DESIGN OF HYBRID LAMINATED COMPOSITE SANDWICH PANELS FOR MAXIMUM BUCKLING LOADS: THE INFLUENCE OF THERMAL ACTIONS

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SUMMARY: In the paper, the better arrangement of sandwich flat panels made with hybrid laminated faces is analysed in order to maximize the local buckling loads in presence of in-plane and/or thermal loads. Laminated hybrid faces, generally unsymmetric and made with outer high stiffness layers and inner low stiffness layers, are modelled as composite plates resting on an elastic two-parameter foundation. Using the First Order Shear Deformation Theory in conjunction with the Rayleigh-Ritz method, buckling loads are obtained by an energetic algorithm as a solution of a standard eigenvalue problem. Results of the analysis evidence as: *i)* the hybridisation of laminated faces furnishes a good improvement of the buckling loads also for low percentage of high stiffness materials; *ii)* the better arrangement of outer high stiffness layers are depending on both in-plane loads combinations and thermal loads; *iii)* thermal actions, in some cases, can drastically reduce the buckling loads.

KEYWORDS: Sandwich panels, hybrid faces, thermal actions, buckling loads

INTRODUCTION

Sandwich structures with composite laminated faces and a core made of soft materials such as foams or low strength honeycombs, are commonly used in engineering applications mainly in the field of aerospace and civil engineering structures [1]. This technological solution is advantageous in terms of weight and manufacturing processes if compared with sandwiches made with metallic faces and rigid core materials like metallic honeycomb. Besides, the use of composite plastic materials allows to have low thermal conductivity through the thickness and, then, high service temperature and high values of temperature strength.
Laminated composite faces of sandwiches can be arranged in order to optimize the structural response and, consequently, to obtain the minimum cost. At this aim the hybridisation of laminated faces, that is the use of layers of high mechanical properties in high stress area and layers of weaker material in low stress area, can guarantee an improvement both from a mechanical and economical point of view[2].
Sandwich structures with hybrid laminated faces and soft core material are, therefore, candidate for a very widespread use in engineering constructions.

From a static point of view these structures, however, involve some problems because of their out-of-plane flexibility and their high anisotropy. In particular, with reference to the instability of sandwich structures, the out-of-plane flexibility due to the soft core material, leads to different behaviour patterns as compared with structures made of rigid core materials. An interaction between local and global instability is, in fact, associated to a compressible core material and in many cases the structural analysis becomes very onerous.

The wrinkling of compressed faces represents the most frequent buckling mode of sandwich panels where the core is made of a low density material; in this case, in presence of flexural actions, the core may be insufficient to stabilize the whole panel, the faces become independent each other and the compressed face can buckle locally because of its small thickness.

The evaluation of wrinkling buckling load must be carried out considering the coupling between global and local instability; however, the use of models that consider the coupling of the two instabilities are founded on a high-order theory and are defined by complex analytical procedures, often very onerous from a computational point of view [3]. However, in many cases, models that suppose the uncoupling between global and local buckling furnish a good provision for the real behaviour of structures with little computational effort.

Starting from these considerations, in the paper, the better arrangement of sandwich panels with hybrid composite laminated faces is analysed in order to maximize the buckling loads corresponding to the wrinkling of compressed faces.

The analysis is carried out modelling the compressed face of panels as a laminate resting on an elastic two-parameter foundation; these parameters are defined on the basis of the geometrical dimensions and mechanical properties of the core material [4].

Using the First Order Shear Deformation Theory, in conjunction with the Rayleigh-Ritz method, buckling loads of laminated faces, generally unsymmetric, are obtained by an energetic algorithm as a solution of an eigenvalue problem [2].

A numerical investigation allows us to define the better arrangement of laminates corresponding to the maximum buckling load in presence of in-plane and/or thermal loads, by varying parameters of both the laminates (fibers orientation of layers, geometrical dimensions, slenderness, etc.) and the subgrade (modulus of subgrade reaction, shear modulus).

BUCKLING LOAD ANALYSIS

Local buckling loads are determined considering the compressed face of sandwich panels as a flat laminate resting on a two-parameter elastic foundation. Parameters that characterize the elastic foundation, the modulus of subgrade reaction k and the shearing interaction between the loaded face and the core G_b, are strictly connected to the geometrical and mechanical properties of the soft core material. In the analysis the core material is assumed to be isotropic, homogeneous and linear-elastic while the face is assumed as an unsymmetric laminate.

Foundation parameters are defined by Vlasov and Leont'ev relationships [5]:

$$k = \frac{E_0\,\gamma}{b\left(1 - \nu_0^2\right)}\beta_1 \qquad G_b = \frac{E_0\,b}{4\gamma h\left(1 + \nu_0\right)}\beta_2$$

(1)

where

$$E_0 = \frac{E_c}{1-\nu_c^2}; \quad \nu_0 = \frac{\nu_c}{1-\nu_c}; \quad \beta_1 = \frac{sinh\psi \cosh\psi + \psi}{sinh^2 \psi}; \beta_1 = \frac{sinh\psi \cosh\psi - \psi}{sinh^2 \psi}; \psi = \frac{2\gamma h_c}{b}$$
(2)

being b the width of the laminate, γ a factor dependent on the properties of elastic foundation, h_c and h the core and face thickness, respectively.

Thermoelastic constitutive law of laminates

Supposing that the material properties are independent on the temperature, the constitutive law of the laminate is expressed as [6]:

$$\begin{aligned} \{N\} &= [A]\{\varepsilon\} + [B]\{k\} - \{N_T\} \\ \{M\} &= [B]\{\varepsilon\} + [D]\{k\} - \{M_T\} \\ \{Q\} &= [F]\{v\} \end{aligned}$$
(3)

being

$$\{N\} = \{N_x, N_y, N_{xy}\} \quad \{M\} = \{M_x, M_y, M_{xy}\} \quad \{Q\} = \{Q_x, Q_y\}$$
$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij} (1, z, z^2) dz \quad (i, j = 1, 2, 6) \qquad F_{ij} = \int_{-h/2}^{h/2} k_{sh} \overline{Q}_{ij} dz \quad (i, j = 4, 5)$$
(4)
$$\{N_T, M_T\} = \int_{-h/2}^{h/2} \overline{Q}_{ij} \alpha_j (1, z) dz$$

where k_{sh} is the shear correction factor, $\overline{Q_{ij}}$, functions of elastic constants and of the ply angle θ, differ from one layer to another, α_j are coefficients of thermal expansion, while

$$\{\varepsilon\} = \{\varepsilon_{x0}, \varepsilon_{y0}, \varepsilon_{xy0}\} \quad \{k\} = \{k_x, k_y, k_{xy}\} \quad \{v\} = \{\varepsilon_{yx}, \varepsilon_{xy}\}$$
(5)
with

$$\varepsilon_{x0} = u_{0,x}; \;\; \varepsilon_{y0} = v_{0,y}; \;\; \varepsilon_{xy0} = u_{0,y} + v_{0,x}; \;\; \varepsilon_{xy} = w_{x} - \psi_x; \;\; \varepsilon_{yz} = w_{,y} - \psi_y$$
$$k_x = -\psi_{x,x}; \;\; k_y = -\psi_{y,y}; \;\; k_{xy} = -(\psi_{x,y} + \psi_{y,x})$$
(6)

being u_0, v_0, ε_{x0}, ε_{y0}, ε_{xy0} displacements and strains at the middle plane of the laminate and ψ_x, ψ_y the shear rotations.

Buckling loads evaluation

The buckling load of a flat laminate is determined by an energetic approach founded on Dirichlet's principle expressed as $\delta_2 E = P + \lambda(L_2^* - L_2)$ where P is the total strain energy of a laminate resting on an elastic foundation, L_2^* is the work of internal stresses in the initial

configuration for the second order strain components, L_2 is the work corresponding to the external loads for the second order displacements components, $\delta_2 E$ is the second variation of the total potential energy and λ is the load multiplier [2].
Considering a flat rectangular laminate of dimensions a and b along the x- and y-axis, respectively, the total strain energy P is expressed as $P = W+U$ being W the strain energy of the laminate furnished by the following relation

$$W=\frac{1}{2}\iint_S \left[\{\varepsilon\}^T[A]\{\varepsilon\}+\{\varepsilon\}^T[B]\{\kappa\}+\{\kappa\}^T[B]\{\varepsilon\}+\{\kappa\}^T[D]\{\kappa\}+ \right.$$
$$\left. +\{\nu\}^T[F]\{\nu\}-\{\varepsilon\}^T\left\{N_T\right\}-\{\kappa\}^T\left\{M_T\right\}\right]dxdy \tag{7}$$

while the strain energy due to the two-parameter foundation is given by

$$U=\frac{1}{2}\iint_S \left[k\,w^2+G_b\left(w_{,x}^2+w_{,y}^2\right)\right]dxdy$$
(8)

The work L_2^* is given by

$$L_2^*=\frac{1}{2}\iint_S \left[(N+N_T)_x\left(u_{0,x}^2+v_{0,x}^2+w_{,x}\right)+(N+N_T)_y\left(u_{0,y}^2+v_{0,y}^2+w_{,y}\right)+\right.$$
$$\left.+(N+N_T)_{xy}\left(u_{0,x}u_{0,y}+v_{0,x}v_{0,y}+w_{,x}w_{,y}\right)\right]dxdy \tag{9}$$

The work L_2 is zero when external loads are constant or indipendent on u,v and w displacements. However, for unsymmetric laminates, in which the geometric middle plane is different from the neutral plane, added conditions (*flatness conditions* [7]) are needed in order to determine the buckling load as bifurcation load; these conditions involve the presence of edge moments and shear forces that make[2] $L_2 \neq 0$.
The condition $\delta_2 E=0$ gives the relations that allow the buckling load of laminates to be determined. The problem solution is obtained by the Raleygh-Ritz method assuming a linear variation of in-plane displacements u and v and a constant transverse deflection w over the panel thickness

$$u(x,y,z)=u_0(x,y)+z\psi_x(x,y);\quad v(x,y,z)=v_0(x,y)+z\psi_y(x,y);\quad w(x,y,z)=w(x,y) \tag{10}$$

For a simply supported flat laminate the displacements that satisfy the geometric boundary conditions are chosen in the form of trigonometric series

$$u_0(x,y)=\sum_m\sum_n U_{mn}\cos(m\pi x/a)\sin(n\pi x/b)\quad v_0(x,y)=\sum_m\sum_n V_{mn}\sin(m\pi x/a)\cos(n\pi x/b)$$
$$\psi_x(x,y)=\sum_m\sum_n X_{mn}\cos(m\pi x/a)\sin(n\pi x/b)\quad \psi_y(x,y)=\sum_m\sum_n Y_{mn}\sin(m\pi x/a)\cos(n\pi x/b)$$
$$w(x,y)=\sum_m\sum_n W_{mn}\sin(m\pi x/a)\sin(n\pi x/b) \tag{11}$$

By this procedure, one obtains a system of linear equations of *5mn x5mn* order and the buckling load is determined as solution of a standard eigenvalue problem [2].

THE OPTIMIZATION PROCEDURE

The optimal configuration of the sandwich panels is defined maximizing the local buckling load, in presence of in-plane and/or thermal loads.
The optimization procedure, followed in the analysis, refers to both the faces arrangement and the core material properties, assuming as objective function the local buckling load. For assigned geometrical dimensions of panels, the local buckling load is expressed as $N_x = min\ N_{mn}(m,n;\ \alpha_r;\ \theta;\ \xi;\ \zeta;\rho;\ k;\ G_b;\ \alpha_j;\ T)$ being N_{mn} the buckling load; m,n the number of Fourier harmonics in the x and y directions that guarantee the solution convergence; $\alpha_r = t_r/h$ is the ratio between the thickness of the outer high-stiffness layer of faces t_r and the thickness of faces h, θ is the layers lamination angle, $\xi = N_y/N_x$ and $\zeta = N_{xy}/N_x$ are the ratio between the in-plane loads, $\rho = h_c/t$ is the ratio between the core thickness and the total thickness t of the panel, α_j are the thermal coefficients of layers, T is the temperature value. Maximum buckling load values are obtained by optimal determination of each parameter influencing N_x; therefore if η represents a generic parameters it will result $max\ N_x\ (\eta) = max\ [min\ N_{mn}(m,n;\ \eta)]$.

NUMERICAL INVESTIGATIONS

Numerical analyses are referred to sandwich panels with hybrid unsymmetric laminated faces made with one outer layer of graphite-epoxy for which $E_{11} = 103.3\ GPa;\ E_{22} = 9.377\ GPa;\ G_{12} = G_{13} = 5.402\ GPa;\ G_{23} = 1.724\ GPa;\ v = 0.33;\ \alpha_1 = 0.31x10^{-6}\ °C^{-1};\ \alpha_2 = 31x10^{-6}\ °C^{-1}$ and inner layers of glass-epoxy for which $E_{11} = 38.6\ GPa;\ E_{22} = 8.27\ GPa;\ G_{12} = G_{13} = G_{23} = 4.14\ GPa;\ v = 0.26;\alpha_1 = 8.6\ x10^{-6}\ °C^{-1};\ \alpha_2 = 22.1x10^{-6}\ °C^{-1}$. Mechanical properties of the core material are defined as: $E_c = \gamma^* E_{22\ glass}$, $\nu_c = 0.35$. k and G_b values are determined by Vlasov's relationships [5] assuming $\gamma = 1.0$ while $k_{sh} = 5/6$ for all calculations.
The Fig. 1 shows curves n'-T being $n' = n/n_0$ and n_0 the buckling load value for the homogenous laminate ($\alpha_r = 0$) subjected to in-plane loads (without thermal loads), varying the amount of high stiffness material (graphite-epoxy), for a *45°/-45°/45°/0°* hybrid laminate.
It is evident that the use of hybrid solutions allows to obtain a remarkable increase of the buckling loads also for low percentage of high stiffness material. On the contrary, the increase of the thermal loads produces a strong reduction of the buckling loads.
The influence of mechanical properties of the core material ($\gamma^* = E_c/E_{22\ glass}$) for a five layer square laminate (Lay-up $45°/0°/90°/0°/\ \theta_{out}$; θ_{out} is the lamination angle of the outer high stiffness layer) is shown in the Fig. 2. In this figure the n-γ^* curves ($n = N_x b^2/E_{22}h^3$) are drawn. Obtained results put in evidence that buckling loads are increasing with the mechanical properties of the core material; maximum buckling loads correspond to $\theta_{out} = 90°$for the lowest γ^*values and to $\theta_{out} = 45°$for higher γ^*values.
Curves T_{cr}- θ_{out} varying the slenderness *a/b,* for a $45°/0°/90°/0°/\theta_{out}$ (with $\alpha_r = 1$) laminate, are drawn in the Fig. 3, being T_{cr} the temperature value corresponding to the thermal buckling load of the laminate. The analysis of results shows that, the best configuration of laminates corresponds always to $\theta_{out} = 0°$.

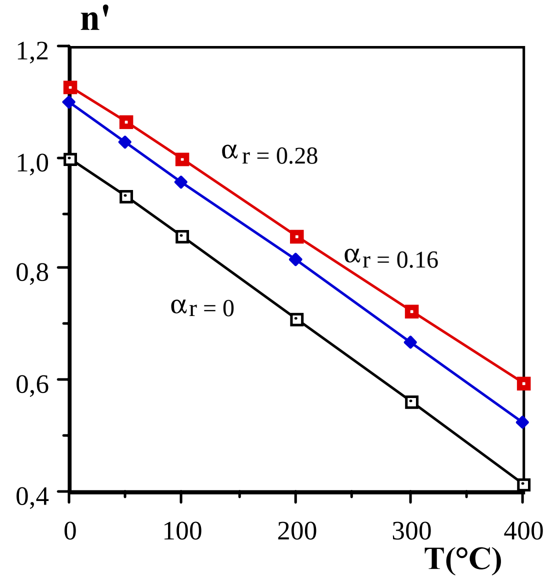

Fig. 1: n'-T - α_r curves for a square laminate
(ξ=1; ζ=0; b/h= 100; γ^*=0.001; *Lay-up: 45°/-45°/45°/0°*)

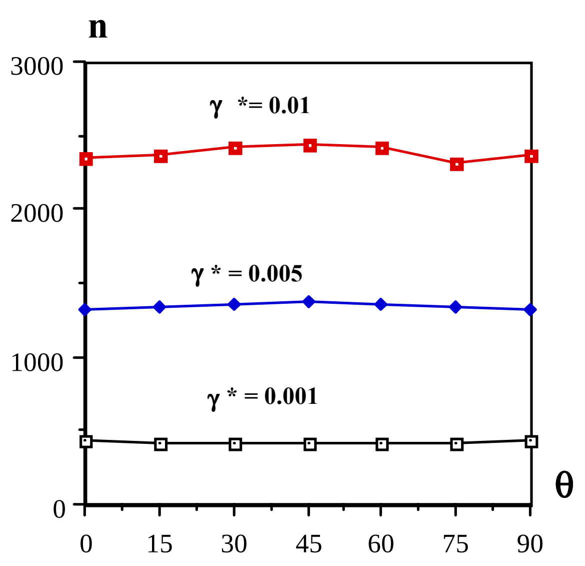

Fig. 2 : n-θ (θ=θ_{out}) curves varying γ^* for a square laminate
(ξ=0;ζ=1; b/h= 80; ρ=0.60; *Lay-up: 45°/0°/90°/0°/* θ_{out})

In the Fig. 4 curves n^*-θ_{out} for a *45°/-45°/45°/*θ_{out} laminate are drawn, varying the temperature value (n^* is the ratio between buckling load values and the buckling load determined in absence of thermal loads). It is possible to evidence as increasing the temperature value a drastical reduction of the buckling loads takes place mostly for some lay-up of laminates. Optimal arrangements correspond to different values of θ_{out} for each thermal load value (θ_{out} = *- 60°* for *T=100 °C*; θ_{out} *=0°* for *T=300 °C*; θ_{out} = *-15°* for *T=500 °C* for cases examined). It clearly appears as the lamination angle plays a relevant role in the optimal design of a sandwich panel in presence of high temperature values. In fact, as one can see from the Figs 3 and 4, an inopportune choice of the θ_{out} could give differences in buckling loads of almost 80% when temperature arise to *500 °C*.

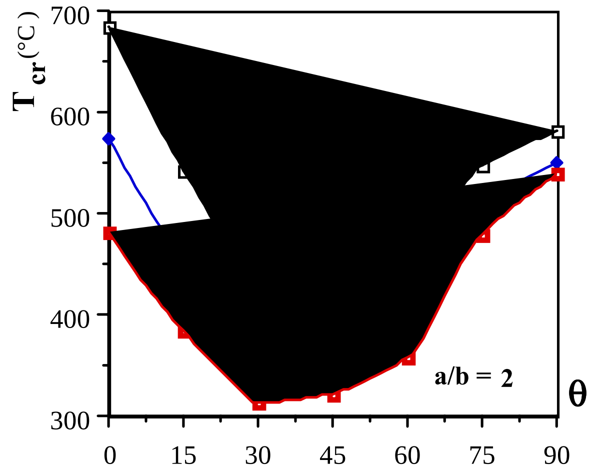

Fig. 3: T_{cr} -θ_{out} (θ= θ_{out}) curves varying *a/b* for a square laminate
(*ξ=ζ=0; b/h=100; γ^{*}=0.001; Lay-up: 45°/0°/90°/0°/ θ_{out}*)

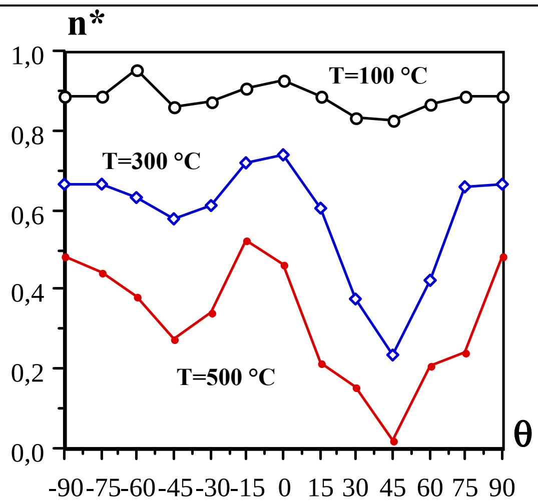

Fig. 4: n^{*}– θ (θ= θ_{out}) curves varying *T* for a square laminate
(*ξ=ζ=0; b/h= 80; γ^{*}=0.001; α_r=0.1; Lay-up: 45°/-45°/45°/θ_{out}*)

CONCLUSIONS

The better arrangement of compressed hybrid laminated faces of sandwich panels for maximum local buckling loads under in-plane and thermal loads was investigated in the paper. On the basis of obtained results it is possible to draw the following concluding remarks:

1. the hybridisation of laminated faces furnishes a good improvement of the local buckling load value also for low percentage of high stiffness materials;
2. the better arrangement of outer high stiffness layers are depending on both in-plane loads combinations and thermal load values;
3. buckling loads are strongly influenced from the core material properties; varying these properties it is possible to optimize sandwich panels for each external loads combination;
4. thermal actions can drastically reduce buckling load values.

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