MULTICRITERION OPTIMIZATION OF LAMINATE STACKING SEQUENCE FOR MAXIMUM FAILURE MARGINS

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SUMMARY: Multicriterion stacking sequence optimization of composite laminates for maximum failure margins with minimum number of layers is considered. Laminate failure margins to different loading conditions are measured in terms of laminate initial failure reserve factors that are computed by applying constant and variable load approach and an iterative line search method. A generalized failure criterion is used to point out that the internal formulation of the failure criterion has no effect on the solution procedure. The actual multicriterion optimization problem to be solved is reduced to a bicriterion problem, where the laminate initial failure reserve factors are combined linearly as the two strongly competing criteria. Discrete design variables for layer orientation identity are utilized in creation of laminate symmetric and balanced stacking sequence permutations. The constraint method is applied to generate maximal solutions. Minimizing distance from the ideal solution in the criterion space identifies the best-compromise solution.

KEYWORDS: composite laminates, design, multicriterion, multiobjective, optimization, failure analysis, failure margin

INTRODUCTION

A weight or a volume is a frequently used criterion in the structural optimization. In practical engineering, however, there are usually several other criteria to measure the performance of a structure. The criteria may be competing, i.e., an improvement of one criterion can only be achieved at the cost of another. The naturally competing criteria in designing composite structures are load carrying capability of a composite laminate against different design loads when the laminate thickness can only be multiples of the layer thickness. Constant and variable load approach for predicting load carrying capability of composite laminates gives a quantitative measure on the margin to initial failure

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[1], [2]. Initial failure is assumed to occur when failure first occurs in one layer or simultaneously in several layers of the laminate. The laminate initial failure analysis is based on the laminate load response computed with the Classical Lamination Theory [3]. Margin to initial failure is measured in terms of a reserve factor RF. In this paper we consider a composite laminate stacking sequence optimization problem with multiple objectives. Given allowable angles 0, 90, and $\pm q$ deg for layer orientations, find a symmetric and balanced laminate stacking sequence with minimum number of layers such that laminate initial failure reserve factors due to m loading conditions subjected to the laminate are maximized. The layer properties as well as the layer thickness t are assumed to be constant. At least some of the loading conditions are assumed to be conflicting, i.e., the optimum stacking sequences corresponding to the different loading conditions are different.

The aim of this paper is to study the stacking sequence optimization problem from the multicriterion optimization point of view. We first represent the laminate analysis formulation appropriate for zero-one programming. In the formulation, zero-one design variables are used to denote the current layer orientation for each layer of the laminate. The discrete design variables are utilized in creation of permutations of different stacking sequences of the laminate. Since the formulations of failure criteria of composite materials may be quite complicated, we apply a derivative-free line search method to solve the laminate initial failure reserve factors [4]. In this method, failure criterion internal formulation has no effect on the solution procedure. In the second part of this work, a multicriterion optimization problem is formulated. The initial problem is reduced to a bicriterion problem, where the vector objective function consists of laminate reserve factors to two conflicting loading conditions or a linear combination of reserve factors to several loading conditions. The objective is to obtain a bicriterion problem with strongly conflicting criteria. The approach has some advantages. The method is efficient from computational viewpoint yet preserving the multicriterion nature of the original problem. Also the graphic illustration of maximal solutions in the criterion space is possible. The payoff table by Benayoun *et. al.* [5] is utilized in forming the bicriterion problem. We apply the constraint method [6] for generating maximal solutions in the criterion space. We select the best-compromise solutions by using the distance function d_∞ to minimize the distance from the ideal solution in the criterion space.

A small selection of papers on the laminate stacking sequence optimization by using discrete design variables comprises [11], [12], [13]. Multicriterion optimization in its present sense originates from an optimality criterion introduced by Pareto in 1896 [7] for the cases where an improvement of one criterion can only be achieved at the cost of another. A comprehensive overview on multicriterion programming can be found in Cohon [6]. Norm methods and partial weighting in multicriterion optimization of structures is dealt with in Koski and Silvennoinen [8]. Defectiveness of weighting method in multicriterion optimization of structures is discussed by Koski [9].

COMPOSITE LAMINATE ANALYSIS

Laminate Load Response

A laminate composed of continuous fibre-reinforced-polymer composite layers in different layer orientations is considered (Fig. 1). Classical Lamination Theory [3] is used to compute properties of laminates and laminate load response, i.e., the laminate is assumed to be in plane stress state.

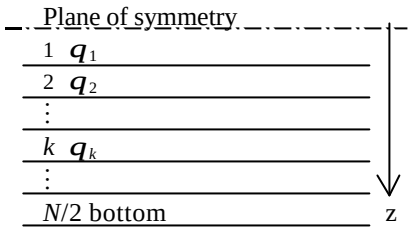

Fig. 1: Layer numbering convention for laminates.

The constitutive equations of the laminate in plane stress state are

$$\begin{Bmatrix} N \\ M \end{Bmatrix}_{xy} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \mathring{a}^0 \\ \hat{e} \end{Bmatrix}_{xy}, \tag{1}$$

where

$$\{N\}_{xy} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}, \quad \{M\}_{xy} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{2}$$

are laminate resultant in-plane forces (forces per unit length) and resultant moments (moments per unit length), respectively, and

$$\{\mathring{a}^0\}_{xy} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}, \quad \{\hat{e}\}_{xy} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{3}$$

are the mid-plane strains and the curvatures of the laminate, respectively, in the laminate *xyz*-coordinate system. Let the laminate be symmetric and balanced, that is, the number of $+\theta$ deg layers is equal to the $-\theta$ deg layers. Due to the symmetry, the laminate coupling stiffness matrix B is identically zero. By using zero-one design variables

$$x^q = \begin{cases} 1 & \text{if the allowable angle } \grave{e} \text{ occurs} \\ 0 & \text{if the allowable angle } \theta \text{ does not occur} \end{cases} \tag{4}$$

to represent binary choice for layer orientations, the laminate in-plane and flexural stiffness matrices A and D may be expressed as

$$A = 2\sum_k \overline{Q}_k\, t, \quad D = \tfrac{2}{3}\sum_k \left[k^3 - (k-1)^3\right]\overline{Q}_k\, t^3, \quad k = 1, 2, \ldots, N/2, \tag{5}$$

in which N is the total number of layers in the laminate and the nonzero elements of $\overline{Q}_k$ are

$$
\begin{aligned}
\overline{Q}_{11} &= U_1 + U_2(x_k^0 - x_k^{90} + c_1 x_k^{+q} + c_1 x_k^{-q}) + U_3(x_k^0 + x_k^{90} + c_2 x_k^{+q} + c_2 x_k^{-q}) \\
\overline{Q}_{22} &= U_1 - U_2(x_k^0 - x_k^{90} + c_1 x_k^{+q} + c_1 x_k^{-q}) + U_3(x_k^0 + x_k^{90} + c_2 x_k^{+q} + c_2 x_k^{-q}) \\
\overline{Q}_{66} &= U_5 - U_3(x_k^0 + x_k^{90} + c_2 x_k^{+q} + c_2 x_k^{-q}) \\
\overline{Q}_{12} &= U_4 - U_3(x_k^0 + x_k^{90} + c_2 x_k^{+q} + c_2 x_k^{-q}) = \overline{Q}_{21} \\
\overline{Q}_{16} &= -\tfrac{1}{2} U_2(c_3 x_k^{-q} - c_3 x_k^{+q}) - U_3(c_4 x_k^{-q} - c_4 x_k^{+q}) = \overline{Q}_{61} \\
\overline{Q}_{26} &= -\tfrac{1}{2} U_2(c_3 x_k^{-q} - c_3 x_k^{+q}) + U_3(c_4 x_k^{-q} - c_4 x_k^{+q}) = \overline{Q}_{62} .
\end{aligned}
\tag{6}
$$

Superscripts 0, 90, $+q$, and $-q$ denote the corresponding allowable angles for layer orientations and $U_1 \ldots U_5$ are the stiffness invariants [3]. Constants $c_1 \ldots c_4$ are defined as $c_1 = \cos 2q$, $c_2 = \cos 4q$, $c_3 = \sin 2q$, and $c_4 = \sin 4q$. The laminate strain state is achieved with

$$
\begin{Bmatrix} \mathring{a}^0 \\ \hat{e} \end{Bmatrix}_{xy} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix}_{xy} . \tag{7}
$$

By computing layer actual strains from the laminate strain state and transforming them to the layer coordinate system, we get layer equivalent strains. By applying the layer constitutive relations, we get the layer actual stresses induced by the equivalent strains to be exploited in layer failure prediction.

Laminate Initial Failure Prediction

The laminate is subjected to a set of simultaneously applied loads that may consist of externally applied mechanical loads, thermal, and moisture induced loads. Each loading condition is partitioned into a constant and variable load vectors [1], [2]. The basic assumption in the constant and variable load approach is that the variable load vector, that represents dynamic loads, varies while the constant load vector representing static pre-loads stays fixed. The probabilistic distributions of the applied loads and the material strength properties can be taking into account by using appropriate factors of safety. Factors of safety are defined separately for constant and variable load vectors. The effective load used in the failure analysis is achieved by multiplying the nominal load with the corresponding factor of safety.

Let the layer actual stresses in a layer coordinate plane 12 caused by the effective load be s. Superscripts c and v denote the constant and variable load vectors, respectively. In a linear analysis, the layer stress vector corresponding to the failure load is

$$
ó^f = ó^c + RF\, ó^v , \tag{8}
$$

where $RF \in R$, $RF > 0$. By using the load criticality factor $l \in R$, $l > 0$, the layer stress vector can be written as

$$
ó(l) = ó^c + l\, ó^v . \tag{9}
$$

The value of a generalized failure criterion F in stress space is obtained from

$$
F(l) = F\left[ó(l)\right]. \tag{10}
$$

The failure criterion indicates failure when $F(\boldsymbol{l}) \geq 1$. For determining margin to failure, an unconstrained minimization problem is formulated [4]

$$\begin{aligned} &\min\ v(\boldsymbol{l}) = \left| 1 - F(\boldsymbol{l}) \right| \\ &\ddot{e} \in [a, b],\ \boldsymbol{l} \in R,\ \boldsymbol{l} > 0 \end{aligned} \tag{11}$$

where the objective function is minimized over the closed bounded interval by iteratively reducing the interval of uncertainty [*a*, *b*]. The interval of uncertainty is reduced each time by a factor of the golden section ratio (≅ 0.618) until the final length of uncertainty is reached after *n* iterations. The point where the failure occurs is determined for each layer as

$$\boldsymbol{l} = (a_n + b_n)/2 \cong RF\ . \tag{12}$$

Laminate failure margin is indicated by the laminate initial failure reserve factor defined as the minimum of the layer reserve factors

$$RF_L = \min_l\ RF_l^{t,b}, l = 1, 2, \ldots, N\ , \tag{13}$$

where *l*=1, 2,…, *N* denotes layers of the laminate from top to bottom and superscripts *t* and *b* denote the top and bottom surfaces of the layer. Accordingly, the load criticality factor can be determined in strain space.

MULTICRITERION PROBLEM

Preliminaries

The problem is formulated as a multicriterion optimization problem

$$\max_{x \in \Omega}\ f(x)\ , \tag{14}$$

where *f* is a vector objective function

$$f(x) = \{ f_i(x) \mid f_i : \Omega \to R, i = 1, 2, \ldots, m \}\ . \tag{15}$$

The components are called criteria and they represent the design objectives by which the performance of the laminate is measured. The design variables

$$x = \{ x_k^{\Theta} \mid x_k^{\Theta} \in \{0,1\}, \sum_{\Theta} x_k^{\Theta} = 1, \sum_k (x_k^{+q} - x_k^{-q}) = 0, k = 1, 2, \ldots, N/2 \}\ , \tag{16}$$

where Θ= (0, 90, +$\boldsymbol{q}$, −$\boldsymbol{q}$), belongs to the feasible set defined as

$$\Omega = \{ x \mid g(x) \leq 0, h(x) = 0 \}\ . \tag{17}$$

The image of the feasible set in the criterion space, called the attainable set, is defined by

$$\Lambda = \{z \in R^m \mid z = f(x), x \in \Omega\}. \tag{18}$$

Usually, there exists no unique solution which would maximize all m criteria simultaneously. For the definition of optimal solution of a multicriterion optimization problem we apply the optimality criterion originally introduced by Pareto [7].

Definition. A solution x^* is Pareto optimal for Eqn 14 if and only if there exists no $x \in \Omega$ such that

$$\begin{aligned} &f_i(x) \geq f_i(x^*) \quad \forall i \in [1, m] \\ &f_i(x) > f_i(x^*) \quad \text{for at least one } i \in [1, m]. \end{aligned} \tag{19}$$

The points $z^* = f(x^*)$ in the criterion spaces are called the maximal points.

Solution Strategy, Criteria, and Constraints

Minimizing the distance from the in general nonfeasible ideal solution in the criterion space by applying the distance function

$$d_\infty(z, y) = \max\{|z_1 - y_1|, |z_2 - y_2|\} \tag{20}$$

optimizes the laminate stacking sequence. First, the optimum initial lay-up configuration is determined for each loading condition separately. Hence, we get m different lay-up configurations. Laminate failure margins to the different loading conditions are computed for each laminate and gathered into a table called the payoff table by Benayoun *et. al.* [5] (Table 1).

Table 1: Payoff table for m criteria.

	$f_1(x)$	$f_2(x)$	$\cdots$	$f_m(x)$
x^1	$f_1(x^1)$	$f_2(x^1)$	$\cdots$	$f_m(x^1)$
x^2	$f_1(x^2)$	$f_2(x^2)$	$\cdots$	$f_m(x^2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x^m	$f_1(x^m)$	$f_2(x^m)$	$\cdots$	$f_m(x^m)$

The *ideal point* is on the diagonal of the payoff table. The minimum value in each column of the payoff table is the *nadir point* denoted by n_i. To find the most competing criterion, the "normalization coefficients" based on the payoff table are computed as

$$m_i = \frac{h_i}{\sum_i h_i}, \text{ where } h_i = \frac{f_i(x^i) - n_i}{\max\{|f_i(x^i)|, |n_i|\}}, i = 1, 2, \ldots, m. \tag{21}$$

The objective is to obtain a bicriterion problem with strongly competing criteria. The criterion producing minimum m_i is selected for the single criterion $f_1(x)$ in the new bicriterion problem. The other criteria are combined linearly as

$$f_2(x) = \sum_i \lambda_i RF_i,\ \lambda_i > 0,\ \sum_i \lambda_i = 1,\ i = 1, 2, \ldots, m-1, \tag{22}$$

where weighting coefficients λ_i are selected arbitrarily by the designer. Hence, all the at least rather non-competing criteria have been combined linearly to yield the problem with the reduced vector objective function

$$f(x) = \{ f_1(x),\ f_2(x) \}\ . \tag{23}$$

The criteria may be non-commensurable, and thus we use a scaled presentation

$$z_i = \frac{f_i(x)}{\max f_i},\ i = 1, 2\ . \tag{24}$$

The feasible set is defined as

$$\Omega = \{ g_i(x) = 1 - RF_i \le 0,\ RF_i \in R,\ RF_i > 0,\ i = 1, 2, \ldots, m \}\ . \tag{25}$$

For determining Pareto optimal lay-up configurations permutations of symmetric and balanced stacking sequences with four allowed layer orientations and fixed initial N are created. Since the convexity of the attainable set in the criterion space is not guaranteed, we apply the constraint method [6], [8], [9] to generate maximal solutions. If none of the permutations is feasible, a new set of laminates is created by adding a layer block $\{(0, 0),\ (0, 90),\ (90, 0),\ (90, 90),\ (+\boldsymbol{q}, -\boldsymbol{q}),\ (-\boldsymbol{q}, +\boldsymbol{q})\}$ deg on the lay-up configuration that minimizes the distance from the ideal solution. Once again the closest solution to the ideal solution is selected. This procedure is repeated until a lay-up configuration that satisfies all the constraints is reached. Finally, the number of layers is tried to reduce, if possible, one layer at a time such that the feasible laminate design is still maintained.

When two conflicting loading conditions are subjected to the laminate, by creating all permutations of the symmetric and balanced stacking sequences, it is possible to generate all maximal solutions. When the reduced problem formulation is used, only a subset of maximal solutions is achieved by fixing the weighting factors. By varying the weighting factors it is possible to generate more maximal solutions to cover the maximal set more completely.

LAMINATE SUBJECTED TO THREE CONFLICTING LOADING CONDITIONS

The application of a multicriterion stacking sequence optimization problem is illustrated by considering a laminate that is subjected to three conflicting loading conditions according to the Table 3. The objective is to determine a laminate lay-up configuration with minimum number of layers such that the laminate initial failure reserve factors are maximized to the loading conditions with $FoS = 1.5$. The laminate is composed of layers having the mechanical properties of AS4 Carbon/epoxy ply [10] (Table 2). The allowable angles for layer orientations are 0, 90, and ±45 deg. Tsai-Hill failure criterion is used.

First, the optimal initial lay-up configuration for each loading condition is determined individually. Maximization of the laminate initial failure reserve factors to the load cases 1, 2, and 3 with the

minimum allowed number of layers $N/2=4$ leads to the unique solutions x^1, x^2, and x^3, respectively. The payoff table is shown in Table 4. Based on the payoff table, $m_1 = 0.3790$, $m_2 = 0.3139$, and $m_3 = 0.3071$.

Table 2: Mechanical properties of the ply.

Ply AS4 Carbon / epoxy (AS4 / 3501-6) $t = 0.25$ mm	
Ply engineering constants	**Ply failure stresses**
$E_1 = 126$ GPa	$X_t = 1950$ MPa $X_c = 1480$ MPa
$E_2 = 11$ GPa	$Y_t = 48$ MPa $Y_c = 200$ MPa
$G_{12} = 6.6$ GPa	$S_{12} = 79$ MPa
$\nu_{12} = 0.28$	

Table 3: The three conflicting loading conditions.

Load case 1	**Load case 2**	**Load case 3**
$N_x = -55$ kN/m	$N_x = -50$ kN/m	$N_x = -280$ kN/m
$N_y = 0$	$N_y = -260$ kN/m	$N_y = -50$ kN/m
$N_{xy} = 100$ kN/m	$N_{xy} = 0$	$N_{xy} = 0$
$M_x = 10$ Nm/m	$M_x = 0$	$M_x = -10$ Nm/m
$M_y = 0$	$M_y = 10$ Nm/m	$M_y = 0$
$M_{xy} = 10$ Nm/m	$M_{xy} = 0$	$M_{xy} = 0$

Table 4: The Payoff table for the laminate design problem.

x	$RF_1(x)$	$RF_2(x)$	$RF_3(x)$
x^1 = [+45/−45/0/90]SE	2.8326	2.0826	2.0172
x^2 = [90/90/90/0]SE	0.8030	5.1215	2.0415
x^3 = [0/0/0/90]SE	0.8101	2.1853	4.8081

Therefore, the problem with the reduced vector objective function is formulated as

$$\begin{aligned} &\max_{x\in\Omega}\{RF_3(x),\ l_1 RF_1(x)+(1-l_1)RF_2(x)\} \\ &\Omega=\{g_i(x)=1-RF_i\le 0,\ RF_i\in R,\ RF_i>0,\ i=1,2,3\}. \end{aligned} \tag{26}$$

Table 5: Pareto optimal lay-up configurations at the maximal points to the case $l_1 = 0.5$ shown in Fig. 1. The best-compromise solution is underlined.

Lay-up	z_1	z_2	RF_1	RF_2	RF_3	d_∞
[90/+45/−45/90]SE	0.2681	1	2.4228	3.9431	1.0080	0.7319
[90/90/+45/−45]SE	0.2694	0.9776	2.0690	4.1545	1.0132	0.7306
[+45/90/−45/0]SE	0.5208	0.7940	2.7212	2.3332	1.9585	0.4792
[+45/90/0/−45]SE	0.5471	0.7789	2.5952	2.3631	2.0573	0.4529
[90/+45/0/−45]SE	0.5679	0.7556	2.3007	2.5092	2.1354	0.4321
[+45/0/90/−45]SE	0.5824	0.7370	2.4828	2.2091	2.1901	0.4176
[90/0/+45/−45/]SE	0.6139	0.7149	1.9922	2.5589	2.3084	0.3861
<u>[0/90/+45/−45]SE</u>	<u>0.6270</u>	<u>0.7020</u>	<u>1.9678</u>	<u>2.5009</u>	<u>2.3576</u>	<u>0.3730</u>
[+45/−45/0/0]SE	0.7872	0.5722	2.5868	1.0557	2.9603	0.4278
[+45/0/−45/0]SE	0.8703	0.5718	2.5834	1.0564	3.2727	0.4282
[+45/0/0/−45]SE	0.9008	0.5368	2.3845	1.0327	3.3874	0.4632
[0/+45/−45/0]SE	0.9529	0.5336	2.3135	1.0831	3.5834	0.4664
[0/+45/0/−45]SE	0.9659	0.4967	2.1011	1.0611	3.6320	0.5033

[0/0/+45/−45]SE	1	0.4756	1.9385	1.0890	3.7604	0.5244

Permutations of symmetric and balanced stacking sequences with allowable layer orientations are created. The maximal solutions in bicriterion problem are computed by using the constraint method. The maximal points achieved with l_1=1, 0.75, 0.5, 0.25, and 0 are shown in Fig. 2. Pareto optimal laminate lay-up configurations corresponding to $l_1 = 0.5$ are given numerically in Table 5. This is the case, in which the weighting coefficients of failure margins to the both load cases are equal.

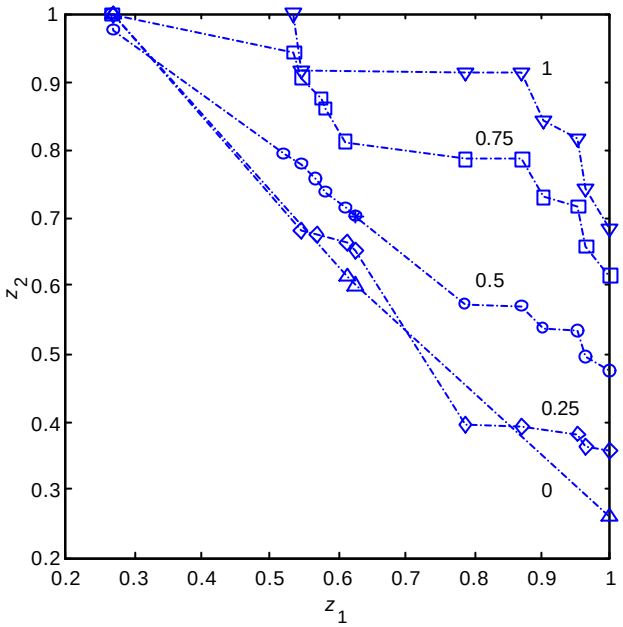

Fig. 2: Maximal points to the bicriterion problem with three conflicting loading conditions, $l_1 = 1$, 0.75, 0.5, 0.25, *and* 0. *The best-compromise solution to the case* $l_1 = 0.5$ *is marked with the star.*

CONCLUSIONS

Generating Pareto optimal solutions for a multicriterion stacking sequence optimization problem of composite laminates subjected to multiple conflicting loading conditions is presented. Laminate initial failure reserve factors are treated as criteria and computed with an iterative line search method. The method is suitable to be used also with complicated failure criterion formulations, since the internal formulation of the failure criterion has no effect on the solution procedure. The optimization scheme is based on the solution of bicriterion subproblems, which enables the designer to obtain results in any accuracy that is relevant. Using the reduced vector objective function is computationally efficient still preserving the multicriterion nature of the problem. Also the graphical illustration of the maximal solutions in the criterion space is possible.

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