture due to biaxial transverse compression is not possible. Accordingly the fracture body is not closed for $\sigma_{II} \approx \sigma_{III}$. Again it has to be emphasised that the Hashin fracture body is independent of the deviation angle δ.

Fracture conditions dependent on d

Action plane related IFF-criterion by Puck

The new criterion for IFF, developed by *Puck* [3, 4, 8], is based on physical foundations. It assumes, that almost all composites in structural applications with a polymer matrix behave brittle; therefore it is essentially based on the hypothesis of *Coulomb* and *Mohr* and its modification by *Paul*. *Puck's* IFF criterion is formulated for a rotatable coordinate system, which is referred to the fracture plane. Using the criterion for any state of stress consisting of the five possible IFF causing stresses (σ_2, σ_3, τ_{23}, τ_{31}, τ_{21}) one can calculate the angle dependent stress exposure factor $f_E(\theta)$ or 'risk of fracture' and is enabled to find the plane with the highest stress exposure factor $f_E = f_E(\theta_{fp})$ (Fig. 5).

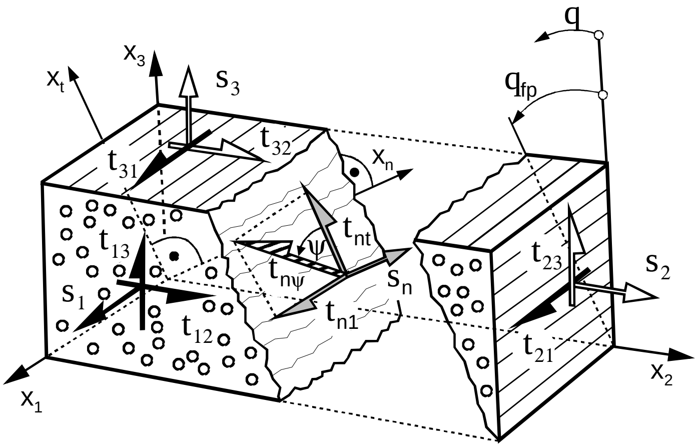

Fig. 5: Stress s_n, t_{nt} and t_{n1} on fibre parallel planes

Following *Hashin´s* suggestions in [2] *Puck* based his IFF criterion on the interaction of the three stress-components σ_n, τ_{nt}, τ_{n1} acting on an inclined fibre parallel plane and not on the usual notation in the (x_1, x_2, x_3)-ply coordinate system. *Hashin* has not pursued this approach because he was afraid of the additional numerical effort for solving the extremum problem. Meanwhile *Puck* has found suitable solutions for every day design practice, which are described in [3] in full detail. Puck's new IFF criterion is based on the following physically reasonable fracture hypotheses [3]:

1. **Inter fibre fracture in a plane parallel to the fibres is exclusively determined by the shear stresses t_{nt} and t_{n1} and the normal stress s_n that are acting in this plane.**
2. **The normal stress s_n promotes fracture if it is a tensile stress in combination with the shear stresses t_{nt} and t_{n1} or even alone for $t_{nt} = t_{n1} = 0$. In contrast to that s_n impedes fracture as a compressive stress by rising the fracture resistance of the fracture plane against shear fracture with increasing compressive stress.**
3. **For $t_{n1} = 0$, the fracture occurs either as tensile fracture due to s_n (the maximum principal stress) or as shear fracture due to t_{nt}, which is impeded by a compressive stress s_n.**

Following these hypotheses, it is necessary to formulate two different fracture conditions, one for tensile σ_n and one for compressive σ_n. These fracture conditions describe the so called Master Fracture Body. Its surface, the Master-Fracture-Surface, is defined by all vectors describing the minimum state of stress leading to IFF.

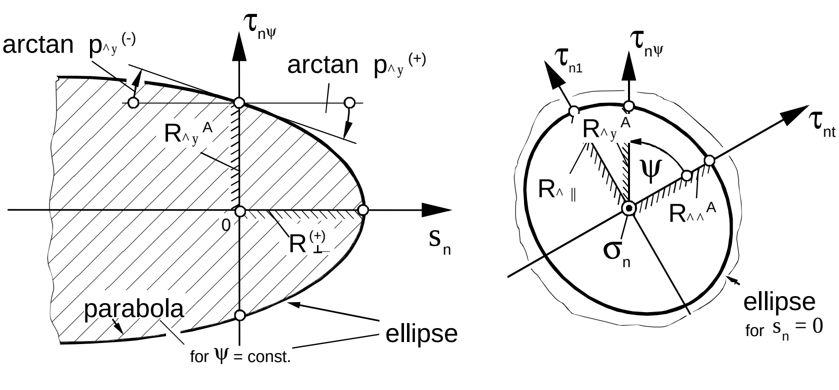

Fig. 6: Sections of the Master Fracture Body

To fulfil the second part of Hypotheses 2 the fracture body may not intersect the negative σ_n-axis, instead the contour lines must ascend to correspond to the rising of the shear fracture resistance. Thus starting from the ellipse at $\sigma_n = 0$, one can follow lines with the shape of parabolas going into the direction of the negative σ_n-axis, which define the fracture surface for $\sigma_n < 0$. The fracture body for $\sigma_n \geq 0$ is defined in a similar way. In this case an elliptic contour line gives a reasonable description for the fracture hypotheses.

In contrast to the conventional fracture criteria, *Puck's* fracture conditions can be used to calculate the stresses leading to fracture only if the angle θ_{fp} of the fracture plane has been determined first. A certain plane is predestined to become the fracture plane; it is the one for which the angle dependent stress exposure factor $f_E(\theta)$ has its global maximum or the angle dependent reserve factor $f_R(\theta)$ a global minimum respectively at given fix ratios of the combined stresses. In order to use the *Puck* IFF criterion one has to determine the stresses

$\sigma_n(\theta)$, $\tau_{nt}(\theta)$, $\tau_{n1}(\theta)$ on the fibre parallel planes first. They can be calculated by the well-known transformation rule shown in Fig. 7. In dependence of σ_n being positive or negative they are inserted into the valid fracture condition. This has to be repeated for all angles θ between -90° and 90° until that cutting angle θ_{fp} is found for which the highest 'risk of fracture' exists, which means the global maximum of *Puck's* fracture conditions, is found.

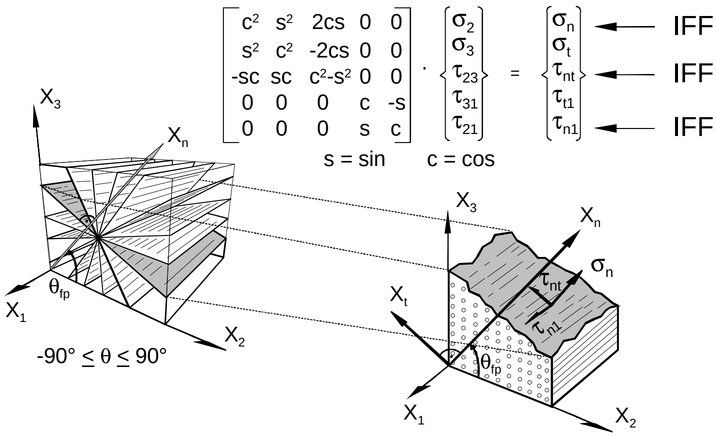

Fig. 7: Determination of the fracture plane

Beyond the usual statement 'fracture will occur or not' the procedure outlined gives additional information about the angle of the fracture plane. With this estimated angle θ_{fp} the interpretation of what will happen after IFF is possible for the first time. A detailed description of the procedure including all equations to apply the action plane IFF-criterion is given in [10, 11]. Fig. 8 displays the fracture body of *Puck's* IFF-criterion in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$)-stress space in dependence on δ. Of course all fracture bodies show the same base in the (σ_{II}, σ_{III})-plane, which has – except of the compression/compression domain – the shape of *Paul's* fracture envelope for intrinsically brittle isotropic material [9]. Comparable to *Hashin's* approach in the third quadrant fracture due to biaxial transverse compression is not possible.

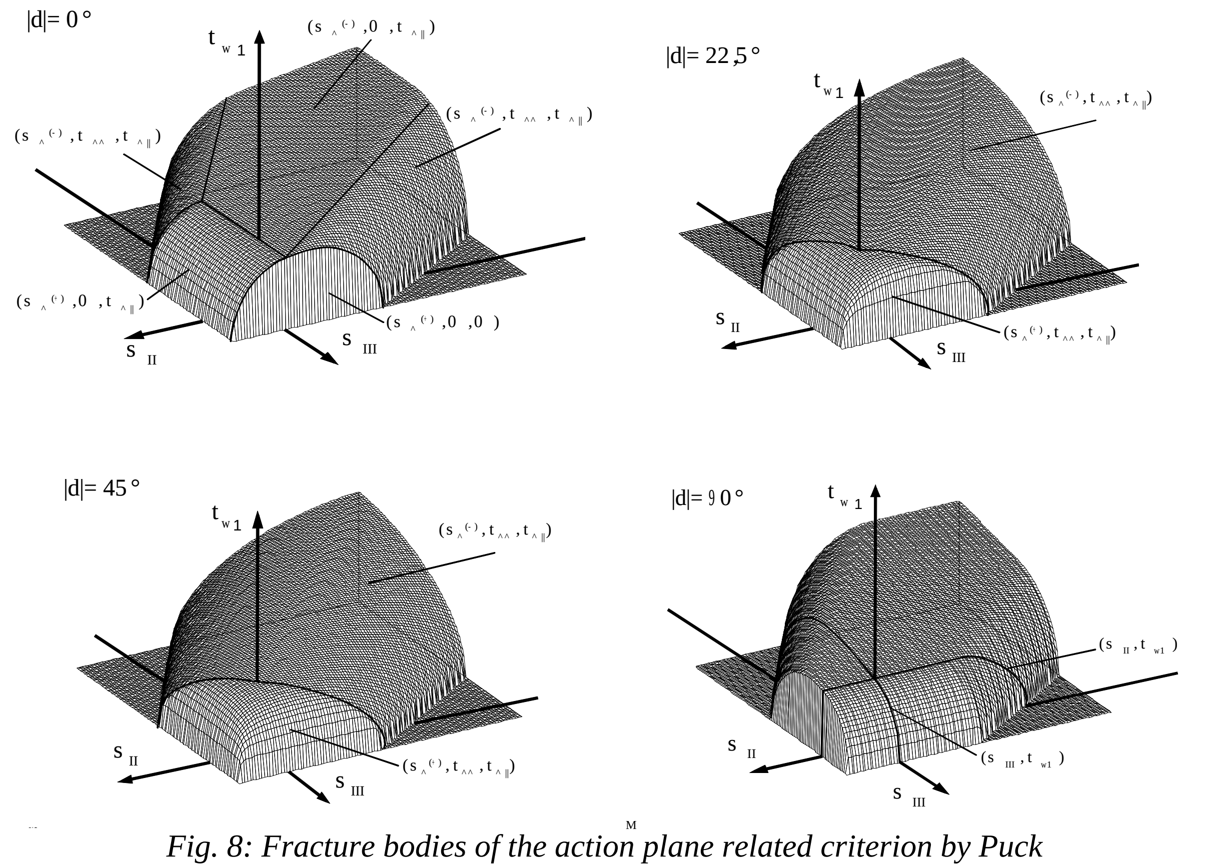

Fig. 8: Fracture bodies of the action plane related criterion by Puck

Particularly interesting is the shape of the fracture bodies for δ = 0° and δ = 90°. Here the components of the fracture surface, which belong to the different failure modes marked in the figure, touch each other in sharp edges. (A failure mode is defined by the combination of stressings acting on the fracture plane [4]). It is remarkable that *Puck´s* fracture conditions, which describe a smooth ellipsoid combined with a smooth paraboloid in the (σ_n, τ_{nt}, τ_{n1})-stress space (compare Fig. 6), have the ability to describe so many fracture surface portions in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$)-stress space. The reason for this is given by the search of the plane with the highest stress exposure factor. Thereby the criterion corresponds in certain sections to a maximum stress criterion, whereas other stress combinations are assessed in the form of a mixed-mode criterion. These effects can be clarified excellently by means of the fracture envelopes for $\sigma_{II} = 0$ and $\sigma_{III} = 0$ which have been drawn onto the fracture body for $\delta = 90°$. Whereas the (σ_{III}, $\tau_{\omega 1}$)-fracture envelope corresponds in its complete extent of validity to a mixed-mode criterion, by far the most of the (σ_{II}, $\tau_{\omega 1}$)-fracture envelope corresponds to a maximum normal stress criterion or a maximum longitudinal shear stress criterion. The explanation for this phenomenon is, that in case of a deviation angle $\delta = 90°$ the stresses $\tau_{\omega 1} = \tau_{III1}$ and σ_{III} are acting on the same fibre parallel plane and therefore promote fracture together. In contrast $\tau_{\omega 1} = \tau_{III1}$ is acting on a plane that is perpendicular to the one σ_{II} is acting on; according to *Mohr* an interaction is therefore not possible until an inclined fracture plane occurs and the transverse shear stress τ_{nt} due to high $\sigma_{III}^{(-)}$ co-operates with the longitudinal shear stress τ_{n1} to produce fracture.

Passing over to deviation angles, which differ some degrees from 0° and 90°, the sections comparable to a maximum stress criterion vanish, except for the (σ_{II}, σ_{III})-base. In case of $\delta = 45°$ the fracture body has to be symmetrical with regard to the ($\sigma_{II} = \sigma_{III}$)-plane, as the action plane of $\tau_{\omega 1}$ is equally inclined towards the action planes of σ_{II} and σ_{III}.

Direct formulations by Cuntze and Jeltsch-Fricker,Meckbach

Based on the approach of *Puck* several attempts have been made to avoid the numerical effort for searching the fracture plane by direct formulation of the δ-dependent fracture bodies with the aid of invariants in σ_2, ..., τ_{21}, especially by *Cuntze* [12] and *Jeltsch-Fricker,Meckbach* [13]. In order to take the dependence of δ into consideration a cubic invariant had to be incorporated, which increases the numerical effort compared to the quadratic polynomials of *Tsai,Wu* and *Hashin*. Inspired by the experiences *Cuntze* obtained in a research project to verify the action plane related IFF-criterion [5], he developed the so called 'fracture-type criteria'. Comparable to *Hashin* and *Tsai,Wu* his invariant approach is referred to the 6 ply stresses σ_1, ..., τ_{21} but *Cuntze* tries to capture the IFF process by formulating three independent fracture conditions. Each condition is scaled in a way that it is primary dominated by one basic strength value $R_\perp^{(+)}$, $R_{\perp\parallel}$ or $R_\perp^{(-)}$. *Cuntze* does not define any extents of validity for the three individual fracture conditions; he is rather convinced that for any stress combination the minimum reserve factor f_R calculated from all three conditions is valid and simultaneously indicates the correct failure mode. Accordingly *Cuntze* only differentiates between failure modes dominated by transverse tensile, longitudinal shear or transverse shear stressing. **This contradicts to the 7 failure modes defined in Fig. 8, which already have been verified by experiment [10, 14]**. *Cuntze's* fracture modes 'pure $\tau_{\perp\perp}$-shear' and 'pure $\sigma_\perp$-tension' can only be found in the basic ($\tau_{\omega 1} = 0$)-plane of the fracture bodies.

In the past five years the 'fracture-type criteria' were modified continuously. Fig. 9 displays the fracture body according to *Cuntze's* latest publication without probabilistic rounding by the so called resultant reserve factor f_R^{res} [12].

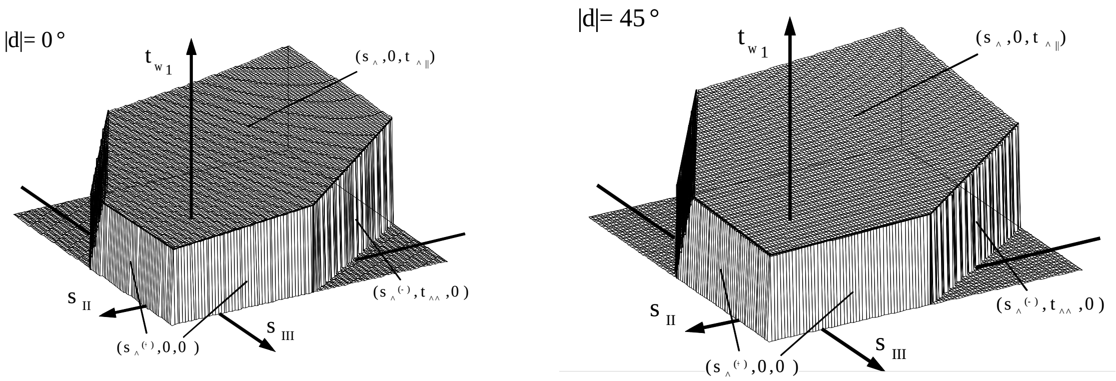

Fig. 9: Fracture bodies of the 'fracture-type criteria' by Cuntze

For the first time Cuntze got rid of the mathematical problems which characterised his former approaches. In his revised version two of the invariant formulations are no longer dependent on $\tau_{\omega 1}$. These fracture conditions are capable of describing the (σ_{II}, σ_{III})-fracture envelope for pure transverse stressings very well (compare Fig. 8). The last fracture condition describes a kind of 'δ-dependent roof' covering failure due to longitudinal shear stressing. Unfortunately these simplifications lead to fracture bodies which do not show the typical asymmetries IFF conditions according to the theory of *Mohr* are supposed to have and the dependence on δ is not very pronounced, too. However the fracture bodies of the 'fracture-type criteria' certainly will have a little more resemblance to those presented in Fig. 8 if the probabilistic rounding by means of f_R^{res} is included in the visualisation technique. This was not succeeded in the run-up to this paper.

Jeltsch-Fricker,Meckbach approximated the fracture body of the action plane related IFF-criterion by means of only two invariant formulations in σ_2, ..., τ_{21}.

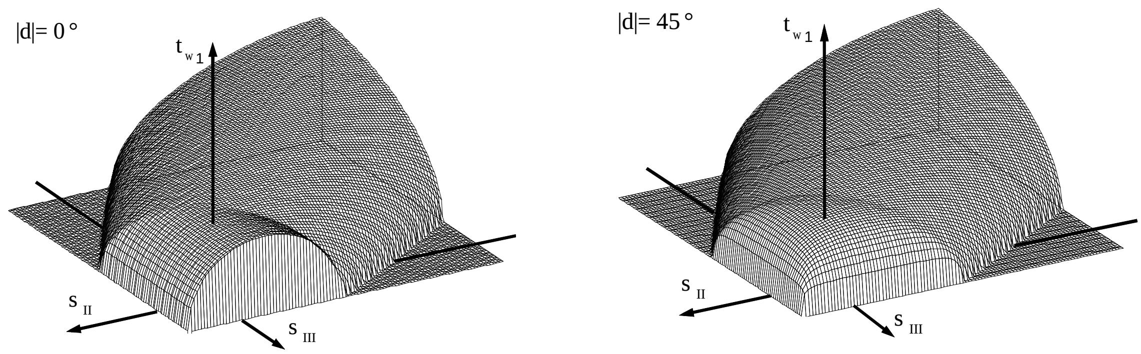

Fig. 10: Fracture bodies of the 'fast solver' by Jeltsch-Fricker,Meckbach

Fig. 10 displays the excellent result they achieved by using one invariant approach of fourth and one of second order. From a geometrical point of view they combined a closed cylinder surface oriented in σ_{III}-direction for δ = 0° with an elliptical paraboloid with its central axis in the direction of $\sigma_{II} = \sigma_{III}$. The elliptical intersection curve of the two bodies can be shifted in the direction of the line $\sigma_{II} = \sigma_{III}$ by a certain parameter, to adapt the criterion to experimental results. The fracture body for δ = 0° shows the typical asymmetries one expects for a IFF criterion according to the theory of *Mohr*. This is especially represented by the (σ_{II}, $\tau_{\omega 1}$)- and (σ_{III}, $\tau_{\omega 1}$)-fracture envelope. Passing over to deviation angles δ, which differ some degrees from 0° or 90°, the asymmetrical sections vanish quite soon and fracture bodies similar to those received indirectly by the action plane related IFF-criterion are obtained for the whole range of possible deviation angles δ. Intensive discussion of [12, 13] can be found in [14].

CONCLUSIONS

For the first time a physically meaningful IFF-criterion for three-dimensional states of stress is available for the dimensioning of laminates stacked up with UD-laminae. In previous papers it has been proven by experiments that this new IFF-criterion by *Puck* with the included analysis of the fracture angle allows a very realistic prediction of the actual fracture process and its consequences. Still more evidence has been found in an extended test program, which is part of a Ph.D. thesis [14]. The comparison of fracture bodies in the (σ_{II}, σ_{III}, $\tau_{\omega 1}$, δ)-stress space clearly demonstrates the shortcomings of criteria that do not account for the deviation angle δ between the action planes of the transverse principle stress σ_{II} and the resulting longitudinal shear stress $\tau_{\omega 1}$. Besides two 'direct formulations' are investigated which represent a δ-dependent approximation of *Puck´s* IFF-criterion. These approximations lower the computational effort, as the numerical determination of the plane with the highest angle dependent stress exposure factor $f_E(\theta_{fp})$ is avoided. However they do not serve the designer with the additional information about the expected fracture angle θ_{fp} against the thickness direction of the layer, which is the only hint to differ harmless from hazardous IFF.

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