DESIGN AND OPTIMISATION OF COMPOSITE PLATES WITH CUTOUTS

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SUMMARY: The optimisation of cutouts in laminated composite plates under a variety of loading conditions has been investigated using the design sensitivity method of MSC/NASTRAN. Two approaches were investigated: optimisation of the cutout shape and optimisation of local cutout reinforcement. For the shape optimisation, a least squares objective function was shown to successfully produce a constant failure index around the hole boundary under biaxial loading. The shape optimisation of holes in laminated composite plates showed that the optimum shape depends on the degree of orthotropy. Quasi-isotropic laminates produced holes of similar shape to the isotropic case while laminates without fibres in the primary loading direction could produce unexpected shapes. A minimum weight objective was used to introduce local hole reinforcement which reduced the failure index in the panel to that of a panel without a hole. Significant weight savings were demonstrated.

KEYWORDS: Design, Optimisation, Cutouts, Neutral Holes, Finite element method.

INTRODUCTION

Much of the previous work performed in the field of structural optimisation has involved the development of solution algorithms and their computational implementation into finite element (FE) codes. However, recent focus has been in the area of optimisation techniques as applied in the structural design environment. Optimisation techniques are effectively the design strategy applied to a given design problem to achieve the intended optimised solution. Given the large number of variables generally involved with structural design, the quality of the optimisation technique can greatly increase the effectiveness of the design.

Generally the technique for structural optimisation revolves around three distinct groups: objectives, constraints and design variables. The choice of optimisation objectives, constraints and design variables effectively controls the type of optimum design that will result. Optimisation of holes in composite laminates has been investigated by a small number of researchers. Backlund and Isby [1] investigated optimum hole shapes in shear panels with the emphasis placed on minimising the weight, rather than obtaining the hole shape that minimises the strains in the panel. Further, the shape of the holes was restricted by the use of curves fitted through the boundary nodes. Vellaichamy et al. [2] investigated the optimum orientation and aspect ratio of an elliptical hole for various laminates and loading conditions. In this case, the optimum hole shape was assumed to be elliptical. A more general study was conducted by Falzon et al. [3] who used evolutionary optimisation techniques to determine optimum hole shapes for quasi-isotropic laminates under several loading conditions. This method resulted in irregular edges due to the optimisation technique implemented. Research was also conducted by Senocak [4] who used closed form analytical methods to determine cutout shapes and reinforcement design of certain composite panels under specific loading conditions.

This paper examines strategies for the optimisation of cutouts in laminated composite plates. Optimisation of the shape of the cutout as well as introduction of local reinforcement around the cutout were investigated making use of existing optimisation algorithms. The general aim was to produce a neutral hole: a hole that results in the same strain field as the panel without a hole. The optimum neutral hole may involve changing the hole shape, as well as adding local reinforcement around the hole. The addition of structural optimisation into commercial FE codes has provided the power to apply optimisation techniques to a variety of structural problems. In this study, the application of MSC/NASTRAN has been made for the optimisation of cutouts in laminated composite plates under a variety of load cases.

GENERAL OPTIMISATION APPROACH

There exist several methods that are well suited for the optimisation of holes in plates, with each possessing unique advantages. Optimisation analyses require the definition of the design variables, the objective function and also constraints. These differ significantly for shape and property optimisation problems.

Analysis Methods

Sensitivity analysis methods require calculation of stress gradient information so that the sensitivity of the design objective function with respect to the design variables can be determined. Most industry-standard FE analysis codes which have an optimisation capability, including MSC/NASTRAN, use the sensitivity analysis method for design optimisation [5]. While sensitivity analysis is a powerful general technique for optimisation, it can be relatively demanding on computer time. Also significant is the fact that typically a great deal of effort is required (compared to standard FE analyses) to generate suitable initial model specifications to obtain successful results. Mesh distortion and appropriate specification of the objective function are typical problems.

Other optimisation techniques exist which are suitable for shape and property optimisation problems. Most of these are gradientless which reduces the complexity of the computations required. The biological growth analogy method proposed by Mattheck and Burkhardt [6] is a gradientless shape optimisation technique which aims to achieve a constant Von Mises stress field along a given free boundary. The Neuber notch stress reduction method, developed by Schnack [7], is also a gradientless approach. This aims to reduce notch stresses to a minimum by producing a constant tangential stress boundary. Another gradientless approach, termed the moving boundary method, has been proposed by Kaye and Heller [8]. This method also aims to produce a constant boundary stress field by appropriate boundary nodal movements.

A technique, termed homogenisation optimisation, was developed by Rozvany et al. [9] which aims to achieve identical strain energy in each element. A similar method, termed evolutionary optimisation, has been developed by Xie and Steven [10]. The principle behind this approach is the removal of elements that have low stress. However, it is not ideally suited to stress concentrator problems as the removal of elements leaves an irregular boundary.

Design Variables

The general term, design variable, refers to geometry or properties that are varied in the optimisation process to achieve the required objective. In the case of shape optimisation of holes, the design variables were the location of the nodes along the hole boundary. To reshape the hole, the nodes are allowed to move finite amounts dependent upon the constraints applied.

Shape Basis Vectors and Auxiliary Models

In MSC/NASTRAN, the design variables in shape optimisation are related to allowable shape variations using shape basis vectors (SBVs). SBVs are the possible changes in location for a group of nodes. The optimisation process determines the best linear combination of the SBVs for a given objective and constraints. The SBVs were generated through another FE model termed the auxiliary model. This model shared the same geometry and element connectivity but had different material properties and boundary conditions. A typical auxiliary model used is shown in Fig. 1, which features individual radial point loads applied to the nodes on the boundary of the hole.

Panel Configuration and Loading

The panel configuration used in this work was that of a 250 x 250 mm plate containing a centrally located 50 mm diameter hole. The sign conventions are shown in Fig. 2. A variety of laminates were considered, all using T300/914C uni-directional carbon/epoxy pre-preg tape, the properties of which are presented in Table 1.

The optimisation of a circular hole in a plate under a uniform biaxial stress field is a common shape optimisation benchmark problem. For isotropic materials, the optimum hole shape is an ellipse with an aspect ratio equal to the biaxial loading ratio (σ_x:σ_y). Both 2:1 and 4:1 biaxial loadings were investigated, the latter representing a more complex situation due to the presence of compressive stresses around the hole boundary. Due to symmetry, only one quarter of the plate was modelled.

SHAPE OPTIMISATION OF CUTOUTS

The optimisation of plates manufactured from laminated fibre composite materials offers more design variables than standard isotropic materials. For design purposes, the individual lamina orientation and thickness can be design variables. In this study, however, the optimisation problem is limited to the hole shape in an existing laminate. The optimisation was conducted on a number of laminates with varying degrees of orthotropy.

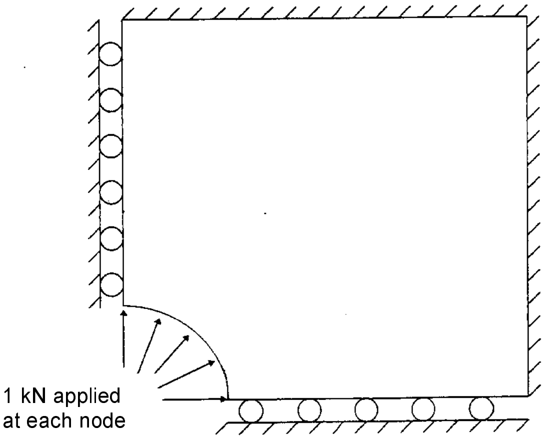


Fig. 1: Typical auxiliary model used for the shape optimisation of a hole on a plate

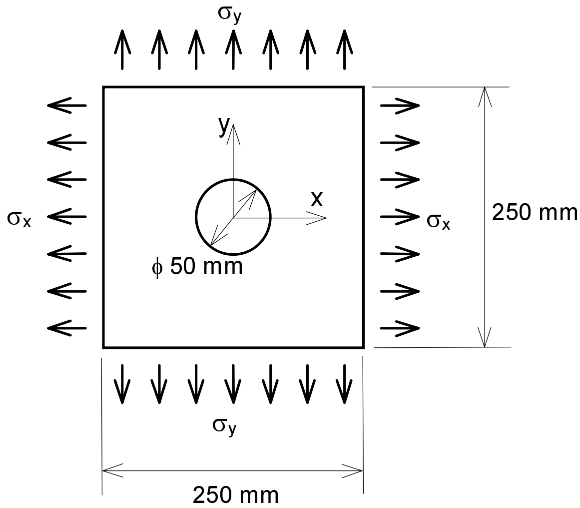


Fig. 2: Panel dimensions and sign conventions

Table 1: Properties of T300/914C uni-directional pre-impregnated carbon fibre epoxy tape

Property	**Value**
Longitudinal Modulus, E_1	130.0 GPa
Transverse Modulus, E_2	4.65 GPa
In-Plane Shear Modulus, G_{12}	4.65 GPa
Poisson's Ratio, ν_{12}	0.3
Ply Thickness	0.127 mm
Long. Tension Ultimate Strain, X_T	9230 $\mu\varepsilon$
Long. Compression Ultimate Strain, X_C	7692 $\mu\varepsilon$
Trans. Tension Ultimate Strain, Y_T	10750 $\mu\varepsilon$
Trans. Compression Ultimate Strain, Y_C	25810 $\mu\varepsilon$
In-plane Shear Ultimate Strain, S	13980 $\mu\gamma$

Objective Function

The least squares objective function applied successfully to isotropic plates [11] was modified for the case of laminated composite materials. Instead of boundary hoop stress, failure indices in the boundary elements were calculated using the maximum strain failure theory. The objective function took the following form:

$$\text{Minimise } \frac{\Sigma\left(FI_i - FI_{av}\right)^2}{k^2} \tag{2}$$

where: FI_i = Element failure index
FI_{av} = Average boundary failure index
k = Number of boundary elements

In this case, the element failure index is the maximum (most critical) failure index of all plies in the element. Also, the average boundary failure index is determined by averaging the maximum element failure indices.

Laminates

The laminates used for the orthotropic cases are as detailed in Table 2, where 0° and 90° correspond with the x-axis and y-axis respectively of Fig. 2. The FE mesh used for the analysis consisted of 200 QUAD4 elements, as shown in Fig. 3.

Table 2: Laminates used for the shape optimisation of flat plates containing holes

Laminate Number	Lay-up
1	$[(45/-45/90/0)_S]_5$
2	$[(45/-45/90/0_2)_S]_5$
3	$[(45/-45/0_2)_S]_5$

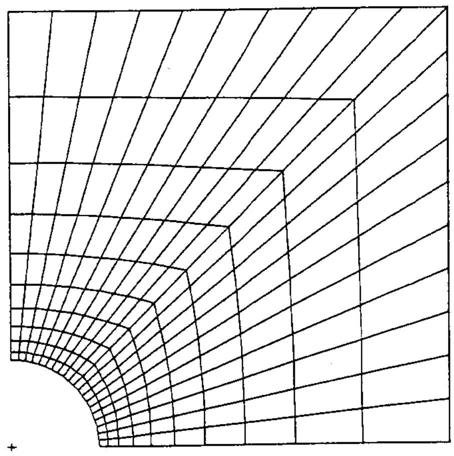


Fig. 3: Finite element mesh used for the shape optimisation.

Shape Optimisation Results

The optimum hole shapes for the laminated plates under 2:1 and 4:1 biaxial loadings are shown in Fig. 4 and Fig. 5 respectively. The optimum hole shape occurs when there is a constant failure index in the elements around the boundary of the hole. As presented in Fig. 6 for Laminate 1 under 2:1 and 4:1 biaxial loadings respectively, the least squares objective function reduced the failure index in regions where it exceeded the average boundary stress and increased it in regions where it was below the average. The failure index in these graphs has been normalised so that the optimised shape gives a boundary failure index equal to one.

The ply orientations of the laminate do affect the optimum hole shape. The aspect ratios of the optimum hole for the various configurations under biaxial loading are presented in Table 3. It was evident that the optimised hole shapes of laminates which contain plies orientated in the four principal directions (0°, 90°, 45°, -45°) was very similar to an isotropic case [11]. However, these holes did not precisely satisfy the equation for an ellipse. Laminate 3, which does not contain any 90° plies, resulted in an hole shape with a lower aspect ratio.

The reduction in peak failure index around the hole for the optimised shape in the various configurations is presented in Table 4. The lowest recorded reduction was a significant 34% for Laminate 3 under 2:1 biaxial loading. These figures demonstrate the value of optimising hole shapes under biaxial loading conditions.

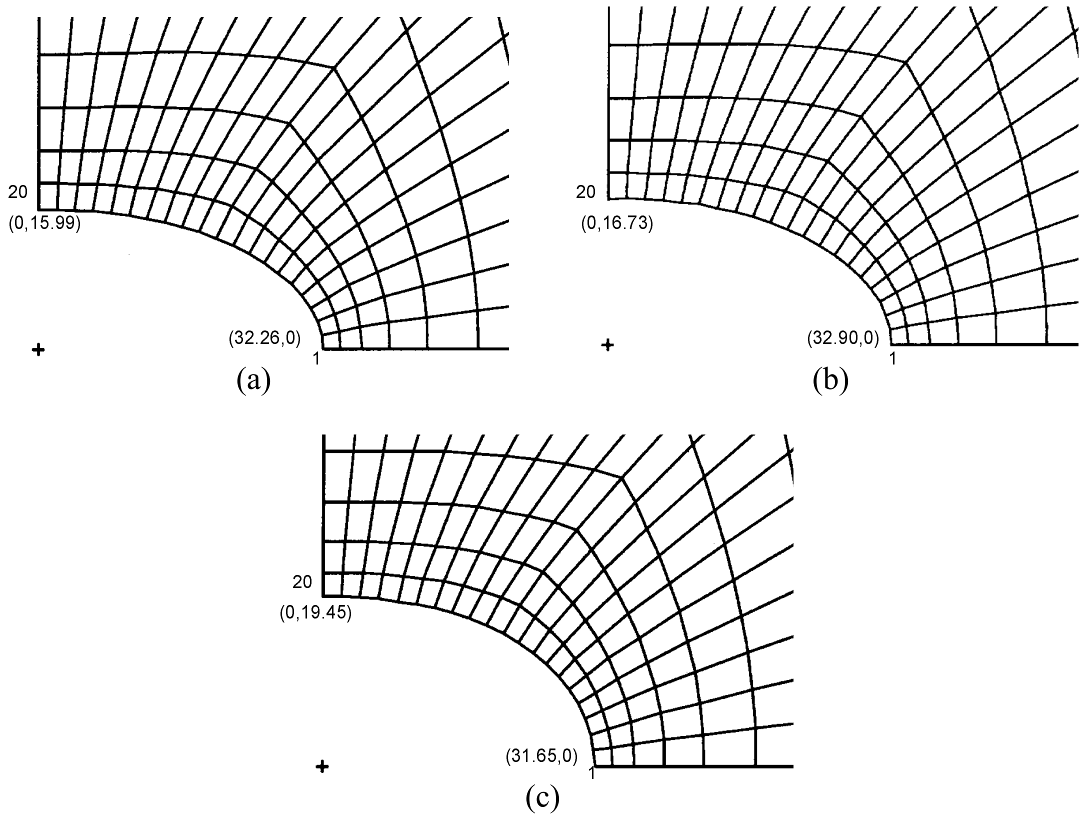


Fig. 4: Optimum hole shapes under 2:1 biaxial loading for
(a) Laminate 1, (b) Laminate 2 and (c) Laminate 3

Table 3: Aspect ratio of the optimum cutout shape for
the three laminates under biaxial loading

Laminate	**Hole Aspect Ratio**	
	2:1 Biaxial	**4:1 Biaxial**
Laminate 1	2.08	5.16
Laminate 2	1.97	4.06
Laminate 3	1.63	2.75

Table 4: Percentage reduction in peak failure index for optimised hole
shape for the various configurations under biaxial loading

Laminate	**% Reduction in Failure Index**	
	2:1 Biaxial	**4:1 Biaxial**
Laminate 1	42	59
Laminate 2	39	54
Laminate 3	34	50

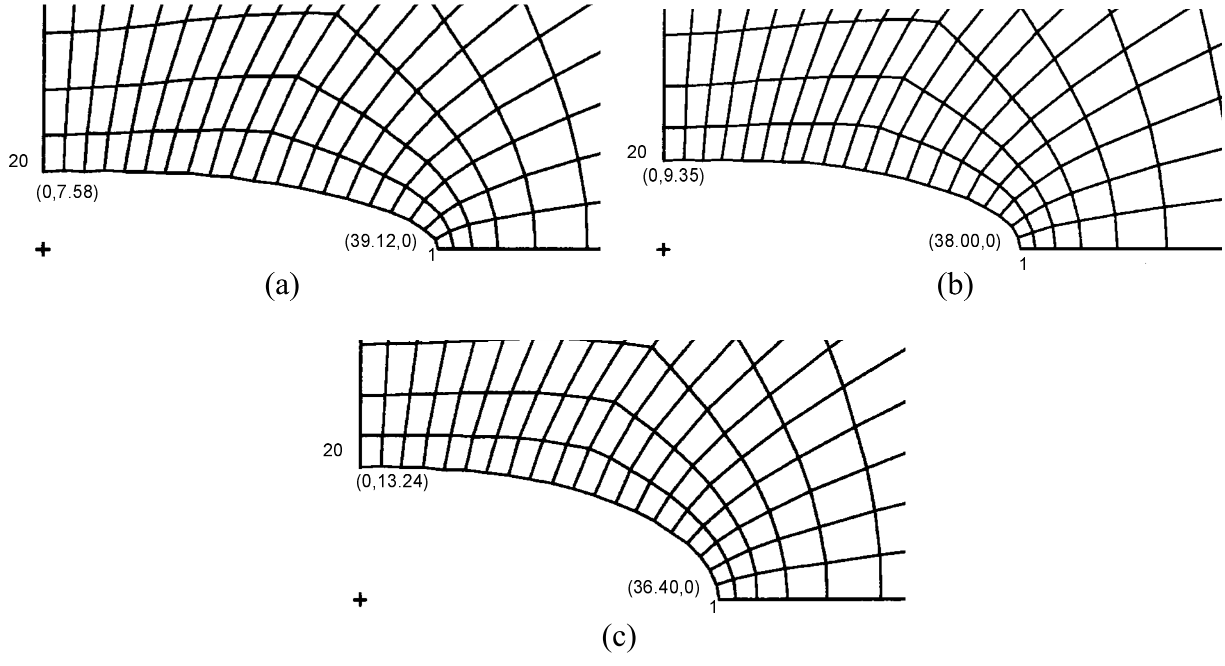


Fig. 5: Optimum hole shapes under 4:1 biaxial loading for
(a) Laminate 1, (b) Laminate 2 and (c) Laminate 3

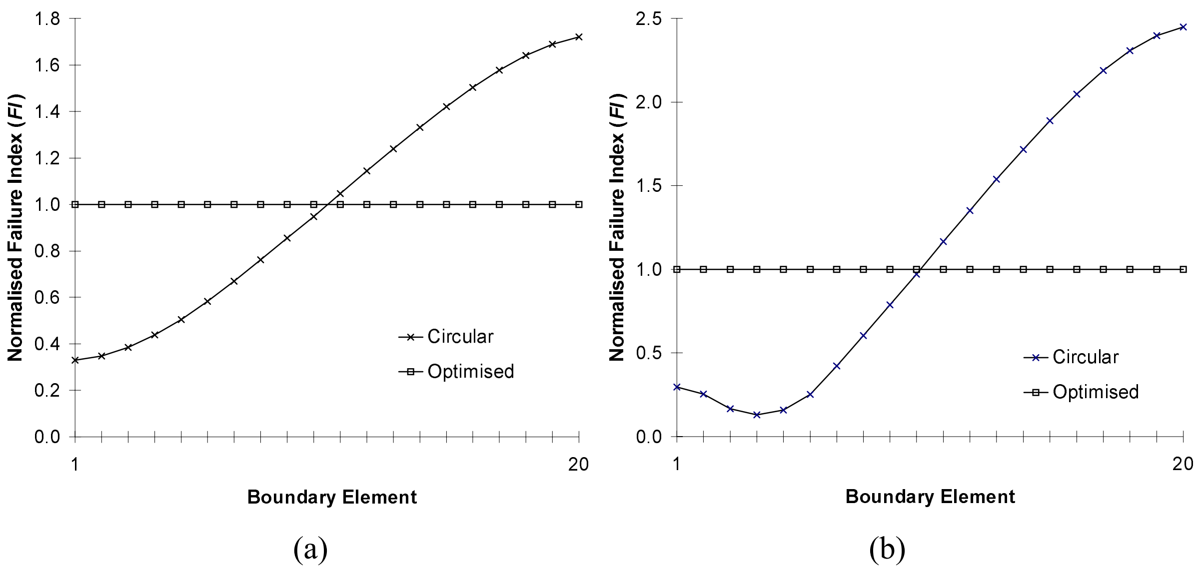


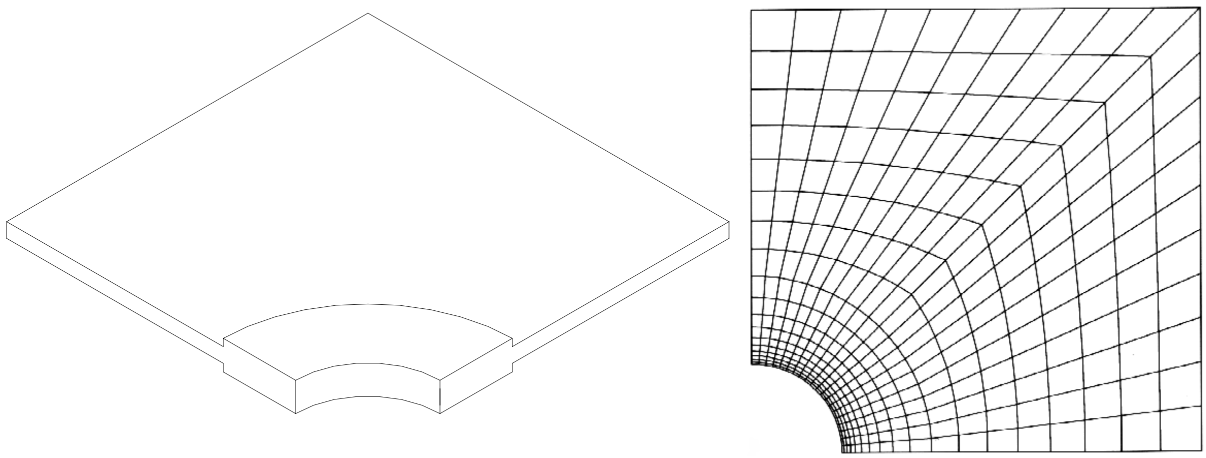
Fig. 6: Comparison of boundary failure indices for Laminate 1 between circular and optimised holes under (a) 2:1 biaxial loading and (b) 4:1 biaxial loading

LOCAL REINFORCEMENT OPTIMISATION

It was shown that the optimisation of cutout shape could achieve a significant reduction in the failure index, however, the failure index still exceeds that of an equivalent panel that does not contain a hole. To create a neutral hole, local cutout reinforcement is required. The procedure for optimisation of this local reinforcement is described for the quasi-isotropic laminate (Laminate 1) under 2:1 and 4:1 biaxial loadings.

Optimisation Definition

The local reinforcement was assumed to be a simple ring around the cutout boundary, as shown in Fig. 7(a). This ring extended a distance of approximately 50% of the cutout diameter from the cutout boundary. The analysis was performed on both the original circular cutout and the optimised cutout shapes for both 2:1 and 4:1 biaxial loadings. The finite element mesh used for the circular cutout is shown in Fig. 7(b). The reinforcement was assumed to consist of [45/-45/90/0] plies added to each side of the panel. The design variables were the thicknesses of these reinforcement plies while the design objective was to minimise weight. Constraints were enforced so that the failure index was equal to or less than the failure index in the plate with no hole under identical loading.



(a) (b)

Fig. 7: Representation of (a) the local reinforcement region and (b) the finite element mesh

Optimisation Results

The predicted optimum local reinforcement of Laminate 1 under 2:1 and 4:1 biaxial loadings is shown in Table 5. It should be noted that the thickness of Laminate 1 is 5.08 mm and the reinforcement is applied symmetrically to both sides of the panel. Significantly less reinforcement thickness is required around the previously optimised hole compared with the circular hole, reflecting the reduction in failure index achieved through shape optimisation. In addition, reinforcement in the 0° direction makes up over 45% of the reinforcement around the circular hole, which reflects the fact that the dominant load path is in this direction.

The effect of various optimisation schemes on the weight of Laminate 1 under 2:1 and 4:1 biaxial loadings is presented in Table 6. The panel weights were calculated so that the maximum failure index for a given panel was equal to the failure index of the panel without a cutout. These panel weights have been normalised with respect to the panel without a cutout. Local reinforcement of the shape optimised cutouts led to an increase in panel weight of only 10% and 4% for the 2:1 and 4:1 biaxial load cases respectively. Local reinforcement of the circular cutout, on the other hand, led to weight increases of 30% and 28% for the 2:1 and 4:1 biaxial load cases respectively. This clearly demonstrates the benefit of the combination of shape optimisation and local reinforcement for the design of neutral holes in laminated composite panels.

Table 5: Optimum local reinforcement of Laminate 1 for both circular and optimised hole shapes under 2:1 and 4:1 biaxial loadings

Thickness	**2:1 Biaxial**		**4:1 Biaxial**	
	Circular Hole	**Optimised Hole**	**Circular Hole**	**Optimised Hole**
45° Ply	1.91 mm	0.84 mm	1.36 mm	0.42 mm
-45° Ply	1.91 mm	0.84 mm	1.36 mm	0.42 mm
90° Ply	1.22 mm	0.84 mm	0.72 mm	0.28 mm
0° Ply	4.19 mm	0.80 mm	5.21 mm	0.51 mm
Total Reinforcement	9.23 mm	3.32 mm	8.67 mm	1.64 mm
Total Reinforced Panel	23.54 mm	11.72 mm	22.42 mm	8.36 mm

Table 6: Normalised weight of Laminate 1 for various configurations under 2:1 and 4:1 biaxial loadings

Configuration	**2:1 Biaxial**	**4:1 Biaxial**
No Hole	1.00	1.00
Circular Hole	2.80	2.80
Optimised Hole	1.63	1.27
Circular Hole + Local Reinforcement	1.30	1.28
Optimised Hole + Local Reinforcement	1.10	1.04

For the design of a practical reinforcement, manufacturing constraints need to be considered. In this case, the ply thickness must be multiples of 0.127 mm (T300/914C single ply thickness). Such limitations cannot be enforced during optimisation but can be imposed afterwards. To incorporate local reinforcement in a conventional autoclave manufacturing process, the reinforcement would be only applied to one side (the bag side) of the laminate. The optimisation of this situation would be more complex due to the presence of offsets and the associated bending of the laminate.

Improved reinforcement designs may also be possible using more than one ring so that, for example, the reinforcement can taper. The same thickness of reinforcement may not be required around the cutout, so this too could be allowed to vary. Further improvement may also be gained by allowing simultaneous optimisation of ply thickness and orientation. These aspects are the subject of current research.

CONCLUSIONS

The work presented demonstrates the suitability of the design sensitivity method of MSC/NASTRAN for the optimisation of cutouts in laminated composite plates. For shape optimisation, a least squares objective function based on element failure indices, was shown to successfully produce a constant failure index around the hole boundary. The results showed that the optimum hole shape does depend on the degree of orthotropy of the laminate. Quasi-isotropic laminates produced holes of very similar shape and aspect ratio to isotropic cases, while less orthotropic laminates affected the resulting optimum cutout aspect ratio. The introduction of local reinforcement surrounding the cutout was shown to be able to produce a neutral hole. The weight penalty was very small when the reinforcement was applied to a cutout that had been previously shape optimised. A greater weight penalty was necessary when local reinforcement was applied to a circular cutout.

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