LOAD-RATIO DEPENDENCE ON FATIGUE LIFE OF COMPOSITES

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SUMMARY:
A model for predicting the fatigue life of composites at different load ratios have been developed. The basic assumption of the model is that the fatigue life of a specimen is governed by delamination growth. The model is based on calculation of delamination growth by integration of Pari's law. It is assumed that fracture for different load ratios, but with the same peak-load magnitude, will occur at the same delamination length. In addition, it is assumed that the energy release rate does not vary with delamination length. The model is able to predict the fatigue life for tension-tension, compression-compression, and tension-compression fatigue. In comparison with experimental results for unnotched specimens the model shows good agreement. Due to the large exponent in Pari's law for composites the model is sensitive to errors in the parameters used.

KEYWORDS: fatigue life, load ratio, delamination growth, notch, life-prediction model

INTRODUCTION

Different composite structures in applications such as aircraft are often subjected to spectrum fatigue loading with different average load ratios, $R = \sigma_{min}/\sigma_{max}$. When characterizing the material for such structures constant amplitude fatigue data of coupons at the specific *R*-values are often used. This results in a large number of fatigue test having to be run which is expensive. Hence, there is a need to develop models for calculating the fatigue life of composites at different *R*-values.

There are a large number of different models available for predicting fatigue life of composites and some of them will be mentioned here. The model which probably is most well known is Miner's rule. It has been found that the model usually overestimates the fatigue life of composites.[1][2] However, for some simple spectra it gives reasonable results.[3] In the strength degradation model fatigue failure occurs when the residual strength has dropped to

the maximum stress of the fatigue loading.[4][5] By using the fatigue failure envelope method to describe the fatigue behavior of composite laminates Rotem was able predict the fatigue life at any load ratio.[6] The fatigue life of a composite can be estimated with fatigue failure functions established on a laminate level.[7] Different *R*-values are then handled with the Goodman correction approach. The model by Ratwani and Kan[8] models delamination growth in a specimen. The delamination growth is predicted from interlaminar shear stresses. They found that if the interlaminar shear stress is plotted against fatigue life data for different *R*-values and specimens the results will collapse into a single scatter band.

The objective of this paper is to develop a model for predicting the fatigue life at different *R*-values and then compare the model with experimental results. The model should be based on material data which can be measured with simple test specimens.

MODEL

The basic assumption of the present model is that the fatigue life of a specimen is governed by delamination growth. When the delaminations have grown to a critical size the specimen will fail. This critical size is dependent on the peak load at failure. All other damage mechanisms, for example matrix cracking and fiber failure, are assumed to be of secondary importance and will only have a minor influence on the total life of the specimen. A consequence of this assumption is that the fatigue life of a specimen can be estimated by calculating the delamination growth rate in the specimen which is what the model in this paper will do. The model will be applicable to all situations which fulfill the basic assumption above. Extensive delamination growth has been observed for unnotched specimens fatigue loaded in tension-tension and tension-compression.[9] Delamination growth of the critical delamination which will cause failure is calculated by integrating Paris law. In fatigue of composites several delaminations in thickness direction are often growing at the same time. But, it is reasonable to assume that the delamination growth of the critical delamination will still be governed by Paris law when the energy release rate of that delamination is used. From the global quantity compliance of notched specimens it has been possible to obtain Paris law.[10] This would suggest that Paris law also is valid on a global scale.

Recently, a model for predicting fatigue delamination growth rate has been developed.[11] The model uses Paris law and predicts the growth rate for different *R*-values and mixed mode ratios. It is based on the observation that the peak energy release rate can not be larger than the critical quasi-static energy release rate and that the change in energy release rate ΔG_r at the threshold is independent on mixed mode ratio and load ratio. This gives two known values from which the constants in Paris law can be calculated. This model will be the basis for calculating the constants in Paris law.

Model for tension-tension and compression-compression fatigue

When the delamination growth rate is measured experimentally with test specimens, for example DCB and ENF specimens, the *R*-value is defined such that $-1 \leq R \leq 1$. For other test specimens such as notched and unnotched coupons there is a clear difference in loading between, for example, R=0.1 and R=10. But, when energy release rates of delaminations in those specimens are compared with those from simple test specimens there is no difference between R=0.1 and R=10. Therefore, a new parameter Q is introduced and defined as

$$Q = R \qquad \text{if } -1 \leq R \leq 1 \tag{1}$$

or

$$Q = \frac{1}{R} \qquad \text{if } R<-1 \text{ or } R>1 \tag{2}$$

To calculate the delamination length $a(N)$ as function of number of cycles in a specimen Paris law is integrated as

$$a(N) = \int_{Ni}^{N} \frac{da}{dN} dN = D \int_{Ni}^{N} \left[\Delta G_r\right]^n dN \tag{3}$$

where N is number of cycles, Ni is number of cycles at which a delamination is initiated, D and n are constants in Paris law, and ΔG_r is the range of change in energy release rate. The difficulty is to determine ΔG_r. It is assumed that the energy release rate as a function of crack length $G(a)$ for the critical delamination of the specimen has been calculated at an applied load P_{ref}. The energy release rate at an applied load P is then

$$G_P(a) = G(a) \frac{P^2}{P_{ref}^2} \tag{4}$$

For the case $Q \geq 0$ the range of change in energy release rate can now be given as

$$\Delta G_r = G_{P\max}(a) - G_{P\min}(a) = \frac{G(a)}{P_{ref}^2} \left[P_{\max}^2 - P_{\min}^2\right] \tag{5}$$

where $P_{\max}$ and $P_{\min}$ are maximum and minimum applied force magnitude. Force magnitude is used since it will make it possible to treat tension-tension and compression-compression loading in the same way in the following derivation. The Q-value is related to the applied force magnitude as

$$Q = \frac{P_{\min}}{P_{\max}} \tag{6}$$

Substitution of Eqn 6 into Eqn 5 gives

$$\Delta G_r = G(a) \frac{P_{\max}^2}{P_{ref}^2} \left[1 - Q^2\right] \tag{7}$$

Substitution of Eqn 7 into Eqn 3 gives the delamination length as

$$a(N) = D \left\{ \frac{P_{\max}^2}{P_{ref}^2} \left[1 - Q^2\right] \right\}^n \int_{Ni}^{N} \left[G(a)\right]^n dN \tag{8}$$

Consider now two specimens subjected to constant amplitude fatigue loading at two different Q-values, Q_1 and Q_2. The fatigue loading is either compression-compression on both specimens or tension-tension on both specimens with $P_{\max}$ equal for both specimens. It is then reasonable to assume that both specimens will break when the delamination has grown to a

critical size a_c, which is equal for both specimens, since $P_{\max}$ is equal for both specimens. This gives

$$a_{c,Q1}(N_{f1}) = a_{c,Q2}(N_{f2}) \tag{9}$$

where N_{f1} and N_{f2} is number of cycles at failure for the specimens loaded at Q_1 and Q_2, respectively. Substitution of Eqn 8 into Eqn 9 gives

$$D_1\left\{\frac{P_{\max}^2}{P_{ref}^2}\left[1-Q_1^2\right]\right\}^{n1}\int_{Ni}^{Nf1}\left[G(a)\right]^{n1} dN = D_2\left\{\frac{P_{\max}^2}{P_{ref}^2}\left[1-Q_2^2\right]\right\}^{n2}\int_{Ni}^{Nf2}\left[G(a)\right]^{n2} dN \tag{10}$$

Assume that N_i is zero. That means a delamination will begin to grow from the first cycle or after only a few cycles which can be neglected. Considering that some damage might be present at the interface from specimen preparation this is not unrealistic. Also, assume that the energy release rate of the critical delamination $G(a)$ is independent of length and do not change mode ratio. For unnotched specimens with a delamination growing from the edge it has been found that the energy release rate increases for short delamination lengths and then remains constant for delamination lengths above 4 ply thicknesses.[12][13][14] The energy release rate mode ratio also remains constant. For a notched specimens a FEM solution found that the total energy release rate decreases slightly with increasing delamination length and the mode ratio was fairly constant.[15] This makes the assumption reasonable for unnotched and notched specimens. In the future it might be possible to obtain $G(a)$ numerically which would make it possible to numerically integrate the delamination growth. The constants D are given from[11]

$$D = \frac{\left(\frac{da}{dN}\right)_{th}}{\left(\Delta G_{r,th}\right)^n} \tag{11}$$

where $\left(\frac{da}{dN}\right)_{th}$ is the threshold delamination growth rate when da/dN versus ΔG_r makes a sharp corner, $\Delta G_{r,th}$ is the threshold change in energy release rate at this point. Both $\left(\frac{da}{dN}\right)_{th}$ and $\Delta G_{r,th}$ are independent of Q-value and mixed mode ratio and as a result

$$\frac{D_1}{D_2} = \left[\Delta G_{r,th}\right]^{n2-n1} \tag{12}$$

Equation 10 now becomes

$$\left[\Delta G_{r,th}\right]^{n2-n1}\left\{\frac{P_{\max}^2}{P_{ref}^2}\left[1-Q_1^2\right]\left[G(a)\right]\right\}^{n1} N_{f1} = \left\{\frac{P_{\max}^2}{P_{ref}^2}\left[1-Q_2^2\right]\left[G(a)\right]\right\}^{n2} N_{f2} \tag{13}$$

It has been observed for unnotched specimens that a fatigue threshold exists.[9] It is reasonable to assume that the fatigue threshold is related to $\Delta G_{r,th}$. Therefore, it is possible to write

$$\Delta G_{r,th} = G(a)\frac{P_{th,1}^2}{P_{ref}^2}\left[1 - Q_1^2\right] \tag{14}$$

where $P_{th,1}$ is the maximum load magnitude at threshold for the specimen loaded at Q_1. Substitution into Eqn 13 gives

$$N_{f,2} = \left[\frac{P_{th,1}}{P_{\max}}\right]^{2(n2-n1)}\left[\frac{1-Q_1^2}{1-Q_2^2}\right]^{n2} N_{f1} \tag{15}$$

This equation makes it possible to calculate the fatigue life at the load ration Q_2 and maximum load magnitude P_{max} if the constant n in Paris law is known at the two load ratios.

Model for tension-compression fatigue

Now, it is possible to estimate the fatigue life for tension-tension and compression-compression fatigue. But, if Q<0, tension-compression fatigue, the situation is more complicated. Assume that the delamination crack is open during the compressive part of the load cycle. This means that G_I>0. During the tension part of the load cycle the crack will be closed which means that G_I=0. Therefore, the energy release rate of the delamination crack during compression $G_c(a)$ will not be equal the corresponding $G_t(a)$ for the tension part of the load cycle. The objective of the following derivation is to find an expression similar to Eqn 15 for the fatigue life when Q<0 which should be based on fatigue lives for Q>0. Following the previous derivation Eqn 5 need to be changed to

$$\Delta G_r = G_{Pt}(a) + G_{Pc}(a) = G_t(a)\frac{P_t^2}{P_{ref}^2} + G_c(a)\frac{P_c^2}{P_{ref}^2} \tag{16}$$

where subscript ”c” and ”t” stands for compressive and tensile loading. Assume that the specimen will fail in compression with a delamination length $a_c(N_{f3})$ after N_{f3} cycles. The specimens fails at the same delamination length as a specimen loaded in compression-compression, Q_1≥0, with the peak load magnitude P_c which fails after N_{f1} cycles. This gives

$$a_c(N_{f3}) = a_{c,Q1}(N_{f1}) \tag{17}$$

If Eqn 16 is substituted into Eqn 3 and part of Eqn 10 is used then Eqn 17 can be written as

$$D_3\left[\left[\frac{P_t^2}{P_{ref}^2}\right]^{n3}\int_{Ni}^{Nf3}\left[G_t(a)\right]^{n3} dN + \left[\frac{P_c^2}{P_{ref}^2}\right]^{n3}\int_{Ni}^{Nf3}\left[G_c(a)\right]^{n3} dN\right] = D_1\left\{\frac{P_c^2}{P_{ref}^2}\left[1-Q_1^2\right]\right\}^{n1}\int_{Ni}^{Nf1}\left[G_c(a)\right]^{n1} dN \tag{18}$$

where n_3 and D_3 are constants in Paris law for the Q<0 case studied. Now, assume that N_i is zero and that both $G_c(a)$ and $G_t(a)$ are independent of delamination length and that the energy release rate ratio is independent of delamination length. Substitution of Eqns 11, 12 and 14 into 18 gives

$$N_{f3} = \left[\frac{P_c^2}{P_{th,1}^2}\right]^{n1-n3} \frac{\left[1-Q_1^2\right]^3 N_{f1}}{\left\{\frac{P_t^2}{P_c^2}\frac{G_t(a)}{G_c(a)}\right\}^{n3} + 1} \tag{19}$$

where $P_{th,1}$ is the threshold load magnitude at load ratio Q_1. In order to continue it is necessary to find an expression for $\frac{G_t(a)}{G_c(a)}$. Consider a specimen loaded in compression-compression at load ratio Q_4 and one loaded in tension-tension at load ratio Q_5 at the same load magnitude $P_{max,4}$. Following the technique used above it is assumed that failure in the specimens occur at the same delamination length. This might not be correct in all situations since $G_c(a)$ is not equal to $G_t(a)$ and local buckling might occur in compression. Equations 9 and 10 now becomes

$$D_4\left\{\frac{P_{\max,4}^2}{P_{ref}^2}\left[1-Q_4^2\right]\right\}^{n4}\int_{Ni}^{Nf4}\left[G_c(a)\right]^{n4} dN = D_5\left\{\frac{P_{\max,4}^2}{P_{ref}^2}\left[1-Q_5^2\right]\right\}^{n5}\int_{Ni}^{Nf5}\left[G_t(a)\right]^{n5} dN \tag{20}$$

Substitution of Eqns 12 and 14 into Eqn 20 gives

$$\left[\frac{G_t(a)}{G_c(a)}\right] = \left[\frac{P_{th,4}^2}{P_{\max,4}^2}\right]^{1-n4/n5}\left[\frac{1-Q_4^2}{1-Q_5^2}\right]\left[\frac{N_{f4}}{N_{f5}}\right]^{1/n5} \tag{21}$$

Substitution into Eqn 19 gives

$$N_{f3} = \left[\frac{P_c^2}{P_{th,1}^2}\right]^{n1-n3} \frac{\left[1-Q_1^2\right]^3 N_{f1}}{\left\{\frac{P_t^2}{P_c^2}\left[\frac{P_{th,4}^2}{P_{\max,4}^2}\right]^{1-n4/n5}\left[\frac{1-Q_4^2}{1-Q_5^2}\right]\left[\frac{N_{f4}}{N_{f5}}\right]^{1/n5}\right\}^{n3} + 1} \tag{22}$$

An alternative to using Eqn 22 would be to run one test at $Q<0$ and then use Eqn 19 to calculate $\frac{G_t(a)}{G_c(a)}$. It would then be possible to calculate N_{f3} for other negative Q-values. If failure occurs in tension instead of in compression, which was assumed above, subscript ”c” should be changed to ”t” and vice versa in equations above.

Comparison with experimental results

To compare the model with experimental results data from the work by Gathercole et al.[16] are used. The data are for unnotched T800/5245 composites tested in fatigue at different R-values, see Fig. 1. Before it is possible to use the model it is necessary to find the n-value in Paris law for the different R-values. This can be done with the delamination growth model by Schön[11] which require that the mix mode ratio and the parameters $\left(\frac{da}{dN}\right)_{th}$, $\left(\frac{da}{dN}\right)_c$, $\Delta G_{r,th}$, and $G_{max,c}$ are known together with a quasi-static failure criteria for mixed mode delamination growth. The parameter $\Delta G_{r,th}$ is the change in energy release rate at which the $\left(\frac{da}{dN}\right)$ versus

ΔG_r curve turns down. In this case this value is assumed to be 80 J/m^2. The parameter $\left(\frac{da}{dN}\right)_{th}$ is the delamination growth rate at which $\Delta G_{r,th}$ is reached. In this case this is assumed to be at 1E-6 mm/cycle. The parameter $G_{max,c}$ is the energy release rate at static failure. In this case a linear failure criteria for the different mixed-mode ratios was used which some experiments would suggest.[17] The parameter $\left(\frac{da}{dN}\right)_c$ is the delamination growth rate at which static failure is reached. In this case it is assumed to occur at 0.1 mm/cycle. It is also necessary to know the mixed mode ratio for the critical delamination in the specimens during tensile and compressive loading. One way to estimate this is by considering the static fracture. In tensile loading fracture occurred at 1.67 GPa and in compressive loading at -0.88 GPa. It is assumed that fracture in both cases will be due to delamination growth. It is reasonable to assume that during tensile loading the delaminations will be closed, G_I=0, since the tensile strength is higher than the compressive one. It is assumed that quasi-static delamination growth will occur when G_{II}+G_{III}=750 J/m^2. During compressive loading there will be a mode I fracture component together with mode II and III. It is assumed that the mode II and III fracture components will be equal to that during tensile loading at a given load magnitude. A linear mixed mode failure criteria is used together with G_{IC}=215 J/m^2.[17][18] This makes it possible to calculate the mode I to mode II+III ratio as $\frac{G_I}{G_{II}+G_{III}} = 1.20$. It is now possible to calculate the *n*-value in Paris law for the different *R*-values used, see Table 1. For *R*=-0.3 and -0.6 fracture is assumed to occur in tension when G_I=0 and for *R*=-1.0 and -1.5 fracture is assumed to occur in compression when all three fracture modes are present. That is the reason for the *n*-value to be larger for *R*=-1.0 and -1.5 than for *R*=-0.3 and -0.6. These values are similar to those in the literature.[19]

Table 1: Calculated *n*-values in Paris law.

R-value	+0.5	+0.1	-0.3	-0.6	-1.0	-1.5	+10
n-value	5.90	5.17	4.76	4.08	5.43	5.97	6.64

It is now possible to compare the theoretical predictions with the experimental results. Using Eqn. 15 and the fatigue results for R=0.1 (Q=0.1) at 1.3 and 1.4 GPa the fatigue lives are predicted for R=0.5 (Q=0.5). As $P_{th,1}$ was 0.55 GPa used which was calculated from the static tensile strength and G_{II}+G_{III}=750 J/m^2 together with $\Delta G_{r,th}$ =80 J/m^2. The results can be seen in Fig. 1. For the case of *Q*<0 two methods are possible for determining $\frac{G_t(a)}{G_c(a)}$. Either it can be determined from tension-tension and compression-compression fatigue results at the same P_{max}-value or from *Q*<0 and *Q*>0 fatigue data at the same P_{max}-value and failure mode, tension or compression failure. Since there is such a large difference between tensile and compressive strength it is not possible to use the first method. There are no fatigue data at the same P_{max}-value. The second method is then used. Based on the fatigue curves it is reasonable to assume

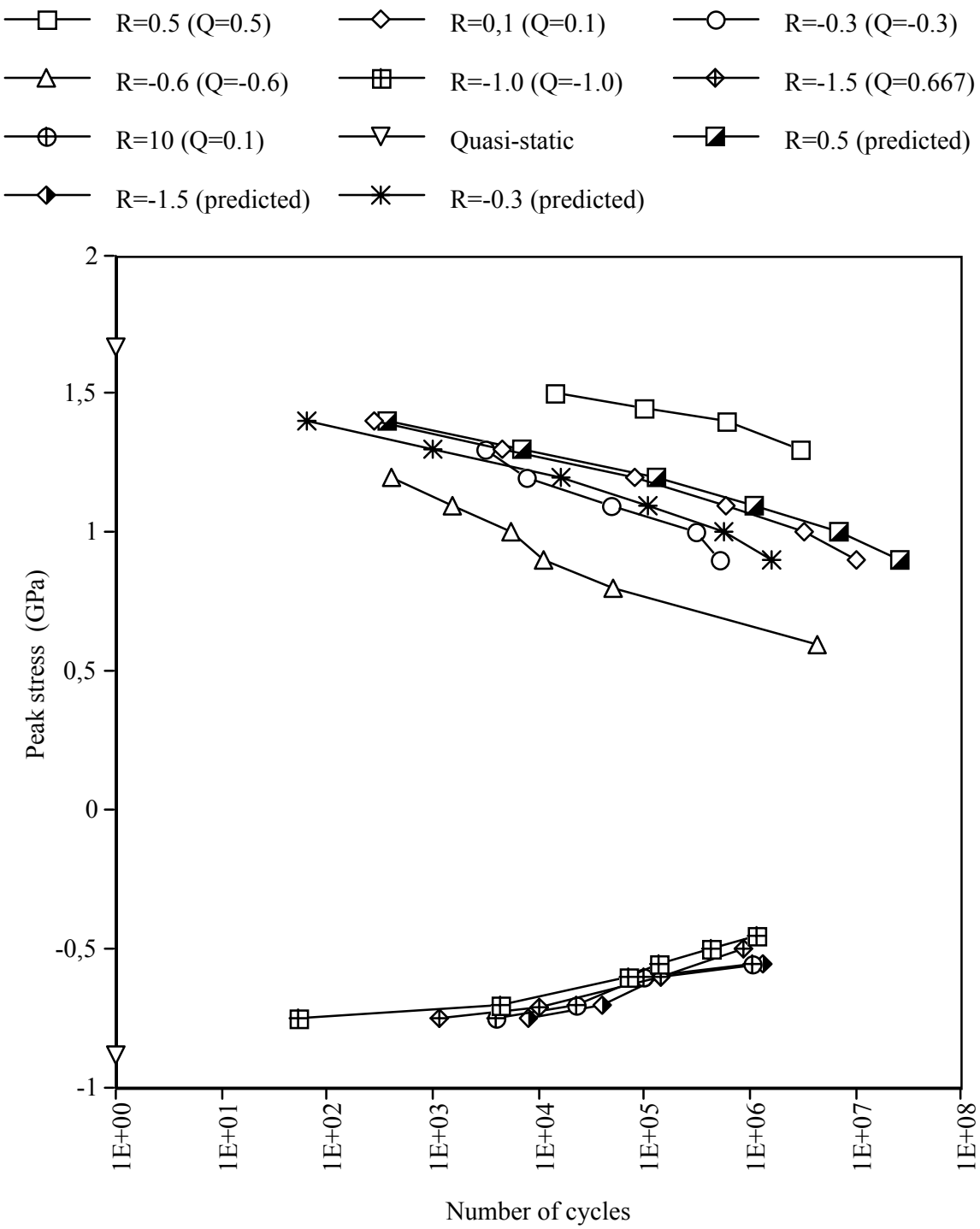

Fig. 1 Experimental [16] and predicted fatigue results

that for R=-1 and -1.5 the failure is compressive. The parameter $\frac{G_t(a)}{G_c(a)}$ was determined from the R=-1 and R=10 curves at P_{max}=0.7 GPa and Eqn. 19 with a $P_{th,1}$ of -0.368 which was determined from quasi-static compressive strength. It was found that $\frac{G_t(a)}{G_c(a)}$=1.77. The fatigue life for R=-1.5 (Q=0.67) was then calculated using Eqn. 19 and the results can be seen in Fig. 1. For the R-values -0.3 and -0.6 it is reasonable to assume that the failure mode is tensile.

After the subscripts ”*c*” and ”*t*” in Eqns 19 and 22 had been shifted the parameter $\frac{G_c(a)}{G_t(a)}$ was determined from the R=-0.6 and R=0.1 curves at P_t=1.1 GPa. It was found that $\frac{G_c(a)}{G_t(a)}$=17.2 which is quite different to what was found for *R*=-1. The predicted fatigue life for *R*=-0.3 can be seen in Fig. 1. There are several possible sources of errors for the comparison with experiments. One is the parameters used for calculating the *n*-value in Paris law for the different *R*-values. Since the *n*-values are large a small error in *n*-value or in the other parameters used to calculate the fatigue life will introduce a large error in predicted fatigue life. This is a basic problem of delamination growth in composites. As tougher matrices are developed the *n*-value will decrease and this problem might be less critical.

CONCLUSIONS

A model for predicting the fatigue life of composites at different *R*-values has been developed. It can be used for tension-tension, compression-compression, and tension-compression fatigue. In a comparison with experimental results the model provides good predictions.

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