DESIGNING WITH ANISOTROPY.

PART 2 :

LAMINATES WITHOUT MEMBRANE-FLEXURE COUPLING.

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SUMMARY : We deal in this paper with a very classical problem in designing laminates : the suppression of coupling between stretching and bending. We formulate a general method to find a large number of solutions, which generalises some of the methods known up to day. The governing equations are found with the help of the polar representation method for plane tensors. A surprising result of the analysis performed is the existence of a very large number of solutions, and the fact that the well known symmetric solutions are in reality a very particular case.

KEYWORDS : laminates, uncoupling, polar representation, plane tensors, composites design, inverse problems.

INTRODUCTION.

One of the problems in using anisotropic laminates is the well known phenomenon of the coupling of bending and stretching effects : an arbitrary laminate has the property that membrane stresses produce also flexural strains, and *vice-versa*. This is generally an undesired effect for composite structures. Moreover, this coupling exists also for thermoelastic effects, which is also of great importance, because laminates are often built up with prepreg layers pressed under high temperature. So, if a coupled laminate is being produced, it will be naturally curved after the fabrication phase.

All the designers of composite structures know that a sufficient condition to have uncoupled laminates is the material and geometrical symmetry of the stacking sequence. Often, and not correctly, this is believed as a compulsory condition to have no coupling effects. Indeed, this is wrong, as already shown in literature, for instance by Caprino & Crivelli-Visconti, [1], Verchery & Vong, [2], Kandil & Verchery, [3]. These authors gave several counter-examples of non symmetric uncoupled laminates.

Questions remain however:

i. the question from theoreticians is : are there a necessary and sufficient condition or more general sufficient conditions to have laminates with no coupling effects ?
ii. from the point of view of the designer, how is it possible to find these laminates, or at least enough of them to satisfy practical applications ?
iii. from the point of view of the sceptical, are these non-symmetrical laminates really uncoupled for their elastic and thermal expansion behaviour ?

The first, theoretical question of a necessary condition cannot receive presently a general solution. It should be emphasised that this is an inverse problem of the classical laminated plate theory, and, just like other inverse problems, no general methods are currently available to attack it.

In the present study, we were able to find a large class of solutions for laminates made of identical laminas with different orientations. This was made possible by using the polar representation of plane tensors as developed by Verchery and co-authors (see Part 1). It is an application to tensors of the complex methods, and has in tensor algebra the same efficiency that these well known methods have in ordinary problems.

A very interesting property of our class of solutions is that the number of solutions is large even for a rather small number of plies, and increases rapidly with the number of laminas. As this number is much larger than the number of symmetrical stacking sequences, our solutions enlarge very much the perspective for the designers, presently restricted by the symmetrical stacking rule. Indeed, we show how the symmetric solution can be considered as a very particular one, the rule being the non-symmetry. This gives a satisfactory answer to the second question, for the designer.

We have manufactured some of such laminates, by hot-pressing. After processing, the specimens were perfectly plane. When tested, they exhibited an uncoupled behaviour, as predicted, which gives an excellent experimental assessment for our research. Is that a satisfactory answer to sceptical ?

THE POLAR REPRESENTATION TECHNIQUE

The general stress-strain relationship for a single ply may be put under the classical form, [4],

$$\boldsymbol{\sigma} = \mathbf{Q}\,\boldsymbol{\varepsilon}\,; \tag{1}$$

it is well known that for the case of a plane stress state, tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ can be represented by "vectors" with three components, and the fourth order tensor **Q** by a 3x3 matrix, which is symmetrical.

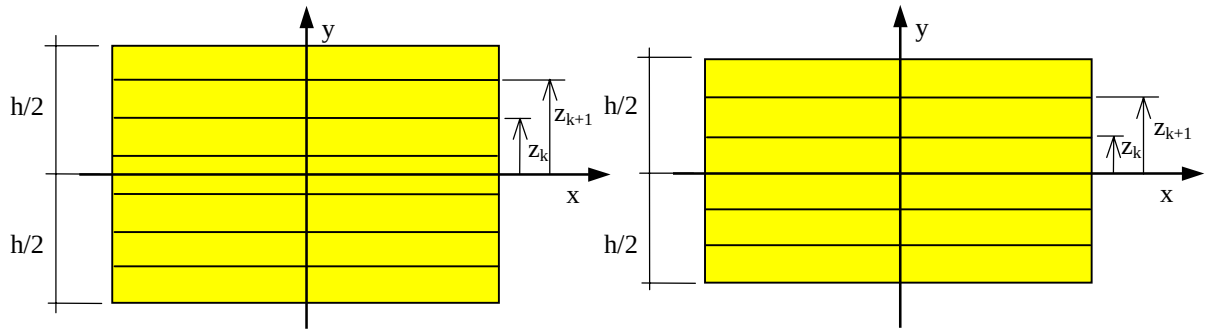
The Classical Laminated Plate Theory (CLPT), based upon the well known hypotheses of Kirchhoff-Love, provides the behaviour for a plate obtained by superposing and bonding together some plies:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^{o} \\ \boldsymbol{\chi} \end{Bmatrix}, \tag{2}$$

where **N** is the vector of membrane forces and **M** that of bending moments, while $\boldsymbol{\varepsilon}^{\circ}$ and $\boldsymbol{\chi}$ are respectively the vectors of middle plane strains and curvatures. **A**, **B** and **D** are symmetric double tensors, and as apparent from equation (2), the tensor **A** describes the membrane behaviour of the plate, while **D** is the flexural one ; the role played by tensor **B** is

quite different, and it does not exist in homogeneous plates : it takes into account for the coupling between membrane and bending. It is evident by equation (2) that the presence of membrane deformations, that is, of stretching and shearing of the middle plane, produces also, and by itself, thanks to **B**, the existence of bending moments ; in the same manner, the fact that the middle plane has some curvature produces the existence of in-plane forces. This is the phenomenon of stretching-bending coupling. Generally, this effect is quite undesired by composite designers.
According to the general scheme of Fig. 1, the tensors components of **A**, **B** and **D** are given, for a N-plies laminate (N= 2p+1 if odd and N= 2p if even), by :

$$\mathbf{A} = \sum_{k=-p}^{p}\left(z_{k+1} - z_k\right)\mathbf{Q}_k;\ \mathbf{B} = \frac{1}{2}\sum_{k=-p}^{p}\left(z_{k+1}^2 - z_k^2\right)\mathbf{Q}_k;\ \mathbf{D} = \frac{1}{3}\sum_{k=-p}^{p}\left(z_{k+1}^3 - z_k^3\right)\mathbf{Q}_k. \tag{3}$$



a) *b)*

Figure 1: General sketch ; a) odd number of plies; b) even number of plies.

In calculating mechanical properties of a laminate composed by anisotropic layers, the tensors in equations (1) and (2) must be expressed in a rotated frame. In Cartesian co-ordinates, their components vary according to a fourth-power law of the director cosines of the angles formed by the new frame with the old one: in other words, this kind of operation gives rise to algebraically complicated expressions.
The polar representation technique for plane tensors makes this kind of manipulation much more easy to do. Verchery and his co-workers (see Part 1 and [2], [3], [6], [7], [8], [9] and [10]) have developed and used this polar representation, extending results such as given by Tsai & Pagano [5] who had already transformed the classical rules for rotating components. For what concern a fourth order tensor **L**, its six independent components are substituted by other six quantities, T_0, T_1, R_0, R_1, Φ_0 and Φ_1 related to the Cartesian ones by the following complex expressions:

$$\begin{aligned}
8T_0 &= L_{1111} + L_{2222} - 2L_{1122} + 4L_{1212};\\
8T_1 &= L_{1111} + L_{2222} + 2L_{1122};\\
8R_0\, e^{4i\Phi_0} &= L_{1111} + L_{2222} - 2L_{1122} - 4L_{1212} + 4i\left(L_{1112} - L_{2212}\right);\\
8R_1\, e^{2i\Phi_1} &= L_{1111} - L_{2222} + 2i\left(L_{1112} + L_{2212}\right).
\end{aligned} \tag{4}$$

These four equations define the so-called *polar components* of the tensor **L**. The converses of equations (4) are :

$$
\begin{aligned}
L_{1111} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 + 4R_1 \cos 2\Phi_1;\\
L_{1122} &= -T_0 + 2T_1 - R_0 \cos 4\Phi_0;\\
L_{2222} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 - 4R_1 \cos 2\Phi_1;\\
L_{1212} &= T_0 - R_0 \cos 4\Phi_0;\\
L_{1112} &= R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1;\\
L_{2212} &= -R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1.
\end{aligned}
\tag{5}
$$

In a rotation θ of the material, the four quantities T_0, T_1, R_0 and R_1 are invariant, as long as the difference Φ_0-Φ_1, while the angles Φ_0 and Φ_1 are changed into $\Phi_0+\theta$ and $\Phi_1+\theta$.
Moreover, it can be easily shown that for an isotropic material, it must be $R_0= R_1= 0$, for an orthotropic one, in any axes, $\Phi_0= \Phi_1+k\,\pi/4$ and for square symmetry, $R_1= 0$.
Evidently, equation (2) may be rewritten as a function of the polar components: it is immediate to see that equations (3) are valid also for the polar components, and in the most general case, for tensor **B**, the results are the following (analogous expressions can be obtained for tensors **A** and **D**, see [10]):

$$
\begin{aligned}
\hat{T}_0 &= \frac{1}{2}\sum_{k=-p}^{p} T_{0_k}\left(z_{k+1}^2 - z_k^2\right);\\
\hat{T}_1 &= \frac{1}{2}\sum_{k=-p}^{p} T_{1_k}\left(z_{k+1}^2 - z_k^2\right);\\
\hat{R}_0\, e^{4i\hat{\Phi}_0} &= \frac{1}{2}\sum_{k=-p}^{p} R_{0_k} e^{4i\left(\Phi_{0k}+\delta_k\right)}\left(z_{k+1}^2 - z_k^2\right);\\
\hat{R}_1\, e^{2i\hat{\Phi}_1} &= \frac{1}{2}\sum_{k=-p}^{p} R_{1_k} e^{2i\left(\Phi_{1k}+\delta_k\right)}\left(z_{k+1}^2 - z_k^2\right);
\end{aligned}
\tag{6}
$$

Herein the angles Φ_{0_k} and Φ_{1_k} are the angles defined for the ply k by its own reference frame, and δ_k is the angle that such frame forms with the global reference frame of the laminate. By equations (5), the components of tensors **A**, **B** and **D** for the laminates may be obtained as functions of the above final components T_0, etc. The great advantage resides in the fact that if the frame of reference for the laminate is rotated of an angle θ, it is sufficient to calculate the above quantities for the angles $\Phi_{0_k} + \delta_k - \theta$ and $\Phi_{1_k} + \delta_k - \theta$ to have the tensors in the rotated frame.

UNCOUPLING CONDITIONS

A laminate is uncoupled when the presence of stretching strains does not provide bending moments, and the middle plane curvatures do not give rise to in plane forces. Mathematically speaking, a quick glance at equation (2) tells us that a laminate is uncoupled if and only if the tensor **B** is the null tensor :

$$\mathbf{B} = \mathbf{0}. \tag{7}$$

Let us consider laminates composed by identical plies; this means that each ply is composed of the same material, in the same original frame, and with the same thickness. In such a case, equations (6) are considerably simplified, and it can be easily seen that :

$$\hat{T}_0 = \hat{T}_1 = 0;$$

$$\hat{R}_0\, e^{4i\hat{\Phi}_0} = \frac{1}{2} R_0 e^{4i\Phi_0} \sum_{k=-p}^{p} e^{4i\delta_k}\left(z_{k+1}^2 - z_k^2\right); \tag{8}$$

$$\hat{R}_1\, e^{2i\hat{\Phi}_1} = \frac{1}{2} R_1 e^{2i\Phi_1} \sum_{k=-p}^{p} e^{2i\delta_k}\left(z_{k+1}^2 - z_k^2\right).$$

Hence, the uncoupling conditions become simply :

$$\begin{aligned} \hat{R}_0\, e^{4i\hat{\Phi}_0} = 0 \quad &\Rightarrow \quad \sum_{k=-p}^{p} e^{4i\delta_k}\left(z_{k+1}^2 - z_k^2\right) = 0; \\ \hat{R}_1\, e^{2i\hat{\Phi}_1} = 0 \quad &\Rightarrow \quad \sum_{k=-p}^{p} e^{2i\delta_k}\left(z_{k+1}^2 - z_k^2\right) = 0. \end{aligned} \tag{9}$$

These last can be rewritten as follows :

$$\begin{aligned} B\left(e^{4i\delta_k}\right) = 0 \quad &\Rightarrow \quad \sum_{k=k^*}^{p} b_k\left(e^{4i\delta_k} - e^{4i\delta_{-k}}\right) = 0, \\ B\left(e^{2i\delta_k}\right) = 0 \quad &\Rightarrow \quad \sum_{k=k^*}^{p} b_k\left(e^{2i\delta_k} - e^{2i\delta_{-k}}\right) = 0, \end{aligned} \tag{10}$$

where k*= 0 if N is odd, and k*= 1 if it is even. The coefficients b_k are given by

$$b_k = k \ \text{ for N odd}, \ b_k = 2k - 1 \ \text{ for N even}. \tag{11}$$

These are the equations to solve in order to obtain uncoupled laminates composed of identical plies. As it can be noticed, such a property does not depend upon the mechanical properties of the single ply, as it was to be expected, since the plies are identical.
To identify a solution, it is necessary to fix the orientation of one ply, in order to prevent from rigid rotations. In the following, we will consider that for the first ply from the bottom it is $\alpha_k = 0$. Obviously, the trivial solution, that is the solution which gives the same orientation for each ply in a laminate, exists for each N. However, the complete solution of system (10) for a given N is not available. Nevertheless, a particular class of solutions is rather easy to be found, once and for all, for each laminate : we will call such class the *set of quasi-trivial solutions* of the problem of uncoupling.

QUASI-TRIVIAL UNCOUPLED LAMINATES

The coefficients b_k are anti-symmetric about the middle plane. This fact suggests immediately the well-known rule of symmetrical stacking sequences for uncoupled laminates : in that case, equation (10) will be automatically satisfied, for any orientation of a symmetric pair of layers. Indeed, this is a sufficient condition, but not a necessary one, and much more solutions can be obtained using a similar approach : we have an uncoupled laminate each time that the sum of the coefficients of the layers having the same orientation will be zero, and such a rule does not apply exclusively to symmetric stacking sequences, making possible to find non-symmetric laminates that satisfy this condition.
So a very general rule to find uncoupled laminate is well established, and the very important feature is that it is not necessary at all to solve equations (10) to find solutions that satisfy this

rule. To recognise this circumstance, we will call *quasi-trivial* these solutions, and saturated group a group of layers having the same orientation. It must be stressed out that for this kind of solutions, the orientation of each saturated group is not defined ; so, a stacking sequence formed by g saturated groups is in reality the collection of g-1 infinities of different uncoupled laminates from a mechanical point of view.

It is quickly recognised that the maximum number of different saturated groups in a N-ply laminate is E[(N+1)/2], where E denotes the integer part of the quotient. Moreover, it is worth noticing that, due to the identical structure of equations in system (10), only one group of coefficients b_k must be considered to find quasi-trivial solutions. Another important feature is that each saturated group of a g-groups solution is a subgroup of another saturated group of a (g-1)-groups solution.

Basing upon the previous properties, we have built up an algorithm able to find all quasi-trivial solutions for a given laminate. In the following, each different saturated group in a laminate will be labelled by a number, beginning from zero ; for instance, for a quasi-trivial solution with four different orientations, each group is referenced by a number from 0 to 3. Each sequence is ordered, in the sense that, proceeding from the bottom of the plate, the first orientation is always labelled by 0, then the second by 1, the third by 2 and so on.

We have also introduced the concept of independent or pure solution : let us consider, for instance, a solution of the kind [0 1 2 3 4 3 2 1 0] ; it is clear that solutions like [0 1 2 3 3 3 2 1 0], or [1 1 1 3 4 3 1 1 1] are a particular case of the first one, obtained by attributing to different saturated groups the same orientation. So we will call pure or independent solution a solution of the first type, which is characterised by the fact that it gives no rise to other solutions with a larger number of saturated groups.

Thanks to the algorithm created, we have analysed up to 18-layers laminates, and the results are summarised in Table 1, which details, for each number of layers, the number of pure quasi-

Table 1: Quasi-trivial uncoupled pure solutions.

N. of plies	7	8	9	10	11	12	13	14	15	16	17	18
N. of sol.	2	2	4	5	11	15	41	57	174	275	1033	1639

trivial solutions. The table starts with a seven plies laminate, because from three to six layers only one symmetric independent solution is possible.

Some surprising results can be noticed : first of all, the very large number of solutions, rapidly increasing with the number of layers. If independent solutions are considered, as in Table 1, only one symmetric pure solution exists for each number of plies, that is, the sequence where the orientation change at each layer. In fact, all the others symmetric solutions are a particular case of this one. Under this point of view, hence, the symmetric solution is not the rule, but the exception : it is apparent from Table 1, in fact, that the number of pure solutions rapidly increases, and in each case only one of them is symmetric. The results are shown, as histograms, in Fig. 2.

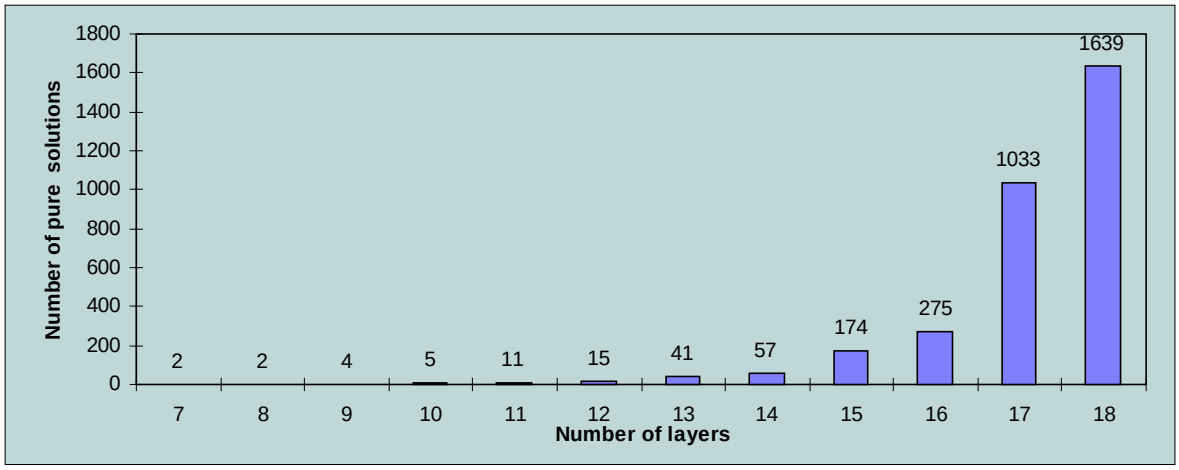


Figure 2: Number of independent solutions.

THERMALLY UNCOUPLED LAMINATES

The previous analysis concerns only mechanical effects due to application of external or body forces, but not thermoelastic effects. It can be inferred from general symmetry consideration that for such laminates thermal coupling vanishes when mechanical coupling vanishes. However, for better understanding, we present hereafter a direct proof. To do this, we will consider the presence of a thermal field acting upon the laminate, linearly varying through the thickness. This is the real situation of a laminate constituted by plies of the same material, subjected to a difference of temperature between the upper and the lower surface, once the thermal flux is stationary. Two things must be remarked : the first one is the importance of having plies with the same physical properties. This hypothesis has already been done in the preceding part, but now it assumes a physical importance, whilst before it was only assumed for simplicity, the theory being anyway perfectly valid also for plies of different materials. The second one is that the assumed hypothesis are those used in the manufacturing process of laminates, so they assume a fundamental importance in practice too.
Let us assume that the temperature τ varies across the thickness according to :

$$\tau(z) = \tau_o + \frac{2\Delta\tau}{h} z \,, \tag{12}$$

where $\tau(z)$ is the value of the difference of temperature between the current state and the reference one, for the level z ; τ_o is the value corresponding to middle plane, z = 0, and $2\Delta\tau$ is the difference of temperature between the two faces. Still remaining in the framework of plane stress theory, the strains corresponding to a free thermal expansion are given by :

$$\boldsymbol{\varepsilon}^{\tau}(z) = \boldsymbol{\alpha}\left(\tau_o + \frac{2\Delta\tau}{h} z\right), \tag{13}$$

where $\boldsymbol{\alpha}$ is the tensor of thermal expansion coefficients of the layer. If $\boldsymbol{\varepsilon}$ is the total strain tensor, the stresses are given by the equation :

$$\boldsymbol{\sigma} = \mathbf{Q}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\tau}\right), \tag{14}$$

and as already done we can obtain force and moment resultants by integrating through the thickness. The generalised expression of equation (2) is so obtained in the following form :

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^{o} \\ \boldsymbol{\chi} \end{Bmatrix} - \tau_o \begin{Bmatrix} \mathbf{R} \\ \mathbf{S} \end{Bmatrix} - \frac{2\Delta\tau}{h} \begin{Bmatrix} \mathbf{S} \\ \mathbf{V} \end{Bmatrix}, \tag{15}$$

where **R**, **S** and **V** are three second order tensors describing thermal stresses for the laminate in stretching, coupling and bending respectively. They are given by :

$$\mathbf{R} = \sum_{k=-p}^{p} (z_{k+1} - z_k)\boldsymbol{\beta}_k;\ \mathbf{S} = \frac{1}{2}\sum_{k=-p}^{p} \left(z_{k+1}^2 - z_k^2\right)\boldsymbol{\beta}_k;\ \mathbf{V} = \frac{1}{3}\sum_{k=-p}^{p} \left(z_{k+1}^3 - z_k^3\right)\boldsymbol{\beta}_k. \tag{16}$$

In equation (16), we have indicated by $\boldsymbol{\beta}_k$ the quantity

$$\boldsymbol{\beta} = \mathbf{Q}\,\boldsymbol{\alpha} \tag{17}$$

for the kth ply. It is evidently the components of a plane second order tensor. It is apparent that a laminate is thermally uncoupled if and only if the tensor **S** is the null tensor. The analysis is performed using again the polar representation of plane tensors. In the case of a second order tensor, like $\boldsymbol{\beta}$, the polar components are the following ones (see Part 1 or [11]) :

$$T = \frac{L_{11} + L_{22}}{2};\quad 2\,Re^{2i\Phi} = L_{11} - L_{22} + 2iL_{12}. \tag{18}$$

The converses of equations (18) are

$$L_{11} = T + R\cos 2\Phi;\ \ L_{22} = T - R\cos 2\Phi;\ \ L_{12} = R\sin 2\Phi. \tag{19}$$

In a rotation θ, T and R are invariant, while the angle Φ is changed into $\Phi + \theta$. Naturally, by equations (19), the components of tensors **R**, **S** and **V** can be expressed as functions of T, R and Φ. Once this done, it is easy to write down the uncoupling conditions for the thermal characteristics in polar components, for laminates made of identical plies :

$$\hat{T} = 0,\quad \hat{R}\,e^{2i\hat{\Phi}} = 0 \quad \Rightarrow \sum_{k=-p}^{p} e^{2i\delta_k}\left(z_{k+1}^2 - z_k^2\right) = 0. \tag{20}$$

Equation (20) corresponds perfectly to equation (9_2) ; this means that a quasi-trivial solution for an uncoupled laminate is a solution also for what concerns the thermoelastic uncoupling. It could be extended easily to linear hygroscopic behaviour, in the same way.

NUMERICAL EXAMPLES

As numerical examples of what is shown above, we have considered laminates made by 7 plies, with a non-symmetric stacking sequence [0 1 1 2 0 0 1], in which layers with the same number have the same orientation. We have taken 0° for 0 and 2, 90° for 1. For the material, we have considered two different possibilities, a boron-epoxy and a glass-epoxy layer, with mechanical characteristics taken from [12], as summarised in Table 2. We have

performed also some experimental tests, which confirm the absolute absence of coupling ; the results of these tests are detailed in Part 3.

Table 2: Thermoelastic properties of the single layer (from [12]).

	E_1 [MPa]	E_2 [MPa]	ν_{12}	G_{12} [MPa]	α_1 [°C^{-1}]	α_2 [°C^{-1}]
Boron-Epoxy	204000	18500	0.23	5590	6.1 10^{-6}	30.3 10^{-6}
Glass-Epoxy	38600	8270	0.26	4140	8.6 10^{-6}	22.1 10^{-6}

It is then an easy task to calculate the components of stiffness, **Q**, and compliance, **S**, tensors in the material frame ; the results are shown in Table 3, along with the polar components of the same tensors ; for the polar components, the capital letters have been reserved to stiffness tensors.

Table 3: Tensorial components of the single ply.
a) stiffness tensor (in GPa).

	Q_{11}	Q_{12}	Q_{16}	Q_{22}	Q_{26}	Q_{66}	T_0	T_1	R_0	R_1	Φ_0	Φ_1
Boron-Epoxy	204.9	4.27	0	18.59	0	5.59	29.67	29.01	24.08	23.30	0°	0°
Glass-Epoxy	39.16	2.18	0	8.39	0	4.14	7.47	6.49	3.33	3.85	0°	0°

b) compliance tensor (in $Pa^{-1}\times 10^{-12}$).

	S_{11}	S_{12}	S_{16}	S_{22}	S_{26}	S_{66}	t_0	t_1	r_0	r_1	φ_0	φ_1
Boron-Epoxy	4.90	-1.13	0	54.05	0	178.9	30.01	7.09	14.71	6.14	45°	90°
Glass-Epoxy	25.90	-6.73	0	120.9	0	241.5	50.23	16.67	10.15	11.87	45°	90°

For the laminate, the tensor components are shown in Table 4 ; here, we show both the membrane and the bending behaviours. Remark that **Q** and **S** are the homogenised tensors, that is, the tensors of an equivalent material, and relate stresses $\boldsymbol{\sigma}$ to strains $\boldsymbol{\varepsilon}$; hence, they are not the proper stiffness and compliance tensors of the plate, which relates **N** and **M** to $\boldsymbol{\varepsilon}°$ and $\boldsymbol{\chi}$. It must be remarked the fact that components T_0 and T_1 do not change for the single ply and the laminate ; this is typical of laminates made of identical layers. In the cases presented, also R_0 does not change, but this is particular to the present case, where angles Φ_0 are always zero and the laminate is orthotropic. In the Tables below, the Cartesian components of the tensor have been determined at 0°, and naturally they change if the angle is changed ; on the contrary, all the polar components do not change by rotation, as long as the difference Φ_0-Φ_1 or φ_0-φ_1.

Table 4: Tensorial components of the whole laminate.
a) stiffness tensor of membrane behaviour (in GPa).

	Q_{11}	Q_{12}	Q_{16}	Q_{22}	Q_{26}	Q_{66}	T_0	T_1	R_0	R_1	Φ_0	Φ_1
Boron-Epoxy	125.1	4.27	0	98.47	0	5.59	29.67	29.01	24.08	3.33	0°	0°
Glass-Epoxy	25.98	2.18	0	21.58	0	4.14	7.47	6.49	3.33	0.55	0°	0°

b) compliance tensor of membrane behaviour (in $Pa^{-1}\times 10^{-12}$).

	S_{11}	S_{12}	S_{16}	S_{22}	S_{26}	S_{66}	t_0	t_1	r_0	r_1	φ_0	φ_1
Boron-Epoxy	8.00	-0.35	0	10.17	0	178.9	24.72	2.18	20.00	0.27	45°	90°
Glass-Epoxy	38.82	-3.92	0	46.73	0	241.5	41.8	9.71	18.51	0.98	45°	90°

c) stiffness tensor of bending behaviour (in GPa).

	Q_{11}	Q_{12}	Q_{16}	Q_{22}	Q_{26}	Q_{66}	T_0	T_1	R_0	R_1	Φ_0	Φ_1
Boron-Epoxy	112.1	4.27	0	111.5	0	5.59	29.67	29.01	24.08	0.07	0°	0°

Glass-Epoxy	23.82	2.18	0	23.73	0	4.14	7.47	6.49	3.33	0.01	0°	0°

d) compliance tensor of bending behaviour (in $Pa^{-1} \times 10^{-12}$).

	S_{11}	S_{12}	S_{16}	S_{22}	S_{26}	S_{66}	t_0	t_1	r_0	r_1	φ_0	φ_1
Boron-Epoxy	8.94	-0.34	0	8.98	0	178.9	24.68	2.15	20.03	0.08	45°	90°
Glass-Epoxy	42.33	-3.89	0	42.49	0	241.5	41.8	9.63	18.62	0.31	45°	90°

CONCLUSIONS

We have given a general rule to find a particular class of solution to the problem of finding an uncoupled laminate. By means of this rule, we have been able to find a large number of different stacking sequences, most of which are totally non-symmetric. Indeed, we have shown that each symmetric solution is a particular case of only one solution for each laminate, while the number of independent non-symmetric solutions is much larger.

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