FREE VIBRATION ANALYSIS OF CLAMPED-FREE COMPOSITE CYLINDRICAL SHELLS WITH AN INTERIOR RECTANGULAR PLATE

Young-Shin Lee [1] and Myoung-Hwan Choi [1]

[1] *Department of Mechanical Design Engineering, Chungnam National University, 220 Kung-Dong, Yusong-Gu, Taejon, 305-764, Korea*

SUMMARY: This paper describes the free vibration characteristics of clamped-free composite cylindrical shells with a longitudinal interior rectangular plate using the receptance method. To formulate the frequency equation of the combined shell, the natural frequencies and mode shape functions for individual components of the plate and shell with clamped-free boundary conditions at both ends of the shell are obtained using Love's shell theory and classical plate theory. The natural frequencies of combined shells can be calculated from the characteristics of the plate and shell two component systems. These calculations are needed for component systems under dynamic force and moment exerted at the joints. The analytical results are compared with those of the experiment and finite element analysis. The effects of the thickness of the interior plate and the fiber orientation angles on the natural frequencies of the combined composite shells are also discussed.

KEYWORDS: Receptance Method, Cylindrical Shell, Free Vibration, Natural Frequency, Dynamic Loading

INTRODUCTION

Cylindrical shells are used to approximate more complex structures such as in aerospace, submarine and nuclear pressure vessels used in many industrial applications. The combined shell with a longitudinal interior rectangular plate seems to be the most realistic model, and an aircraft fuselage with its floor structure may be the idealized model of a combined shell and plate [1]. The vibration problems of combined shells with interior plates are being studied by several researchers using various analytical methods such as extended Rayleigh-Ritz, transfer matrix, dynamic stiffness and Rayleigh-Ritz methods [2-5]. Recently, as one of the analytical methods for vibration analysis of combined systems, the receptance method has been applied to the analysis of free vibration for continuous rectangular plates [6] and the cylindrical shells with circular plate attached at arbitrary axial positions [7,8]. In reference [9] a simply supported composite cylindrical shell with an interior plate is analyzed using the receptance method and experiment.

In this study, in order to analyze the free vibration of a clamped-free composite cylindrical shell with a longitudinal interior plate, the receptance method is used. For two plate and shell structures with clamped-free edge conditions, the natural frequencies and mode shape functions are obtained through the Rayleigh-Ritz procedure based on the energy principle. The dynamic force and moment exerted at the joints due to the constraint of the displacements of two systems can be assumed using the Dirac delta function. The natural frequencies of combined shells can be calculated from characteristics of the component systems of the shell and plate. Those have to be calculated for component systems under dynamic forces and moments exerted at the joints. The analytical results are compared with those of the experiment and finite element (FE) analysis. The effects of the thickness of the plate and the fiber orientation angles on the natural frequencies of the combined shells are also discussed.

FORMULATION

The geometry of the composite circular cylindrical shell with a longitudinal, interior plate and the coordinate systems are shown by Fig. 1. The displacement components of the plate and shell in each direction are presented as u_1^P, u_2^P, u_3^P and u_1^S, u_2^S, u_3^S, respectively. Where, subscripts (or superscripts) s and p indicate the shell and the plate, respectively. The plate is attached at θ_1^* and θ_2^* position of the shell based on the vertical centerline.

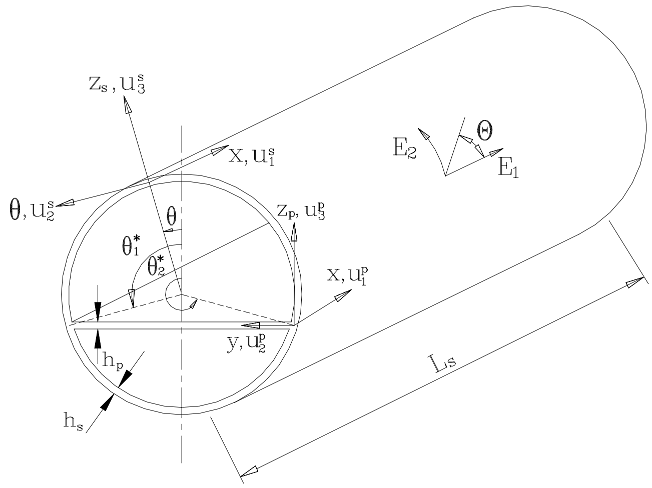


Fig. 1: Geometry of the composite circular cylindrical shell with a longitudinal, interior plate

Receptance of Combined Structure

A receptance is defined as the ratio of a displacement or slope response at a certain point to a harmonic force or moment input at the same or different point. When the two systems are joined and no force (or moment) external to the two systems are applied, it must be equal because of displacement (or slope) continuity. Thus, the natural frequencies of combined structure can be found from [10]

$$|\alpha_{ij} + \beta_{ij}| = 0. \tag{1}$$

Where, the α_{ij} and β_{ij} are the receptance of the shell and plate, respectively.

The displacement of the shell subjected to dynamic loading can be expressed by modal displacement and mode participation factor.

$$u_i(\alpha_1, \alpha_2, t) = \sum_{k=1}^{\infty} \frac{F_k^*}{(\omega_k^2 - \omega^2)} U_{ik}(\alpha_1, \alpha_2) e^{j\omega t} \tag{2}$$

Where, U_{ik} is the mode components of the plate and shell in three principal directions, and

ω_k is the natural frequency of two independent systems which are calculated by Love's shell theory and classical plate theory, respectively. The dynamic forcing terms are given as below.

$$F_k^* = \frac{1}{\rho h N_k} \int_{\alpha_1} \int_{\alpha_1} f_i^* U_{ik} A_1 A_2 \, d\alpha_1 d\alpha_2 \tag{3}$$

$$N_k = \int_{\alpha_1} \int_{\alpha_1} U_{ik}^2 A_1 A_2 \, d\alpha_1 d\alpha_2 \tag{4}$$

Where, f_i^* (i=1,2,3) are the input forcing functions applied along two joint lines. The transverse mode shape satisfying clamped-free edge conditions of the shell is used as following from neglecting the displacement components in axial and circumferential directions.

$$U_3^S = \phi(x)\cos(n\theta) \tag{5}$$

Where $\phi(x)$ is the beam modal function which satisfy clamped-free edge conditions of the shell and plate in the axial direction, and can be expressed as following;

$$\phi(x) = \cosh(\lambda_i x / L) - \cos(\lambda_i x / L) - \sigma_i \{\sinh(\lambda_i x / L) + \sin(\lambda_i x / L)\} \tag{6}$$

and λ_i and σ_i can be obtained from equation (7) and (8) below.

$$\sigma_i = \frac{\sinh \lambda_i - \sin \lambda_i}{\cosh \lambda_i + \cos \lambda_i} \tag{7}$$

$$\cosh \lambda_i \cos \lambda_i + 1 = 0 \tag{8}$$

The mode shapes of the plate used in width and normal directions are expressed as below.

$$U_2^p = \phi(x)\cos(n\pi y / b) \tag{9}$$

$$U_3^p = \phi(x)\sin(n\pi y / b) \tag{10}$$

Displacements Due to Dynamic Loading

When a rectangular plate attached at θ_1^* and θ_2^* position of the shell, the transverse dynamic excitations exerted at the joints due to the constraint of the displacements of the shell by the plate are shown in Fig. 2, and can be assumed as Eq.(11) using the Dirac delta function.

$$f_i^*(x, \theta^*, t) = F_i^S \phi(x) \delta(\theta - \theta_i^*) e^{j\omega t} \qquad i = 1,2 \tag{11}$$

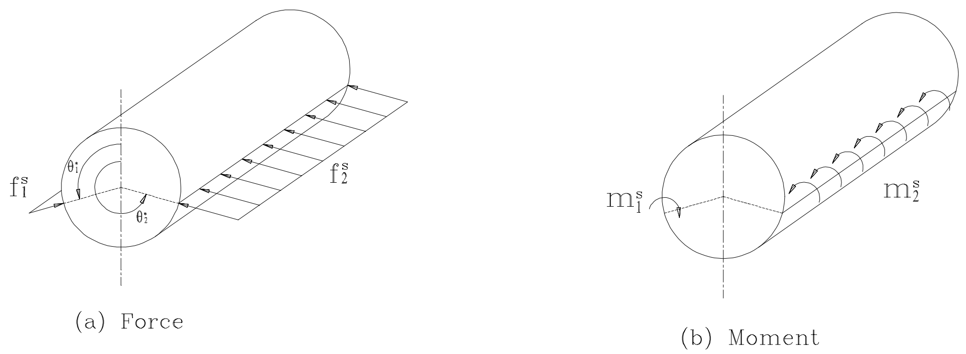


Fig. 2: Force and moment applied on the shell at the joints

Substituting Eq.(11) to Eq.(3) allows us to calculate the dynamic forcing function F_k^*. Using Eqs.(3)~(5), the displacement of the shell can be expressed as following form.

$$u_3^s(x,\theta) = u_3^s(x,\theta)|_{F_1} + u_3^s(x,\theta)|_{F_2} \tag{12}$$

The circumferential slope of the shell can be obtained from Eq.(12) by differentiation with respect to the circumferential coordinate

$$\beta_\theta^s(x,\theta) = \beta_\theta^s(x,\theta)|_{F_1} + \beta_\theta^s(x,\theta)|_{F_2} \tag{13}$$

The dynamic moment loadings exerted at the joints due to the constraint by the plate can be assumed as following form.

$$m_i^*(x,\theta^*,t) = M_i^S \phi(x)\delta(\theta-\theta_i^*)e^{j\omega t} \qquad i=1,2 \tag{14}$$

The dynamic force due to the moment loadings can be obtained with [10]

$$F_k^* = \frac{1}{\rho h N_k}\int_{\alpha_1}\int_{\alpha_1} U_{ik}\left[\frac{1}{A_1A_2}\left\{\frac{\partial(M_1A_2)}{\partial\alpha_1}+\frac{\partial(M_2A_1)}{\partial\alpha_2}\right\}\right]A_1A_2\,d\alpha_1 d\alpha_2 \tag{15}$$

Therefore, the displacement of the shell by moment loadings can be expressed as follows;

$$u_3^s(x,\theta) = u_3^s(x,\theta)|_{M_1} + u_3^s(x,\theta)|_{M_2} \tag{16}$$

and the slope of shell by moment loadings at the joints can also be obtained from Eq.(16) by differentiation with respect to the circumferential coordinate

$$\beta_\theta^s(x,\theta) = \beta_\theta^s(x,\theta)|_{M_1} + \beta_\theta^s(x,\theta)|_{M_2} \tag{17}$$

As a similar method, when the two jointed edges of the rectangular plate are simply supported and the other edges have a clamped-free in the axial direction, one can consider the receptances for a rectangular plate by forces and moments exerted at the joints, (x, y_1^*) and (x, y_2^*). To formulate the receptances that one needs, the displacement and slope as the function of axial coordinates due to forces and moments are calculated. Detailed procedures of the formulation will not be described here.

Frequency Equation for Combined Shell

For the combined shell with an interior rectangular plate, considering the continuity condition at the shell/plate joining points can derive the frequency equation. Here, only the slope in the width direction of the plate and the normal displacement of the shell due to dynamic forces are considered. Also the normal displacement of the plate and the slope of the shell in the circumferential direction due to dynamic moments are taken into consideration because the other components of displacement can be ignored. By applying the continuity condition at the joints, the frequency equation can be expressed as Eqn. (18).

$$\begin{bmatrix} \alpha_{11}+\beta_{11} & \alpha_{12}+\beta_{12} & \alpha_{13}+\beta_{13} & \alpha_{14}+\beta_{14} \\ \alpha_{21}+\beta_{21} & \alpha_{22}+\beta_{22} & \alpha_{23}+\beta_{23} & \alpha_{24}+\beta_{24} \\ \alpha_{31}+\beta_{31} & \alpha_{32}+\beta_{32} & \alpha_{31}+\beta_{33} & \alpha_{34}+\beta_{34} \\ \alpha_{41}+\beta_{41} & \alpha_{42}+\beta_{42} & \alpha_{43}+\beta_{43} & \alpha_{44}+\beta_{44} \end{bmatrix} \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = 0 \tag{18}$$

Neglecting the in-plane displacement due to moments (u_2^p / M) and the slope in the normal direction due to forces (β_3^p / F) for the rectangular plate, the frequency equation of the combined shell will be equation (19).

$$\begin{bmatrix} \alpha_{11}+\beta_{11} & \alpha_{12} & \alpha_{13}+\beta_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22}+\beta_{22} & \alpha_{23} & \alpha_{24}+\beta_{24} \\ \alpha_{31}+\beta_{31} & \alpha_{32} & \alpha_{31}+\beta_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42}+\beta_{42} & \alpha_{43} & \alpha_{44}+\beta_{44} \end{bmatrix} = 0 \tag{19}$$

RESULTS AND DISCUSSIONS

To compare the analytical results, a modal test using the impact exciting method and FE analysis using a commercial code ANSYS [11] are performed. Specimens tested are fabricated in plain weave glass/epoxy composite with $[0^o_3/\pm 45^o_3/90^o_3]_S$ stacking sequences. Table 1 presents the dimensions of the combined shells, and the shells have the same dimensions as the plate. The plate is attached at the center ($\theta_1^* = 90^o$) of the shell and fabricated with the same material properties as the shell. The material properties of the GFRP plain weave composite are obtained by uniaxial tensile tests and are as follows;

$$E_1 = E_2 = 26.2\ GPa,\quad G_{12} = 4.9\ GPa,\quad \rho = 1880\ kg/m^3,\quad \upsilon_{12} = 0.12$$

For the vibration test, the excitation method by impact hammer is used with the Fast Fourier Transform (FFT) analyzer, an impact hammer and an accelerometer. The schematic diagram for the experimental modal test is shown in Fig. 3.

Table 1: Dimensions of combined composite shells

(Unit : mm)

Material	Shell			Plate		
	Length (L_S)	Radius (a)	Thickness (h_S)	Length (L_P)	Width (b)	Thickness (h_P)
GFRP	360	109	3.5	360	218	3.5

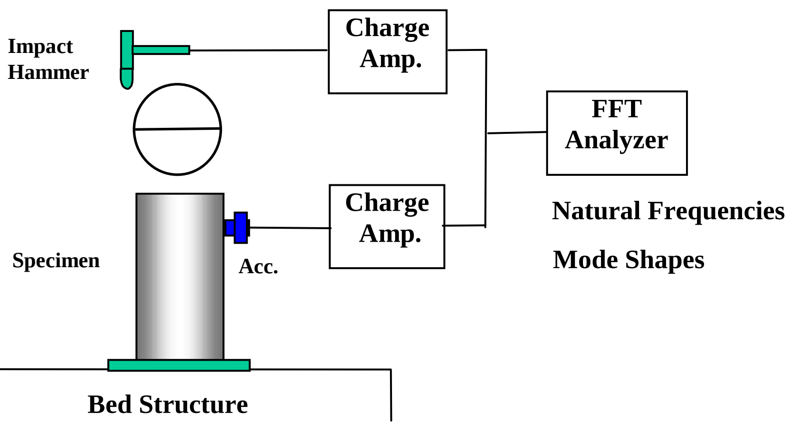


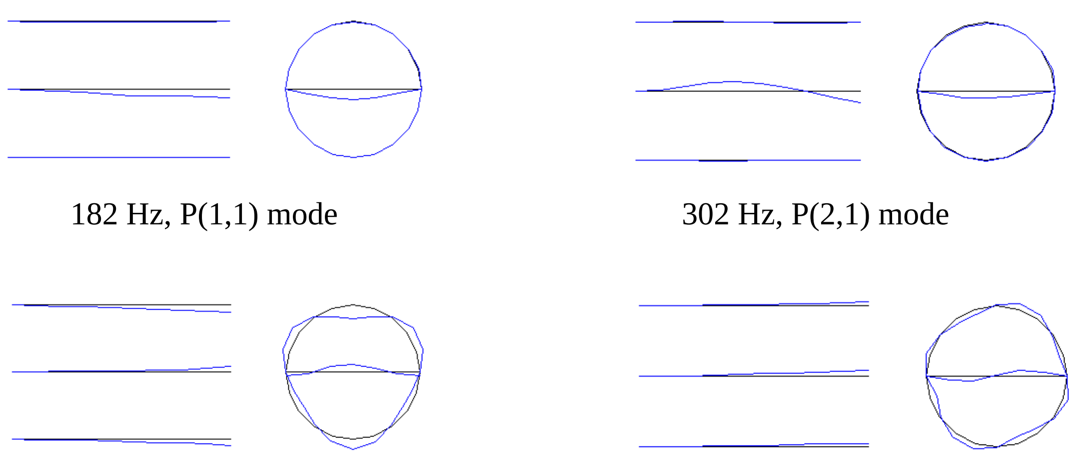
Fig. 3: Schematic diagram of the experimental modal test

Table 2 presents results of the first eight natural frequencies from analytical, experimental and FEM analysis. The experimental results are represented using the mode sequence number of the combined shell, where the P and S indicate the mode of the plate and shell, respectively. The fundamental frequency of the combined shell with clamped-free edge conditions is 208.7 Hz, and it shows the first bending mode, P(1,1) of the interior plate. The discrepancy between the analytical and FEM results is 1.4% for the lowest fundamental frequency, but the experimental result is about 12% lower than that of the analytical method due to the uncertainty of the experimental procedure and the specimen tested. The first frequency for the shell is 362 Hz at S(1,2) mode, but it does not appear in the experiment. According to increasing frequencies, the modes of the plate and shell couple with each other. Fig. 4 shows the experimental mode shapes of the GFRP composite combined shell with clamped-free edge conditions.

Table 2: Comparison of the natural frequencies of analytical, experimental and FEM results of the GFRP plain weave composite cylindrical shell with interior plate at $\theta_1^* = 90^o$ *location*

Method / Mode*	Natural frequency (Hz)				
	Analysis	Exp.	Mode		FEM
1st	208.7	182.0	P(1,1)	-	205.7
2nd	281.9	302.0	P(2,1)	-	292.9
3rd	362.0		-		333.1
4th	409.5	442.0	P(3,1)	-	444.0
5th	439.9	460.0	P(1,3)	S(1,3)	465.4
6th	663.4	590.0	-	S(1,3)	593.4
7th	689.6	610.0	P(1,2)	S(1,4)	623.5
8th	714.5	660.0	P(4,1)	-	675.4

* : Frequency ascending order



182 Hz, P(1,1) mode   302 Hz, P(2,1) mode

460 Hz, S(1,3) mode   610 Hz, S(1,4) mode

Fig. 4: Typical experimental mode shapes of the GFRP composite combined shell with clamped-free edge conditions

In the case of the GFRP plain weave composite combined shell with interior plate at $\theta_1^* = 90^o$ location, the effect of the thickness (h_p) of the plate on the frequencies are studied and shown in Fig. 5 for two modes of the plate and shell. For this calculation, the thickness of the plate varies from 2.0mm to 8.0mm. The fundamental frequency of the combined shell primarily exhibits plate motion with one half wave in each direction. In this case, the frequencies of the first two modes of the plate are linearly increased as the thickness of the plate increased, even if the frequency increment in the case of the thick plate is slightly smaller than the thin plate. If the thickness of the plate is larger than 5.0mm, the frequency of the plate mode, P(2,1) is higher than the one of the shell mode, S(1,2). For the shell modes, the first frequency appears at the circumferential wave number *n*=2, and the frequencies become less sensitive to changes in plate thickness.

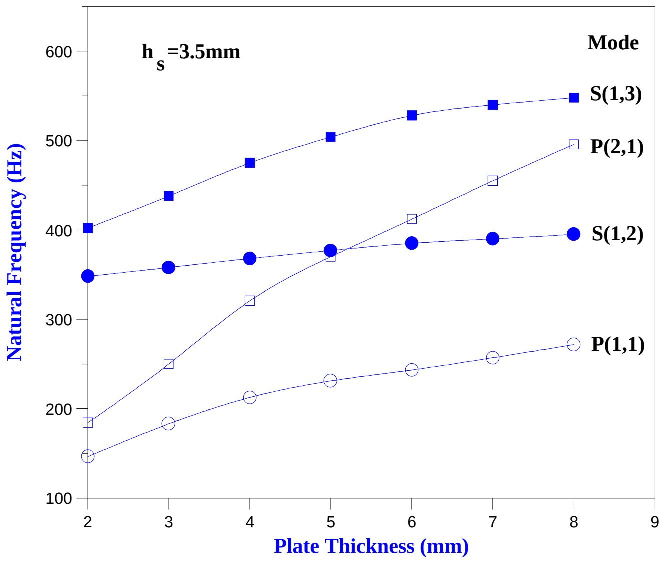


Fig. 5: Effects of the plate thickness(h_P) on the frequencies of the GFRP plain weave composite cylindrical shell with interior plate at $\theta_1^* = 90^o$ location

To study the influence of fiber orientation angles (Θ) of the composite on the frequencies of the combined shell, the CFRP composite made of T300/N5208 is chosen and the material properties are as below [12]:

$$E_1 = 181.0\ GPa,\quad E_2 = 10.3\ GPa,\quad G_{12} = 7.17\ GPa,\quad \rho = 1600\ kg/m^3,\quad \upsilon_{12} = 0.28$$

The composite shell considered is laminated with the $[(\Theta/\text{-}\Theta)_2]_s$ stacking sequence, and the thickness of each ply is assumed to be 0.125 mm. The plate is attached at the center of the shell, and the geometrical parameters of the shell are: L_s/a=2 and a/h_s =100.

Figure 6 shows the fundamental frequency of the CFRP composite combined shell with various fiber orientation angles. The frequency of the plate alone with simple support at the two joints and clamped-free at the other two edges has the highest value at the fiber orientation angle, Θ= 90^o. For the shell with the clamped-free boundary condition, the highest frequency is 650 Hz at Θ= 40^o. Parenthesized numbers in Fig. 6 indicate the circumferential wave number (*n*) for the axial mode, *m*=1. The circumferential wave number on the fundamental frequency decreases as the fiber orientation angle increases. This indicates that the fiber angle has a greater effect on the stiffness in the circumferential direction. When the plate and shell are joined, the fundamental frequency of the combined shell is of a slightly higher value than the frequency of the interior plate alone. Also, increasing the fiber orientation angle increases the difference in the fundamental frequency between the plate alone and the combined shell. In the case of the fiber angle, Θ= 90^o, the combined shell has a highest frequency, 203.4 Hz, which is 80 Hz higher than that of the plate alone.

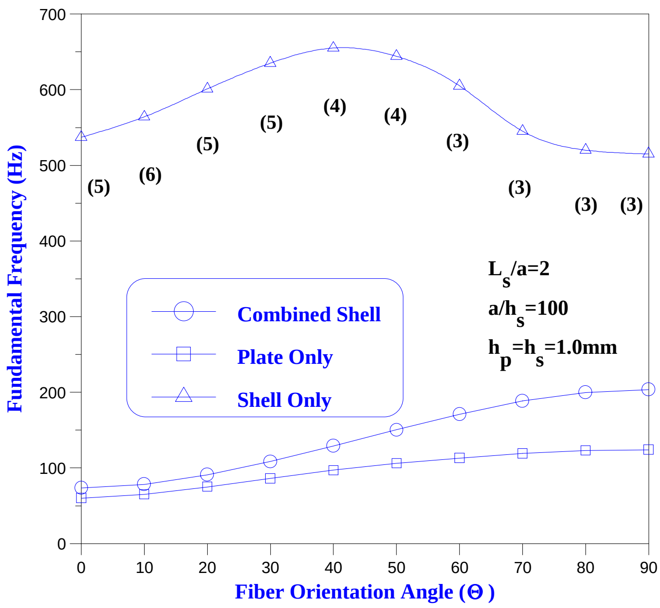


Fig. 6: Fundamental frequencies of the CFRP laminated composite cylindrical shell with the $[(\Theta/-\Theta)_2]_s$ stacking sequence for various fiber orientation angles

CONCLUSIONS

The major conclusions from this study are as follows:

1. The fundamental frequency of the combined shell with clamped-free edge conditions is the first bending mode of the interior plate, and the plate vibrates as a simply supported plate at two joints.
2. For the effect of the thickness of the interior plate, the frequencies are linearly increased as the thickness of the plate increases. Those of shell modes become less sensitive to change as plate thickness increases.
3. The combined shell has the highest fundamental frequency in the case of the fiber orientation angle, $\Theta=90^o$, and the fundamental frequency of the combined shell is of a slightly higher value than the frequency of the interior plate alone.

REFERENCES

1. Petyt, M. and Wei, J., "Free Vibration of an Idealized Fuselage Structure", *Proceeding of the 15th International Modal Analysis Conference*, Japan, 1997, pp. 647<653.

2. Peterson, M.R. and Boyd, D.E., “Free Vibrations of Circular Cylinders with Longitudinal, Interior Partitions”, *Journal of Sound and Vibration*, Vol. 60, No. 1, 1978, pp.45<62.

3. Irie, T., Yamada, G. and Kobayashi, Y., "Free Vibration of Non-Circular Cylindrical Shells with Longitudinal Interior Partitions", *Journal of Sound and Vibration*, Vol. 96, No. 1, 1984, pp.133<142.

4. R.S. Langley, "A Dynamic Stiffness Technique for the Vibration Analysis of Stiffened Shell Structures", *Journal of Sound and Vibration*, Vol. 156, No. 3, 1992, pp. 521<540.

5. Missaoui, J., Cheng, L. and Richard, M.J., "Free and Forced Vibration of a Cylindrical Shell with a Floor Partition", *Journal of Sound and Vibration*, Vol. 190, No. 1, 1995, pp.21<40.

6. Azimi, S., Hamilton, J.F. and Soedel, W., "The Receptance Method Applied to the Free Vibration of Continuous Rectangular Plates", *Journal of Sound and Vibration*, Vol. 93, No. 1, 1984, pp. 9<29.

7. Huang D.T. and Soedel, W., "Study of the Forced Vibration of Shell-Plate Combinations Using the Receptance Method", *Journal of Sound and Vibration*, Vol. 166, No. 2, 1993, pp. 341<369.

8. Yim, J.S., Shon, D.S. and Lee, Y.S., "Free Vibration of Clamped-Free Circular Cylindrical Shell with a Plate Attached at an Arbitrary Axial Position", *Journal of Sound and Vibration*, Vol. 213, No. 1, 1998, pp. 75<88.

9. Lee, Y.S. and Choi, M.H., “Free Vibration of a Composite Cylindrical Shell with a Longitudinal, Interior Rectangular Plate”, *The First Asian-Australian Conference on Composite Materials*, Osaka, Japan, 1998, pp. 320-1<320-4.

10. Soedel, W., *Vibrations of Shells and Plates*, 2nd ed., Marcel Dekker, Inc., 1993.

11. Swanson Analysis Systems Inc. : *ANSYS User's Manual: Volume Elements*, 1992.

12. Tsai, S.W., *Composite Design*, Think Composite, Dayton, Ohio, U.S.A., 1988.

13. Lam, K.Y. and Loy, C.T., “Influence of Boundary Conditions and Fiber Orientation on the Frequencies of Thin Orthotropic Laminated Cylindrical Shells”, *Composite Structures*, Vol. 31, 1995, pp. 21<30.

14. Jones, R.M., *Mechanics of Composite Materials*, Hemisphere Publishing Co., New York, 1975.

15. Whitney, J.M., *Structural Analysis of Laminated Anisotropic Plates*, Technomic Publishing Co., Lancaster, Pennsylvania, 1987.