COMPUTING UNCERTAINTIES IN A DELTA WING COMPOSITE STRUCTURE USING INTERVAL-BASED METHODS

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SUMMARY: This paper deals with interval-based methods used to perform structural analysis of structures submitted to uncertainties. Recently, intervals have been introduced as an alternative to more classical statistical approaches such as Monte-Carlo analysis or stochastic finite element methods. In a linear static context, a discretized structural analysis resumes to the resolution of linear systems of equations. Using interval parameters in those systems requires the development of new computational methods dedicated to interval arithmetic. We compare here usual techniques to an efficient new method, called NAVS, based on series development and optimisation schemes, through the study of a composite delta-wing. This application shows one of the benefits which structural engineers can get from using non-deterministic approaches.

KEYWORDS: uncertainties management, interval computations, fuzzy logic, vertex method, composite delta wing.

INTRODUCTION

In the context of uncertainty analysis our purpose is to develop computational methods based on interval arithmetic. As a matter of fact, uncertainties, whatever their origins are (loadings, material characteristics, geometrical data, …), must now be taken into account in almost every kind of finite element analysis. In uncertainty analysis, there is two main goals: to quantify the dispersions of structural responses and to perform reliability analysis. For the computation of the responses' dispersions, simulation methods (Monte-Carlo) or stochastic finite element methods (SFEM) can be employed [7]. As an alternative, we introduce the fuzzy logic-based approach which consists in considering uncertainties as fuzzy numbers, offering a different point of view compared to the classical probabilities [5]. After discretisation of the fuzzy numbers, such an approach leads to the resolution of linear systems of equations whose coefficients are intervals [3]. The methods related to the resolution of these systems and their applications to a composite delta-wing structure are addressed in this paper.

DATA UNCERTAINTIES

Classically, when statistical methods (Monte-Carlo simulation techniques or Stochastic Finite Element Methods for instance) are employed, input data need to be described by a probabilistic law and its statistical moments (mean and standard deviation). As an alternative, usage of *possibilistic* techniques is convenient since poor or sparse information is available about parameters dispersions whereas statistical (probabilistic) distributions require lots of information describing the behaviour of uncertain data. Basic to possibility theory, fuzzy numbers are often considered as ‘weak’ distributions. In the context of structural uncertainties, they are a convenient way to describe economically parameters scattering.

Considering a linear static structural analysis problem, three main steps can be defined to proceed to a ‘possibilistic analysis’. First, a fuzzification step is realised, giving a fuzzy representation of each uncertain parameter. A discretisation of these fuzzy numbers is done afterwards, leading to intervals for every discrete degree of membership (Fig. 1). The second step, repeated for each discrete degree, consists in the resolution of an interval linear system of equations extracted from the previous step (Fig. 2). The last step, not addressed in this paper, is just an interpretation step, to ‘rebuild’ a fuzzy number describing the responses variabilities (Fig. 3).

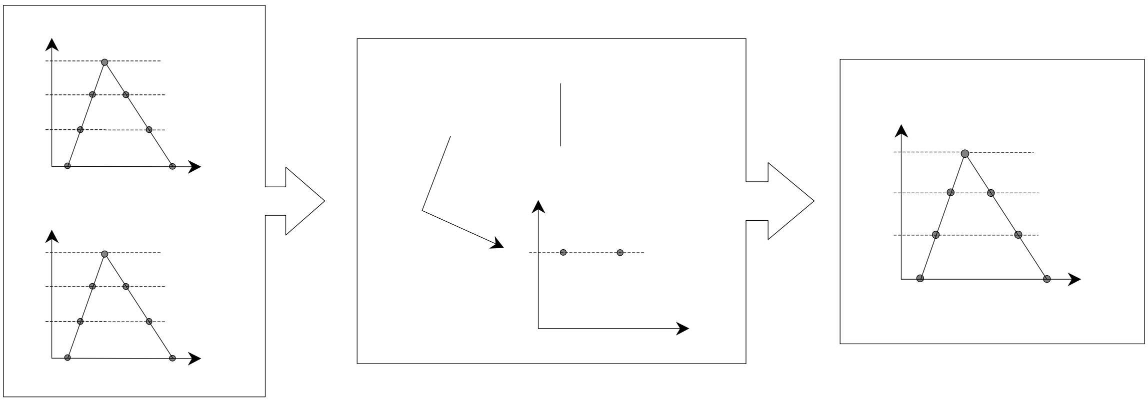

Fig. 1 : Fuzzification *Fig. 2 : Resolution* *Fig. 3 : Interpretation*

Fuzzy data interpretation

More than an alternative way to manage data uncertainties, fuzzy numbers must be seen as useful tools for design. Considering the membership degree introduced on Fig. 1 as an index describing a ‘cost/satisfaction’ ratio, the designer can - using his knowledge of responses’ scattering at every degree - choose the most cost-effective design within a controlled cost range (Fig. 4).

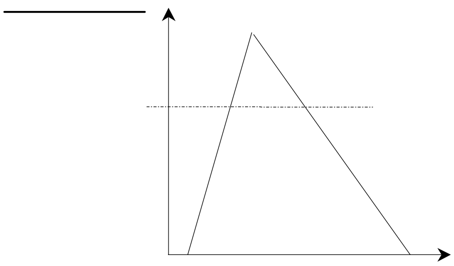

Fig. 4 : Fuzzy cost-related design

Interval approach

The second step of the main process is dedicated to the resolution of the problem extracted at every discrete degree of membership. A fuzzy number $\mu(x)$ can then be defined as a set of intervals ordered by increasing values of the membership (satisfaction/cost ratio) degree. At a given degree α, the variable x lies in the interval $[x_\alpha^{min}; x_\alpha^{max}]$. In fact, still considering a linear static problem, we face a linear system of equations whose parameters are intervals. Thus, the heart of the method resumes to the management and the resolution of such interval systems.

INTERVAL-BASED METHODS

Interval arithmetic and systems of equations

The previous section introduced the link between the main problem and interval variables. Dealing with intervals means using adapted tools (specific arithmetic, algorithmic particularities, …). Intervals were introduced first in 1966 by Moore [1] and since were mainly used to treat round-off errors problems. More recently some works trying to apply interval techniques to structural mechanics under uncertainties have been proposed with limited success since interval arithmetic generates many algorithmic limitations.

Bold-italic letters representing interval quantities, a real interval is a subset of $\Re$ defined by: $\boldsymbol{a} = [a_1 ; a_2] = \{x : a_1 \le x \le a_2 ; a_1, a_2 \in \Re\}$. These intervals follow arithmetic rules listed below.

Addition	$\boldsymbol{a} + \boldsymbol{b} = [a_1+b_1 ; a_2+b_2]$
Substraction	$\boldsymbol{a} - \boldsymbol{b} = [a_1-b_2 ; a_2-b_1]$
Multiplication	$\boldsymbol{a} \times \boldsymbol{b} = [\min(c) ; \max(c)]$ with $c = \{a_1.b_1, a_1.b_2, a_2.b_1, a_2.b_2\}$
Division	$\boldsymbol{a} / \boldsymbol{b} = [a_1 ; a_2] \times [1/b_2 ; 1/b_1]$ if $0 \notin [b_1 ; b_2]$

All the classical arithmetic tools exist and other rules, such as subdistributivity and monotonic inclusion are used. An extension of interval computations operators to matrices is derived with $\boldsymbol{A} = (\boldsymbol{A}_{ij}) = ([a_{ij}^{min} ; a_{ij}^{max}], 1 \le i \le n, 1 \le j \le n)$.

If we represent a $n \times n$ linear system of interval equations $\boldsymbol{Ax}=\boldsymbol{b}$ as the family of n real equations Ax=b, with $A \in \boldsymbol{A}$ ($\Re^{n\times n}$ interval matrix) and $b \in \boldsymbol{b}$ ($\Re^n$ interval vector) then the solution set is defined as:

$$\Sigma(\boldsymbol{A},\boldsymbol{b}) = \{x \in \Re^n / Ax=b , A \in \boldsymbol{A} , b \in \boldsymbol{b}\} \qquad (1)$$

The structure of $\Sigma(\boldsymbol{A},\boldsymbol{b})$ is known to be complex and practically only *estimators* of the solution set (bounding boxes) will be obtained [2,6].

Direct methods for resolution of linear systems

The first method proposed is based on Hansen's algorithm which is an extension of the classical Gauss-Seidel algorithm to interval computations [8]. The complete Hansen's method includes a preconditioning step and an interval-intersection operator to minimise the systematic increase of the intervals' sizes. The second method was introduced by Rump and is a Newton-like (fix point search) algorithm designed to solve interval linear equations. In most of the cases, the results obtained are close to those obtained with Hansen's algorithm which provides good results for small structures (in terms of number of degrees of freedom), and fails for high size structures.

New original method

The third method, called NAVS (standing for Neumann Approximation for Vertex Solutions), was originally developed by the authors [6] to suppress the major drawback of usual interval methods: interval linear system resolution only gives a hull including the solution set, but often this box is too wide to be exploited. This drawback of interval computations is due to the specificity of interval arithmetic operators: inclusion and subdistributivity properties lead to a huge increase of interval width when a same parameter occurs many times in an expression (known as "dependency phenomenon"). Although algorithms take care of this problem it is impossible to overcome it for any given structure. The proposed NAVS method uses the Vertex approach [4], a Neumann series expansion and heuristic resolution of optimisation sub-problems.

The classic Vertex method is a combinatorial approach: it leads to the exhaustive resolution of 2^n linear systems for n interval uncertain coefficients, since every vertex of the n-dimensional design space is reached. This method is highly improved by the NAVS method in terms of computational cost since the discrete optimisation phase decreases the amount of computations necessary to reach the vertices of the hull. In terms of results quality, as shown by the test-cases, the NAVS method always gives the smallest box including the whole solution set.

NAVS formulation

Let d_i be an uncertain parameter. The stiffness matrix K and the load vector f are affected by those uncertain parameters through the following relations:

$$\boldsymbol{K} = \left[K^{min};K^{max}\right] = \tilde{K} + \sum_{i=1}^{n_1} \mu_i \Delta \boldsymbol{K}_i(d_i) \tag{2}$$

$$\boldsymbol{f} = \left[f^{min};f^{max}\right] = \tilde{f} + \sum_{i=n_1}^{n} \mu_i \Delta \boldsymbol{f}_i(d_i) \tag{3}$$

Where mean (or nominal) values are denoted by tildes and where μ_i is a normalised (+/- 1) additional design variable. The terms $\Delta\boldsymbol{K}_i$ (resp. $\Delta\boldsymbol{f}_i$) denote variations of $\boldsymbol{K}$ (resp. $\boldsymbol{f}$) relative to the i^{th} parameter.

Considering for simplicity only uncertainties on the stiffness matrix, the NAVS equations are obtained if the global discrete equilibrium problem is restated as:

$$\boldsymbol{Kq} = \boldsymbol{f} \tag{4}$$

$$\boldsymbol{q} = \left(\tilde{K} + \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}_1} \mu_{\boldsymbol{i}} \Delta \boldsymbol{K}_{\boldsymbol{i}}\right)^{-1} \boldsymbol{f} \tag{5}$$

$$\boldsymbol{q} = \sum_{\boldsymbol{r}=0}^{+\infty} \left(-\sum_{\boldsymbol{i}=1}^{\boldsymbol{n}_1} \mu_{\boldsymbol{i}} \tilde{\mathrm{K}}^{-1} \Delta \boldsymbol{K}_{\boldsymbol{i}} \right)^{\boldsymbol{r}} \boldsymbol{q}_{\boldsymbol{o}} \quad \text{with} \quad \boldsymbol{q}_{\boldsymbol{o}} = \tilde{\mathrm{K}}^{-1} \boldsymbol{f} \tag{6}$$

or recursively,

$$\boldsymbol{q}^{(\mathrm{k}+1)} = \boldsymbol{q}_{\mathrm{o}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_1} \mu_{\boldsymbol{i}} \tilde{\mathrm{K}}^{-1} \Delta \boldsymbol{K}_{\boldsymbol{i}} \boldsymbol{q}^{(\mathrm{k})} \tag{7}$$

This last recursive relation gives – according to the selection of optimal μ_i through an optimisation process – the extreme values of $\boldsymbol{q}$. The optimisation process can be driven to achieve extreme values of design criterion such as $\boldsymbol{c}=\mathrm{B}^{\mathrm{T}}\boldsymbol{q}$. This algorithm, although efficient, has a major limitation: using discrete additional variables (μ_i) to reach the vertices means that we deal with monotonic functions.

APPLICATIONS

Aims

The previously presented method is applied to composite structures: a 120 degrees of freedom box beam and a delta wing structure composed of a titanium frame and a composite skin. These examples are used to illustrate and compare the methods: Through these examples, we can show that the enhancements introduced by the NAVS method allow one to compute economically the smallest box including the solution set while classical methods such as Hansen's algorithm fail to produce a meaningful answer. As a matter of fact, these test cases are intractable for classical interval-based methods.

Box Beam

The next figure (Fig. 5) presents a thin plate structure made of composite panels. It is composed of 108 elements: 18 orthotropic composite membranes (quadrangular) made of 4 layers (with 0°, 90°, 45° and –45° respective orientations), 18 shear panels and 18 aluminium bars.

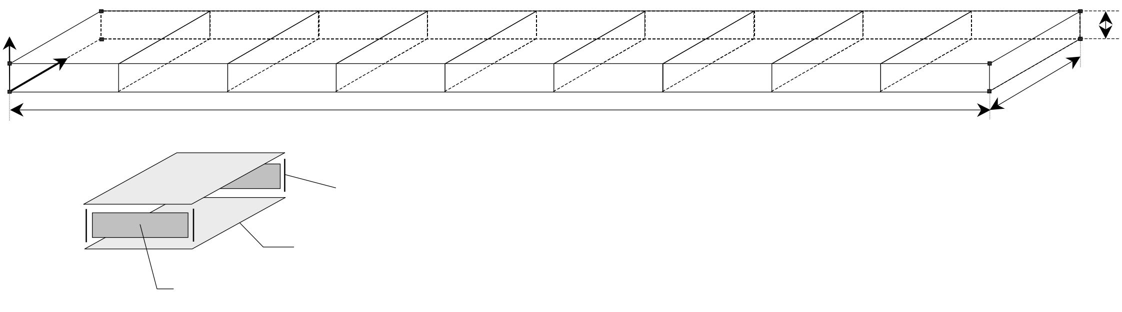

Fig. 5 : Box beam model definition

The following table resumes the material properties of the various components of this box-beam structure:

Material	*Aluminium*	*Composite Boron-Epoxy*
Young’s modulus E_{11}	68948 MPa	206843 MPa
Young’s modulus E_{22}	68948 MPa	18616 MPa
Poisson’s coefficient	0.3	0.21

Shear modulus	27807 MPa	4826 MPa

Table 1 : Box-bean material definition

One load case is considered, consisting of four nodal forces (intensities: –4448 N) along Z-axis on nodes 1,2,3 and 4.

The uncertainties affect bars sections (nominal value: 6.45 cm^2) and layers thicknesses (nominal value: 0.635 cm) with a magnitude of +/- 10% : uncertain parameters are ranging in the interval [0.9N ; 1.1N] where N is the nominal value.

The following table presents NAVS results for vertical displacement of extreme node 4. The last column displays for comparison purposes the results obtained using an optimisation method (MDQA algorithm within the BOSS Quattro environment [9]).

This reference result uses classical optimisation schemes: the only constraints concern uncertain design variables which are supposed to vary in their user-defined range, the objective function is either the minimisation or the maximisation of the targeted response according to which bound is computed. This optimisation based-approach encounters convergence problems since it happens to find smaller interval bounds than those obtained by NAVS.

Nodal displacement	*Nominal value*	*NAVS result*	*Optimisation*
w_4	-2.719	[-3.035, -2.461]	[-3.035, -2.462]

Table 2 : NAVS results for Box-beam test case

For a +/- 10% specified dispersion on input parameters, the responses' bounds are respectively obtained at –11.6% and +9.49% around the median value.

Delta-Wing composite structure

The structure presented on Fig. 6 is a high performance delta-wing made up of graphite/epoxy composite skins lying on a titanium web.

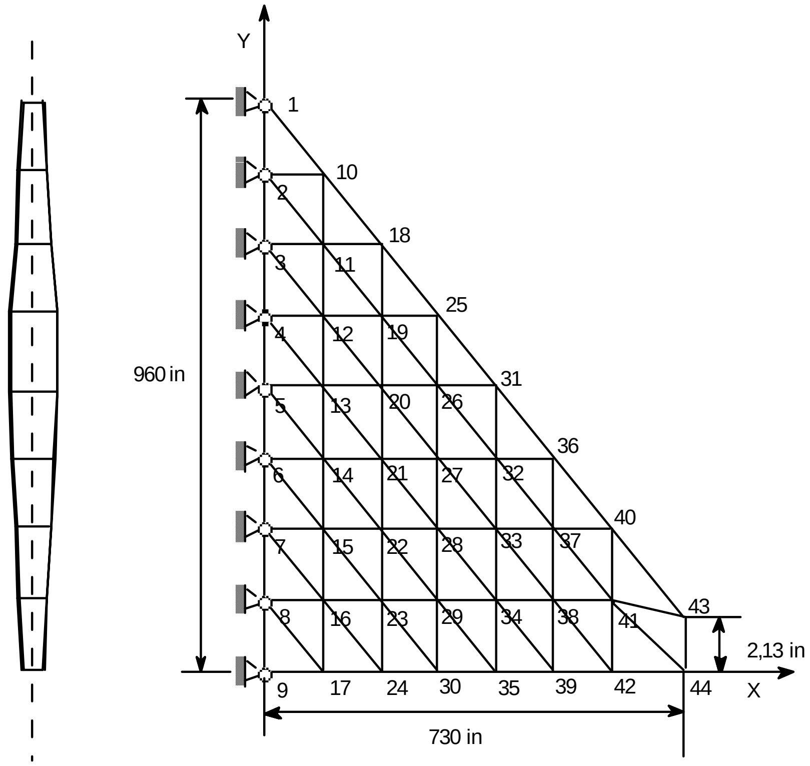

Figure 6 : Delta-wing model definition

The upper and lower skins are composed of four stacked graphite-epoxy laminates respectively oriented at 0°, +45°, -45° and 90° with respect to X-axis. These laminates are modelled by stacking four triangular orthotropic elements in each region defined on Fig. 6. Hence, the model is composed of 252 elements for each skin and 70 symmetric shear panels for the titanium web.

Table 3 resumes material properties of the different parts of the wing

Material	*Titanium (web)*	*Graphite-Epoxy (skins)*
Young's modulus E_{11}	113074 MPa	206843 MPa
Young's modulus E_{22}	113074 MPa	18616 MPa
Poisson's coefficient υ	0.3	0.21
Shear modulus G	43437 MPa	4481 MPa

Table 3 : Delta-wing material properties

The parametrization of the model leads to consider 140 variables:

- First, the web is decomposed in 12 parts and we define for each of them a variable thickness with mean (that is "nominal") value 0.38 cm;

- then, the skins are divided into 16 parts, so, considering 2 skins and 4 laminates thicknesses per part, it leads to 128 variables (nominal values in the range 0.25 to 0.76 cm). The splitting of the model into these groups is defined on Fig. 7.

The boundary conditions are described as follows: the structure is supposed to be fixed on nodes 1 to 9 and is submitted to 35920 N nodal forces along Z-axis on nodes 10 to 44. This static load case is almost equivalent to a uniformly distributed load of 6895 Pa.

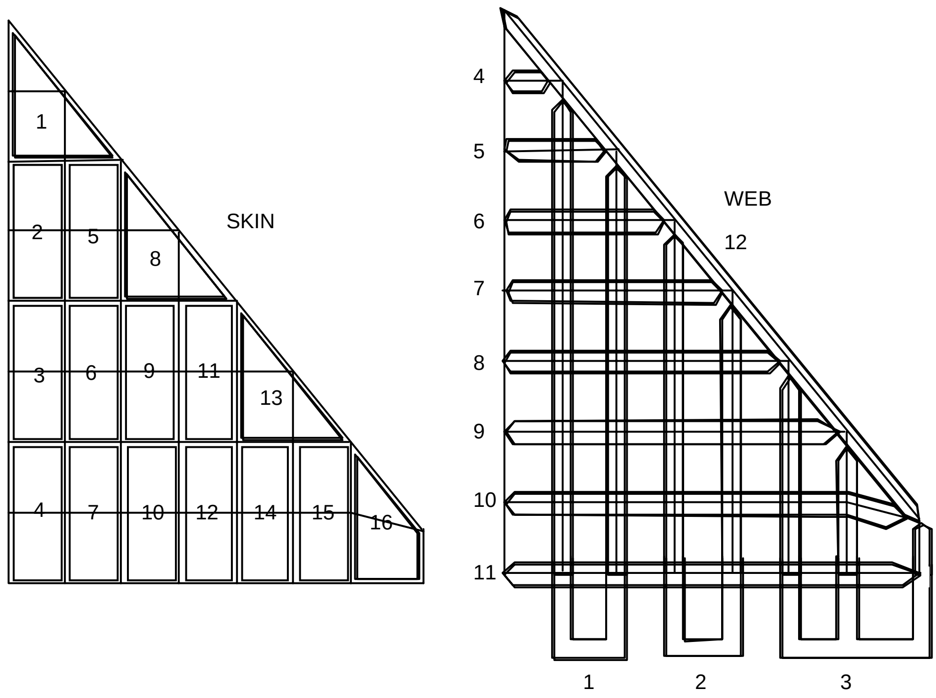

Fig. 7 : Delta-wing - Group parameters definition

The uncertainties analysis through the NAVS process is realised on the 140 previously defined variables. As for the previous test case, the uncertain variables are supposed to vary in a +/- 10% range around their nominal values. The results considered here are the 3 displacement components at extreme node 44.

The table 4 resumes the results obtained and offers comparison with the Monte-Carlo reference results.

Response	*Median*	*NAVS*	*Monte-Carlo*	*Optimisation*
U_{44}	-0.0418	[-0.0520, -0.0324]	[-0.0454, -0.0381]	[-0.0464, -0.0380]
V_{44}	0.0245	[0.0075, 0.0417]	[0.01980, 0.0297]	[0.0223, 0.0272]
W_{44}	4.070	[3.698, 4.523]	[3.975, 4.199]	[3.699, 4.522]

Table 4 : Delta-wing results

These results just confirm the excellent position of the NAVS method: the results obtained here on this multi-variable test case are better than those obtained with optimisation (a good agreement can be found on vertical displacement W_{44}). It should be pointed out here that optimisation require at least n+1 finite element analysis while NAVS methods only generates 2 analysis: NAVS is more accurate and much less time consuming than the optimisation approach.

The comparison with Monte-Carlo results may lead to consider NAVS responses as pessimistic ones since the intervals are much wider. But it would be a false interpretation: NAVS method gives extreme reachable bounds while Monte-Carlo results – just as every statistical result – can't reach those bounds, unless a huge amount of computation is realised.

For both examples, with much less pain than with classical approaches (the ‘heavily’ computational Monte-Carlo method or the optimisation-based approach), NAVS method offers:

- cost effective responses ;
- meaningful responses (compared to other interval methods on such big problems) ;
- always conservative bounds.

These conclusions validate the possibilistic NAVS method as an efficient alternative to more classical tools dedicated to uncertainties management.

CONCLUSIONS

Direct conclusions

In this paper we have introduced a new method to deal with structural uncertainties in the framework of the so-called possibilistic approach. This new method, based on the combinatorial Vertex scheme, a Neumann series development and an optimisation sub-process allows the designer to achieve almost all uncertain studies in linear static analyses of truss, bars and membranes based structures. The application of such techniques to two composite structures was shown to be successful.

Moreover, we pretend that the possibilistic approach is an economic, realistic and efficient alternative to classical probabilistic methods: Economic since just a few finite element computations have to be realised; realistic in the sense that – in an industrial context – statistical data may be out of reach for the designer and so interval description may be really a suitable tool; and efficient since comparison with reference methods are positive.

Perspectives

The NAVS method is new and thus needs to be strengthen. For instance, in a close future we might be able to give not only displacements ranges but also stresses ranges and more generally we want to control uncertainties propagation in structural computations. But, over uncertainties management, a more global design problem is opened here. Using fuzzy representation and efficient computational tools, we can address inverse problems: for a given dispersion on the responses, what is the dispersion admitted on the input parameters ?

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