A STANDARD CHARACTERISATION OF SATURATED AND UNSATURATED FLOW BEHAVIOURS IN POROUS MEDIA

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SUMMARY: A large number of investigations on permeability measurement in fibre reinforcements have been done to establish the behaviour of the macroscopic flow through a porous medium. However, these measurements often depend on large uncertainties related to material heterogeneity, and experimental artefacts that are not easy to control. In most cases, the different measurement techniques used lead to results that are very method-dependent. Thus, a new approach is suggested here to characterise the morphological granulometry of composite samples, in direct relation with a complete description of the macroscopic momentum equation of the fluid phase. The permeability of a porous medium depends on many parameters such as porosity, tortuosity, and the shape factor of a specific section. These parameters describe the geometrical configuration of the void space. The permeability is an intrinsic characteristic of the reinforcement material and does not change with the characteristics of the fluid, although several scales of drag flow should be considered. Our micro-structural approach integrates the local geometrical characteristics of the reinforcement into the global properties. The difference of results between saturated and unsaturated flow conditions is clarified through a description of the saturation degree in the porous medium. These analyses are also compared with structural and phenomenological approaches.

KEYWORDS: Porous Media, Permeability, Saturated-Unsaturated, Macro-micro, Deformable, Image Processing, Connexity, Morphology

INTRODUCTION

The LCM (Liquid Composite Moulding) consists in injecting a fluid (usually resin) into a fibre reinforcement that constitutes a porous media. Hence the study of resin flow can be generalised to flow in porous media. The need for efficient simulation softwares such as LCMFLOT developed in our laboratory is increasingly important for industrial applications. If injection time and pressure simulations are now accurate, considering non-isothermal problems in 3D, the next step is to be able to control and optimise the process. The aim is thus

to determine the fastest injection command that will lead to a part without defaults. The main one is the presence of air bubbles within the part.
The flow conditions that lead to air bubbles are not well understood yet. This is why a new approach has been undertaken to reach a better understanding of microscopic flows. This study concentrates on the key parameter for numerical simulation of the LCM process : permeability.

FLOW IN SATURATED AND UNSATURATED POROUS MEDIA

Flow through porous media are described by Darcy's Law [1]:

$$q = -\frac{K}{\mu}\nabla p \qquad (1)$$

Where q is the average velocity, μ the viscosity and ∇p the pressure gradient of the fluid. It is valid in permanent regime. Unsaturated flows are usually considered as quasi-stationary phenomena, a succession of stationary states. Considering a one-dimensional flow and the continuity equation

$$\nabla \cdot q = 0 \qquad (2)$$

the pressure space-distribution is linear. This is validated in saturated flows, with the flow rate Q increases, as seen on Fig. 1.

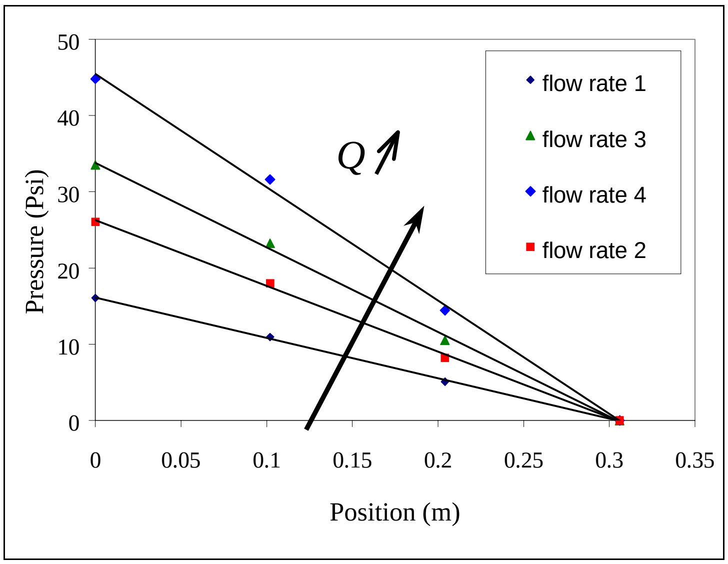

Fig. 1: The pressure profile in saturated porous medium

The unsaturated porous medium

The experiments were carried out on a rectangular mould used to measure unidirectional permeability. Additional pressure transducers were placed along the preform. In this case Darcy's Law can be assimilated to a purely resistive phenomenon and K to a hydraulic conductivity. In that case the preform was completely filled by the fluid. Similar measurements were made in unsaturated flows. As seen on Fig. 2, the pressure distribution is not linear anymore but parabolic.

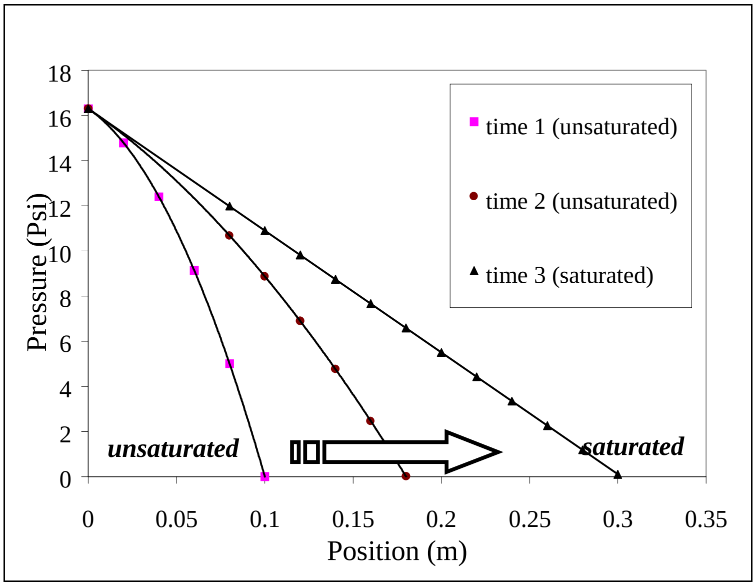

Fig. 2: The pressure profile in unsaturated porous medium

This can be easily explained modifying the continuity equation and Darcy's equation as follow:

$$q = -\frac{K(S)}{\mu}\nabla p \tag{3}$$

$$\nabla \cdot q = -\frac{\partial Tr\varepsilon(S)}{\partial t} \tag{4}$$

where $Tr\varepsilon$ is the trace of the skeleton strain tensor that depends on the saturation. In fact, the permeability now depends on the saturation degree S since the skeleton's deformation depends on the amount of fluid in the pores (wet and dry preform). This saturation phenomenon is also taken into account in the continuity equation. In unsaturated flows the porosity variation term associated to fibre rearrangement within the fabric is neglectible regarding the first term involving the saturation degree variation. Hence in flow through a non-deformable porous medium, the problem simply is

$$q = -\frac{K(S)}{\mu}\nabla p \tag{5}$$

$$\nabla \cdot q = -\phi\frac{\partial S}{\partial t} \tag{6}$$

Solving this problems leads then to a parabolic pressure distribution as seen on Fig. 2. Permeability dependence on saturation degree has been often discussed in the soil mechanics context, by Bear [2] and is also observed in the LCM process by Spaid [3]. Permeability values measured in unsaturated preform differ from those obtained on a saturated preform by a ratio that ranges between 0.4 for unidirectional reinforcement and 0.8 for fibre mats. This shows clearly the dependence of K on the saturation degree (Fig. 3).

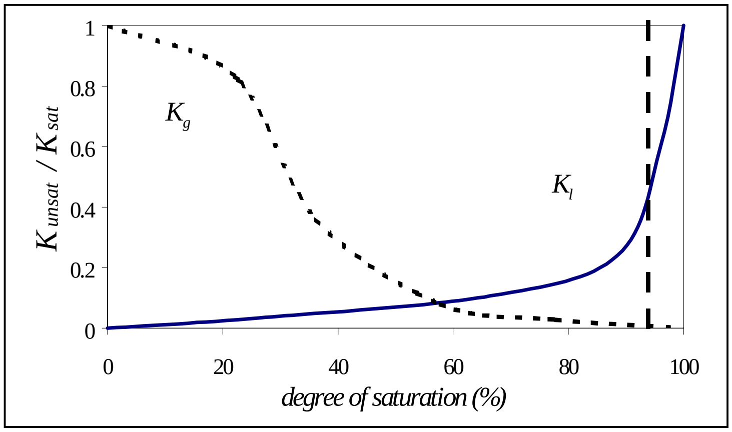

Fig. 3: The saturation degree development

Hence K that was assimilated to a constant hydraulic conductivity should, in unsaturated flows, be considered as a variable conductivity depending on the saturation degree. The medium is defined by three components with the fibre element (*f*), the liquid element (*l*) and the gaseous element (*g*) representing the air bubbles. So, we can identify the liquid conductivity (K_l) and the gaseous conductivity (K_g) in Fig.3 . The vertical line indicates that complete saturation of the porous medium is impossible. The flow through the porous medium is biphasic and hence defines a biphasic saturation. However, we suppose that the capillarity effect is neglected and we consider that the flow is a veritable monophasic behaviour with a saturation. The fact that $K_{sat}>K_{unsat}$ also explains qualitatively the convex shape of the pressure distribution in unsaturated flow in Fig. 2. Since the permeability is smaller at the flow front, the local pressure gradient will be higher.

The saturated porous medium

It is also interesting to study flows on a saturated preform. Darcy's Law considers only permanent regime. We concentrated our study on transient phases in saturated flows, where analysis have been developed in Bréard [4]. As seen on Fig. 4, following a flow rate change the pressure transient seems to be an exponential relaxation in unidirectional experiment.

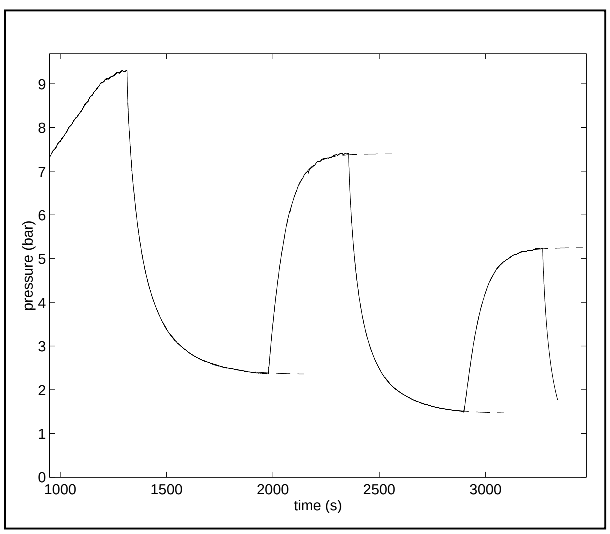

Fig. 4: The double-scaled relaxation in saturated porous medium

We showed in fact that it is a double time-scaled dynamics consisting of the sum of two exponential functions.

$$p(t) = P_\infty - k_1 e^{-t/\tau_1} - k_2 e^{-t/\tau_2} \tag{7}$$

which represent flow in macro (τ_1) and micropores (τ_2) respectively, the faster transition corresponding to flow in macro-pores. It was also noted that these transient are not symmetrical which we think is the consequence of micro deformations of the porous media. Such experiments allow to quantify flow in macro/micropores with a macroscopic measurements. This form of exponential transients can now be obtained with the following equations, neglecting now the term of saturation degree variation (saturated porous medium) in the continuity equation and considering only effects due to a deformable porous medium:

$$q = -\frac{K}{\mu}\nabla p \tag{8}$$

$$\nabla \cdot q = -\frac{\partial Tr\varepsilon}{\partial t} = -C_1(\tau_1, \tau_2)\frac{\partial p}{\partial t} - C_2(\tau_1 \cdot \tau_2)\frac{\partial^2 p}{\partial t^2} \tag{9}$$

Where the constant C_1 and C_2 depend on the micro deformation of the structure. The total stress of the medium, due to compaction, is assumed to be constant during experiments. Moreover, there are non-linear effects in those behaviours that are integrated into constants C_1 and C_2 in saturated regime. However these disappear when stationary state is reached in a saturated medium.

The intrinsic permeability

Permeability in Darcy's Law should, in terms of modelling be considered as the product of the intrinsic geometrical permeability K_{geo} and a relative permeability k_{rel}(S) depending on the saturation degree. It is, in fact, an intrinsic characteristic of the fibrous structure.

$$K = k_{rel}(S) K_{geo} \tag{10}$$

This highlights the point that, in fact, one should consider a triphasic medium: fibre, fluid, air. Indeed, the saturation degree is of primal interest for modelling, it can be a good prediction for default localisation since the less saturated the preform is, the more defaults are likely to happen. We can resume by the scheme on the Fig. 5:

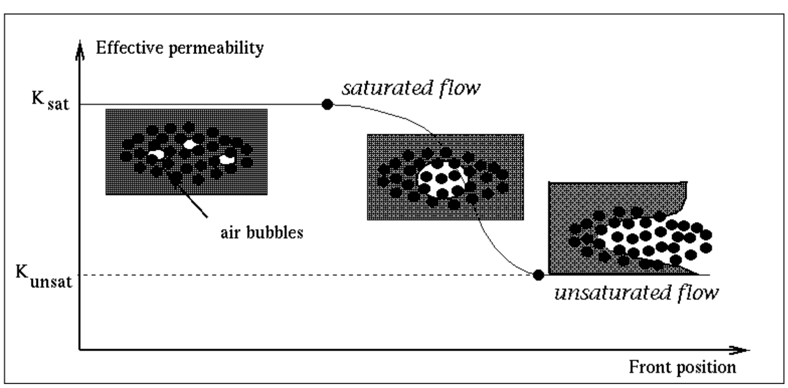

Fig. 5: The flow behaviour through a dry preform

Now, it is important to explain the macro-microscopic behaviour that it governs the flow. In fact, we will define the macroscopic characteristic with a microscopic analysis.

MORPHOLOGICAL CHARACTERIZATION

The phenomena to be studied obviously lay on the microscopic scale as well as on macroscopic phenomena. Thus a purely macroscopic approach seems not appropriate to identify mechanisms such as micro-bubble formation. This could be done considering local geometrical characteristics of reinforcements. Mathematical morphology and granulometry are a way to get the relevant information so that an Representative Elementary Volume (REV) can be identified.

The notion of REV comes from the fact that reinforcement are characterised by three scales. The macroscopic scale (cm) includes tows and spaces between tows which are called macropores. The microscopic scale (micrometer) takes into account fibres within tows and spaces between them (macropores). The intermediate scale is called mesoscopic.

These various scales will allow to identify, on the one hand the mechanism of air bubble generation and, on the other hand, to propose a characterisation of the conductivity (permeability) term with a morphological approach. This is the main originality of this work since the aim is to develop a new approach for permeability characterisation. Indeed, it is usually determined, in the literature, through semi-empirical or even numerical methods using equivalent networks. These works have proved somewhat efficient yet they remain costly in terms of time and experiments because of the multitude of fibrous structures used today and also of their increasing complexity. This is why this works is done in a desire to reach a standard permeability term characterisation but also could be adapted to identify thermal conductivities and visco-elastic properties of porous media. Only the basic elements of this characterisation shall be exposed through some results obtained with image processing analysis.
In a first step, this expertise consists in obtaining a series of cuts of the composite materials with the different families of fibrous structures in order to characterise the double scale of pore sizes. On the picture shown on Fig. 6, the different sizes of pores can be identified as well as an air bubble.

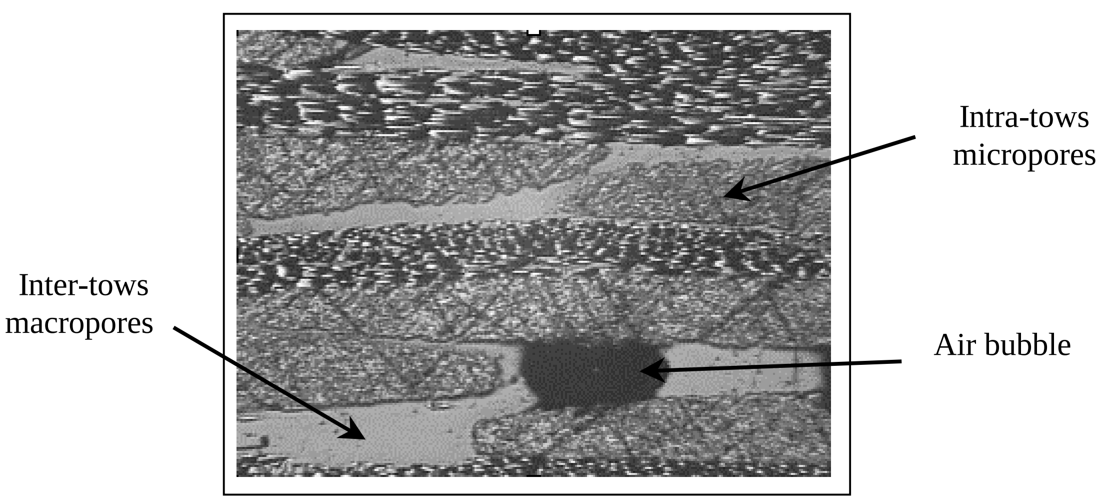

Fig. 6: Visualisation of a reinforcement sample

We find here a bimodal distribution of pore sizes that already had been observed by Binétruy [5] and Bréard [6]. With such images a rigorous analysis has been carried out for the different structures. In the case of biphasic materials (not taking into account the presence of air bubbles) linear and bi-dimensional overture techniques can be used when stereologic characteristics are available. Here only linear granulometry will be presented. It leads to a granulometric distribution curve. This techniques allows to calculate pores size granulometry in numbers and in sizes as well as interesting additional stereologic informations. These concepts can be found in Coster [7] and permit to define all the previous elements from the Euler-Poincaré Characteristics (EPC). These are in fact the connexity numbers (N_i) defined for each analysis space (R^i).

The first step was to compute the $P(l,\alpha)$ function that is need for the granulometric analysis. It allows to characterise the evolution of the eroded structure of the image of the biphasic structure when the length of the structuring element (l) varies in the direction α. This function is actually the probability that a line segment of length l, oriented in the direction α, is included in one of the two phases. It is directly linked to the EPC.

Granulometry in numbers consists in number objects in the image and to link these numbers with the length of the structuring element and is represented by the repartition function $F(l,\alpha)$. This information can be deducted from the $P(l,\alpha)$ function. Differentiating, the number-granulometric density function $f(l,\alpha)$ is obtained. In the same way, granulometry in measure can be performed and repartition function $G(l,\alpha)$ and distribution $g(l,\alpha)$ are computed. In the practice, it is impossible to obtain global information about micro and micro-pores with one single image. Indeed, it is impossible to get images that have good resolution on both scales. Hence the analysis is performed separately and micro and macro-pores and the results are then put together to reach the global characterisation. An example of this characterisation technique is given with a study on micro-pores (Fig. 7).

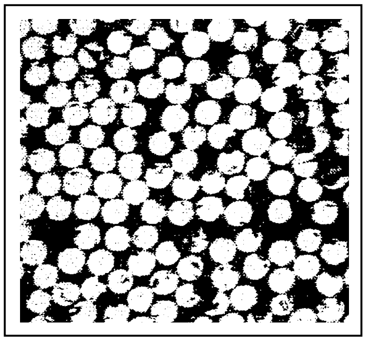

Fig. 7: Microscopic sample of a reinforcement (micropores)

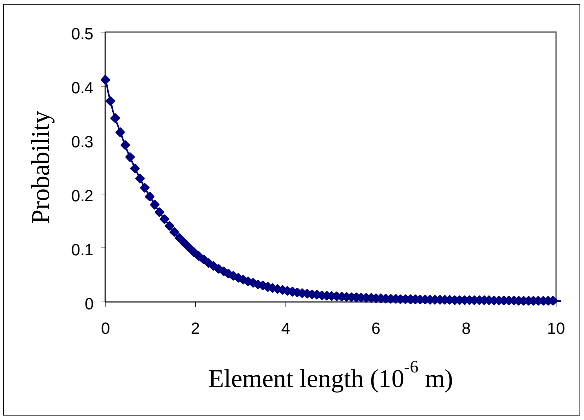

Fig. 8.1: P(l) function of the micro-porous structure

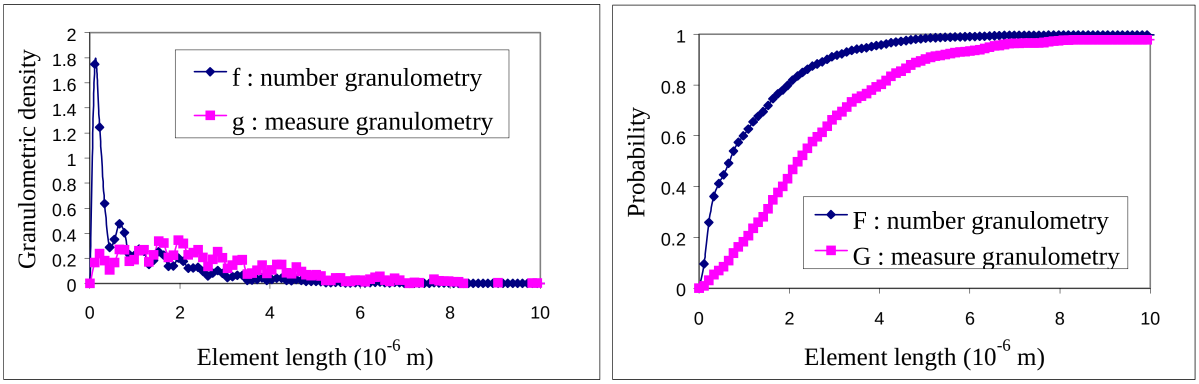

Fig. 8.2: Granulometric density function and distribution

Moreover, it is possible to calculate distribution moments and the star function from the $P(l,\alpha)$ function. The main moments are proportional to stereologic quantities called stars. This can be illustrated in the case of the R^2 space (Fig. 8.1 and 8.2). Considering the connexity number in R^0 is simply the porosity of the medium ϕ, and the star function Δf^2 can be is defined as:

$$\Delta f^2 = \frac{1}{\phi}\int_0^{2\pi} d\alpha \int_0^{\infty} lP(l,\alpha)dl \qquad (11)$$

This function is an important data since it will be introduced in the permeability characterisation. Indeed, according to Bear [8], an approximation of the motion quantity conservation motion can be obtained for the fluid phase. In our case the fluid entirely fills the void space of the porous medium. Distinguishing the various forces by unit volume, it can be described in the following way, all the variables being macroscopic:

$$\phi\frac{dq}{dt} = -\phi T\nabla p + \mu\nabla^2 q - \mu\frac{c_f}{\Delta\tilde{f}^2}q \qquad (12)$$

where c_f is a coefficient characterising the solid-fluid interface configuration (shape factor), and T the tortuosity tensor. In our case we will simplify this expression. Indeed, the fluid flow is such that viscous stress due to motion quantity transfer at the solid-fluid interface are much bigger than inertia and viscous stress within the fluid itself. The equation then becomes equivalent to generalised Darcy's Law and then permeability can be linked directly to macroscopic representations of the microscopic configuration of the solid-fluid interface within the REV.

$$K_{geo} = \phi\frac{\Delta\tilde{f}^2}{c_f}T \qquad (13)$$

The permeability of a porous medium hence depends on three parameters: the porosity ϕ, the tortuosity tensor T and the global specific section Δf^2 that describes the geometrical configuration of the void space. This quantity is intrinsic indeed and does absolutely not depend on any fluid's characteristics. Permeability is hence a symmetric second order tensor and the study asks for a tri-dimensional expertise as shows on the diagram on Fig. 9.

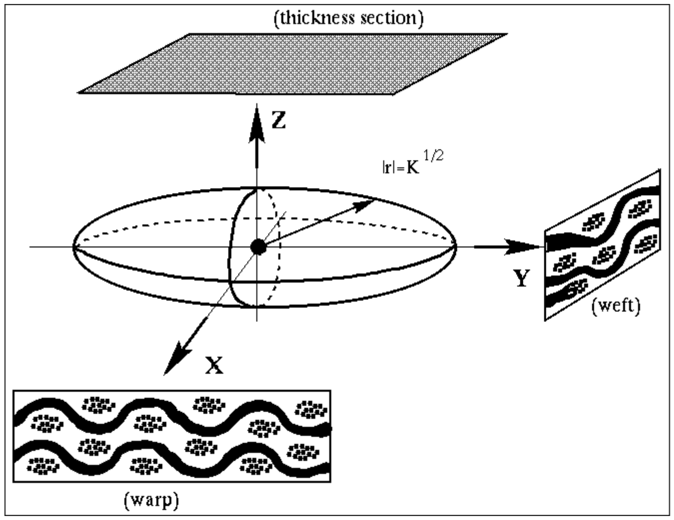

Fig. 9: Representation of REV with the sample relation

We are now developing a procedure that will allow to characterise the global morphology of a porous structure describe as a dual scaled porous medium (micro and macro-pores).

Conclusion

This study started with a study of saturated/unsaturated permeability both on experimental and theoretical approaches. We show that permeability is an intrinsic characteristic of the porous medium even though it depends on the saturation degree. This is shown on unsaturated pressure measurements and also when considering the ratio between K_{sat} and K_{unsat}. Both experimental observations go in the same direction. The difference of permeability values sometimes observed using different fluids is hence a consequence of the saturation ratio that differs and does not come from the permeability term itself. We also show that, flow laws must take fibre rearrangement for transient flows in both unsaturated and saturated porous media.

The saturation ratio is a macroscopic entity yet it is clearly the consequence of microscopic phenomena and especially of air entrapments within tows, hence the presence of micropores. This is why our study concentrated on geometrical characterisation of the porous medium. We chose a granulometric approach using image processing. This is one way among others. We think that a good knowledge of the fibrous structure will allow to build accurate numerical models based on homogenisation techniques. This will the permit to comprehend the local flow behaviours leading to bubble formation and also to calculate permeability numerically.

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