DESIGN OF SMART COMPOSITE LAMINATES AND THIN-WALLED BEAMS

C. Kaiser, H. Baier

Chair of Lightweight-Structures, Technische Universität München, D-85747 Garching

SUMMARY: The behavior of active anisotropic laminates and thin-walled composite beams is characterized by a complex interaction between active layers and anisotropic substrates. CLT shows the influence of actuator-type (isotropic, orthotropic and kind of actuation), actuator position, and actuator orientation onto shear-deformation and twist of tension-shear- and tension-torsion-coupled smart laminates. A linear theory for analysis of smart tension-torsion-coupled thin-walled beams is also presented which takes into account out-of-plane warping, inplane warping and inplane actuation effects. Additional effects of boundary conditions and type of cross-section have significant influence on the optimum design of such smart beams to achieve maximum twist.

KEYWORDS: composite structures, composite laminates, smart structures, thin-walled beams, anisotropy, elastic coupling, piezoelectric actuator.

INTRODUCTION

Smart structures consisting of composite structures in combination with integrated piezoelectric actuators meanwhile have a wide field of applications. In most cases the active elements are used to measure, influence, and control the static and especially dynamic behaviour of lightweight-structures. In combination with the anisotropy of composite structures a new dimension in actuating technology is possible. Areal actuators embedded in anisotropic structures containing elastic couplings or anisotropic actuators allows the actuation of virtually any kind of deformation in structures like shear-deformation or twist. The potential of anisotropic actuator technology meanwhile is recognized and for example slipped in active wing design [1] with Directionally Attached Piezos called DAP or in development of active fibers [2]. An example for combining actuators with elastic couplings of composite structures is the design of an extension-torsion-coupled composite tube actuating a trailing edge flap of a rotor blade [3].

Studying the status quo of technology of active anisotropic structures a lack of general investigations and design rules of active orthotropic or anisotropic composite structures can be identified. Basic topic of this work is the analysis of structural behavior of anisotropic plates and thin-walled beams with isotropic or anisotropic actuators integrated. The possibility and potential of actuating shear-deformation and twist in such structures will be analysed. Several

aspects for a better understanding of the complex behavior and for conclusions to optimize the lay-up of smart composite structures will be outlined. The theoretical analysis of smart laminates will be done by using CLT and the analogy between thermal and electrical field. For active thin-walled tension-torsion-coupled beams with closed and open cross-section a theory will be outlined for calculating the induced twist by various areal actuators in the flanges of the beam. The results will be compared with results from FEM-analysis.

SMART COMPOSITE ANISOTROPIC LAMINATES

Basic equations and fundamentals on active laminates

The plate stress field of a general composite laminate including active layers basing on CLT is:

$$\begin{Bmatrix} n \\ m \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} - \begin{Bmatrix} n_\Lambda \\ m_\Lambda \end{Bmatrix} \Rightarrow \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} n + n_\Lambda \\ m + m_\Lambda \end{Bmatrix} \tag{1}$$

with $A_{ij} = \sum_{k=1}^{n} C_{ij_k}(t_k - t_{k-1})$; $B_{ij} = \frac{1}{2}\sum_{k=1}^{n} C_{ij_k}(t_k^2 - t_{k-1}^2)$; $D_{ij} = \frac{1}{3}\sum_{k=1}^{n} C_{ij_k}(t_k^3 - t_{k-1}^3)$

and $\{n_\Lambda\} = \sum_{k=1}^{n} [C]_k \{d\} (t_k - t_{k-1}) E_{el}$; $\{m_\Lambda\} = \frac{1}{2}\sum_{k=1}^{n} [C]_k \{d\} (t_k^2 - t_{k-1}^2) E_{el}$

$\{n_\Lambda\}$ and $\{m_\Lambda\}$ are equivalent actuator force and moment resultants. C_{ij_k} are elements of the transformed stiffness matrix $[C]_k$ of the k-th layer. $\{d\}$ is the tensor of the piezoelectric coupling coefficients, and E_{el} is the applied electrical field. A PZT-actuator generates strains. Forces are generated only if these strains are constrained. The connection between actuator strains and actuator forces bases on following assumption: The induced strains $\{\varepsilon_\Lambda\} = \{d\} \cdot E_{el}$ inside an active layer are converted into equivalent mechanical force and moment resultants by multiplying them with active layer stiffnesses.

There are three different kinds of planar actuation (fig.1). Because of the availability of many different isotropic actuators, isotropic actuation is mostly used in smart structures. Orthotropic actuation can be achieved for example by using active fibres embedded in a matrix-system [2] or by proper orientaton of PZT-stripes called Directionally Attached Piezos (DAP) [1]. In all those cases polarisation direction and applied electrical field lays perpendicular to layer-plane and the strains are induced via the d_{31}−coefficient. For shear-actuation polarisation direction must lay in layer-plane and the electrical field must be applied perpendicular to it. The induced shear-deformation is caused by the d_{15}−coefficient.

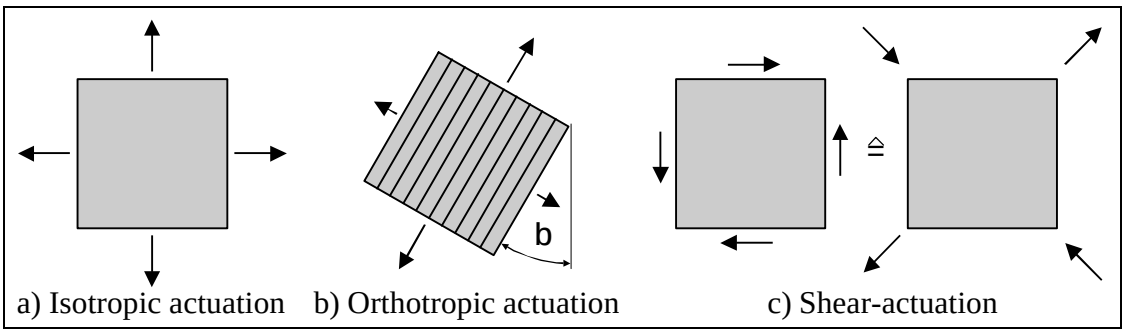

Fig. 1: Principal kinds of areal actuation

In case of orthotropic actuation the equivalent actuator force of a single active orthotropic layer of thickness t oriented with b regarding to the reference axis x is:

$$\begin{Bmatrix} n_{\Lambda x} \\ n_{\Lambda y} \\ n_{\Lambda xy} \end{Bmatrix} = \begin{Bmatrix} \cos^2\beta + k_{12}^{-1}\sin^2\beta + \nu_{p12}k_{12}^{-1} \\ \sin^2\beta + k_{12}^{-1}\cos^2\beta + \nu_{p12}k_{12}^{-1} \\ \left(1 - k_{12}^{-1}\right)\sin\beta\cos\beta \end{Bmatrix} \frac{E_{p1}}{1 - \nu_{p12}^2 k_{12}^{-1}} d_{31}\frac{E_{el3}}{t} \tag{2}$$

with k_{12} as the anisotropic ratio E_{p1}/E_{p2} of the Young‘s moduli of the active layer. From (2) the equivalent actuator force of an active isotropic layer can easily be derived with $k_{12} = 1$:

$$\begin{Bmatrix} n_{\Lambda} \\ n_{\Lambda} \\ 0 \end{Bmatrix} = \frac{E_{p1}}{1 - \nu_{p12}} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} d_{31}\frac{E_{el3}}{t} \tag{3}$$

Table 1: Used material data of PZT and CFRP

	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	n_{12}	d_{31} [mm/V]	d_{15} [mm/V]
CFRP	132.7	9.3	4.6	0.28	-	-
PZT	66	66	26.5	0.3	$-250\text{x}10^9$	$1000\text{x}10^9$

In fig. 2 the equivalent actuator forces for PZT with material data taken from table 1 and an applied electrical field of 800V/mm are compared. For DAP $k_{12} = 50$ and thus is extremly high. One can see that $n_{\Lambda x}$ of DAP for $\beta = 0$ is about 25% lower comparing to isotropic actuation because of dissapearing lateral contraction using DAP. If orienting DAP in 45° a maximum shear force can be induced, but also additional forces $n_{\Lambda x}$ and $n_{\Lambda y}$ of same magnitude are acting with negative influence on the induced shear-actuation because of the elastic couplings. This effect is important for optimum orientation of orthotropic active layers inside a laminate to induce shear-deformation or twist in laminates as one can see later.

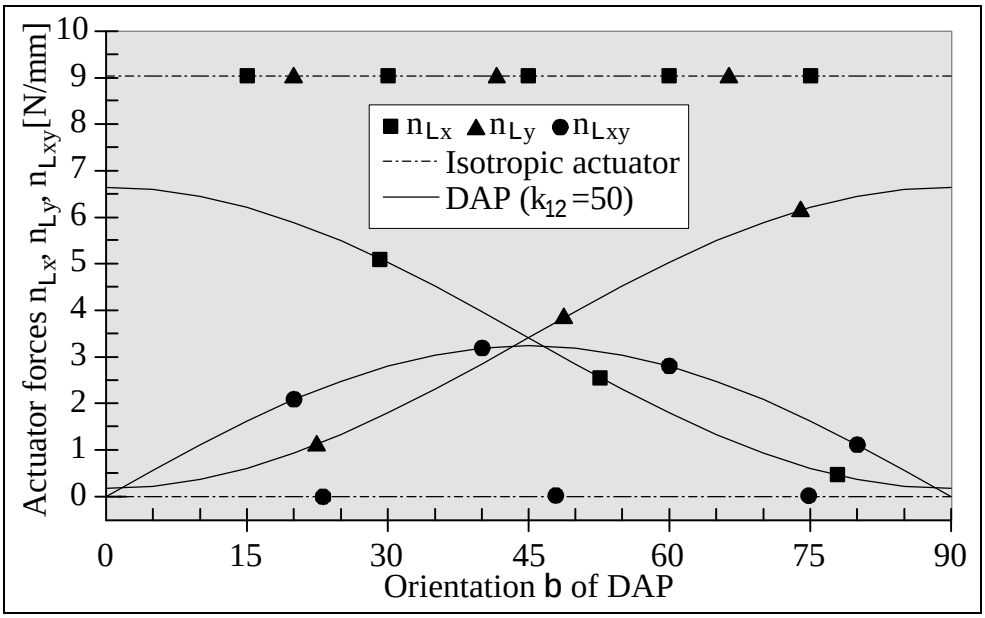

Fig. 2: Comparsion of actuator forces induced by isotropic actuator or DAP

The equivalent actuator shear-force in case of using shear-actuation $(n_{\Lambda x} = n_{\Lambda y} = 0\,!)$ is:

$$n_{\Lambda xy} = G_{p23}\, d_{15} \frac{E_{el2}}{b} \tag{4}$$

Combining the different actuator types with anisotropic laminates all kind of deformation can be induced directly by the actuators and/or by using the elastic couplings of the anisotropic substrate. To outline the design possibilities the two cases inducing shear-deformation and twist will be focussed here. The general results of interdependence of active layers and substrate can be transfered to all other induced deformations. On the other hand the results are relevant for the later discussion of active anisotropic thin-walled composite beams.

Inducing shear and twist in anisotropic active laminates

To achieve shear deformation by isotropic actuation one has to use tension-shear-coupling of the composite substrate. To get maximum tension-shear-coupling all layers must be oriented at 45°. With rising number of substrate layers and thus increasing of tension-stiffness the tension-shear-coupling decreases. On the other hand a minimum number of anisotropic layers is necessary. This leads to an optimum number of substrate layers between the active layers for maximum induced shear-deformation, which turns out relatively flat however.

Since for inducing shear deformation via isotropic actuation only the membrane part is relevant, there is no influence of position of active layers on the resulting deformation. But to avoid other couplings a symmetric stacking is required. This influence becomes important in case of inducing twist by tension-torsion-coupling as shown in fig. 3. The results are set in proportion to the one regarding to the laminate with actuators laying on the outside assuming same thickness of all layers (substrate and active ones) and a constant applied electrical field.

The active layers should be positioned in the center of laminate to minimize the negative influence of their isotropy on the tension-torsion-coupling. If all substrate layers have the same orientation, again one gets maximum coupling for 45°. Contrary to tension-shear-coupling one can rise induced twist by free orienting of layers either in pairs or every layer separately. The reason is the better tension-torsion-coupling by orienting the single layers towards the reference axis *x* and *y*.

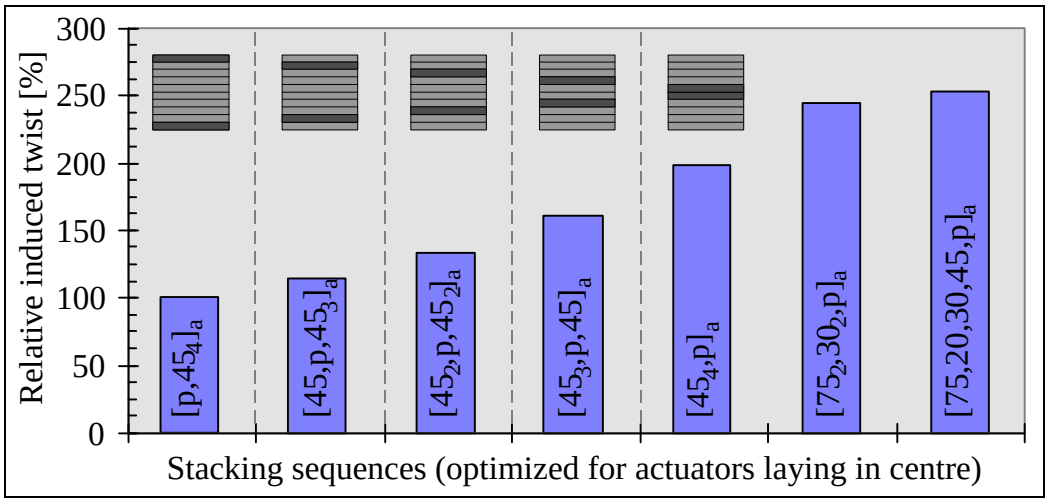

Fig. 3: Optimum of tension-torsion-coupling with isotropic actuation in antisymmetric CFRP-laminates with different stacking sequences

Coming to orthotropic actuation, fig. 4 shows the different possibilities of actuation with DAP in combination with the anisotropic substrate laminate. Important to note is that maximum induced shear-actuation under orientation of 45° can only used optimal in case of 0/90- or quasiisotropic laminates. Using orthotropic laminates leads to anisotropic behaviour of the active laminate and thus to another optimum orientation than 45° of the DAP for maximum deformation. This should be taken into account when designing active laminates with DAP.

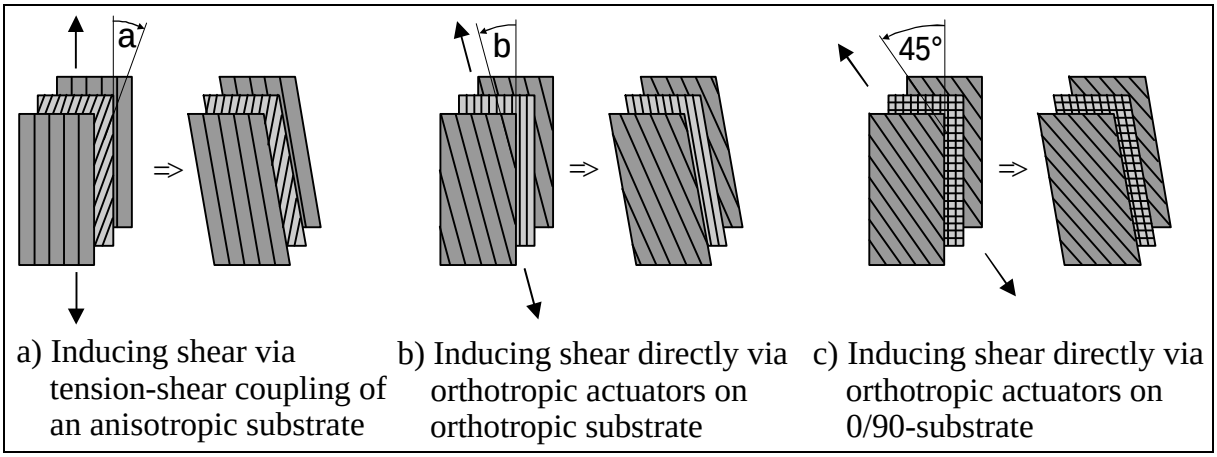

Fig. 4: Possibilities to induce shear with DAP

Fig. 5 shows the relative induced shear-strains using DAP in comparsion to the results with isotropic actuation and with shear-actuation for different laminate lay-ups. In general directly inducing of shear by orienting DAP on the substrate is more effective than inducing shear-strain via elastic coupling (by isotropic actuation or DAP). The optimum orientation of DAP combined with the choosen CFRP-material here is 30°. The reason are the mentioned additional tension forces which leads to disturbing elastic couplings. As expected inducing shear by shear-actuators is most effectively because of the significant higher *d*-coefficient. But the induced

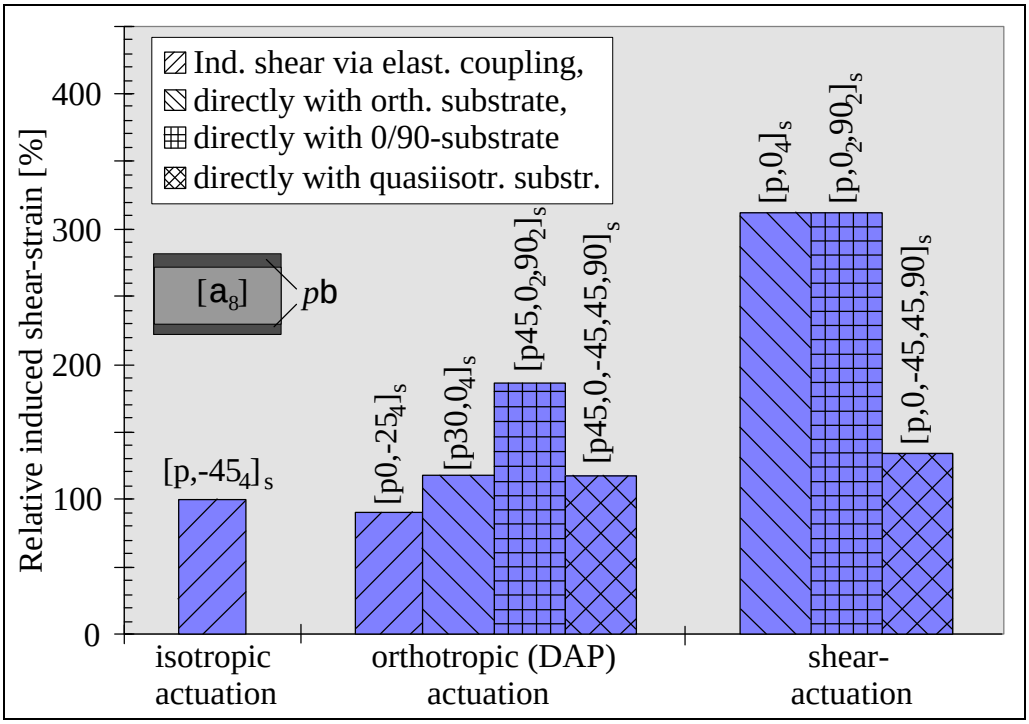

Fig. 5: Comparsion of active symmetric CFRP-laminates inducing shear

shear strain depends directly from the shear-modulus and it is all the more higher the lower the shear-modulus of the laminate is.

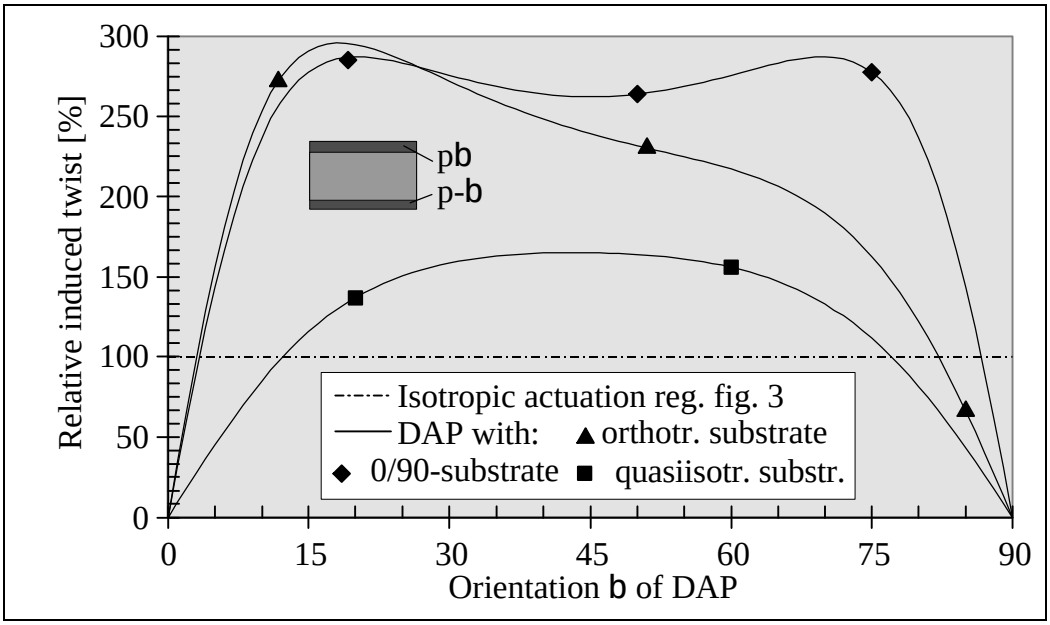

Fig. 6: Comparsion of active antisymmetric CFRP-laminates inducing twist

Similar results can be found if analysing actuation of twist in active laminates with DAP at the outer lamina. Fig. 6 shows the relative induced twist of different active CFRP-laminates actuated by oriented DAP comparing to isotropic actuation. In case of quasiisotropic stacking DAP oriented in 45° are the best choice of course. Otherwise one gets another optimum orientaton of DAP. Here the disturbing additional elastic coupling appears clear in the graphs. Comparing the results with those from fig. 3 one can see, that the advantage of using DAP is not as significant as expected if optimizing the stacking sequence and thus the elastic coupling of active laminates in case of isotropic actuation. Using shear-actuation combined with an orthotropic substrate again maximum twist can be achieved and the relative induced twist rises up to 600%, which is not shown in fig. 6.

SMART COMPOSITE ANISOTROPIC THIN-WALLED BEAMS

Actuation of twist in active anisotropic thin-walled beams

Elastic couplings of anisotropic structures can be used for passive and active displacement control for example regarding to aeroelastic tailoring [4]. To generate twist in composite thin-walled beams anisotropic webs and flanges have to be used. One example of active inducing twist is the actuation of a trailing edge flap of a rotor blade [3]. In general one has to distinguish between beams with closed and open cross-section. In first case the flanges have to be tension-shear-coupled, in case of open cross-section they have to be tension-torsion-coupled as indicated in fig. 7. For optimum design of active anisotropic thin-walled beams one can use directly the general results of the former section regarding to active anisotropic laminates in principal if assuming that all flanges as straight elements behave like thin laminated plates gouverned each by CLT. To compare the potential of twist of open and closed cross-section beams a single-celled beam and an I-beam both tension-torsion-coupled will be analysed.

Theory of thin-walled active composite beams

Basing on the Vlasov-theory of Chandra and Chopra [5] a modified linear beam theory is developed which takes into account shear deformation, out-of-plane warping and inplane warping effects [6]. It is assumed that the cross-section of the beam does not deform in its own plane and inplane deformation is neglected. Thus strain and stress in contour direction is neglected in comparsion to the normal strain and stress in direction of the beam axis. By skillfull convertation of the complete plate stress field (1) into a modified plate stress field with boundary condition $n_y = m_y = 0$ one fullfils on the one side the assumption of rigid cross-section and now takes into account the influence of the inplane strains (including also the active ones!) onto the other strains. One gets a modified stiffness-matrix $[K^*]$ and modified equivalent actuator force and moment resultants. Experimental analyses with anisotropic thin-walled beams under mechanical load showed the need of investigation of this theory [6].

Modifying the active loads for active composite beams regarding to the inplane actuation by embedded actuators for example here leads to a modified active torsional and tension load basing on the inplane actuation. Thus the differential equation system for an active anisotropic thin-walled tension-torsion-coupled beam is [3]:

$$\begin{Bmatrix} N_\Lambda^* \\ T_{s\Lambda}^* \end{Bmatrix} = \begin{bmatrix} K_{11}^* & K_{15}^* \\ K_{15}^* & K_{55}^* \end{bmatrix} \begin{Bmatrix} u' \\ \varphi_x' \end{Bmatrix} \quad ; \quad M_{\omega\Lambda}^* = K_{44}^* \varphi_x'' \tag{5}$$

The elements of stiffness matrix and the modified active loads are listed in appendix. With $T_\Lambda^* = T_{s\Lambda}^* - M_{\omega\Lambda}^{*\,\prime}$ one gets the differential equation for the anisotropic smart composite tension-torsion coupled beam:

$$-\frac{\varphi_x'''}{\lambda^2} + \varphi_x' + N_\Lambda^* \frac{K_{15}^*}{K_{11}^* K_{55r}^*} - \frac{T_\Lambda^*}{K_{55r}^*} = 0 \quad \text{with} \quad K_{55r}^* = K_{55}^* - \frac{K_{15}^{*2}}{K_{11}^*} \quad \text{and} \quad \lambda^2 = \frac{K_{55r}^*}{K_{44}^*} \tag{6}$$

Depending on boundary conditions one gets the solution for tip twist of beams with clamped end and warping free at tip:

$$\varphi_x = \frac{\left(N_\Lambda^* K_{15}^* - T_\Lambda^* K_{11}^*\right) l}{K_{11}^* K_{55}^* - K_{15}^{*2}} \left(\frac{\tanh \lambda l}{\lambda l} - 1 \right) \tag{7}$$

and for warping constrained at tip:

$$\varphi_x = \frac{\left(N_\Lambda^* K_{15}^* - T_\Lambda^* K_{11}^*\right) l}{K_{11}^* K_{55}^* - K_{15}^{*2}} \left(2 \frac{\cosh \lambda l - 1}{\lambda l \sinh \lambda l} - 1 \right) \tag{8}$$

Fig. 7 shows the results of induced tip twist of active composite single-cell beams and active composite I-beams with different actuators depending on boundary conditions and FRP-material each with optimum laminate lay-up. All beams are tension-torsion coupled and each have length $l = 1\text{m}$ and identical flanges and webs of width $b = 30\text{mm}$ and height $h = 30\text{mm}$ respectively and same thickness $t = 2.5\text{mm}$. The applied electrical field is 800V/mm. The results obtained by this approach are in high agreement with those obtained by FEM.

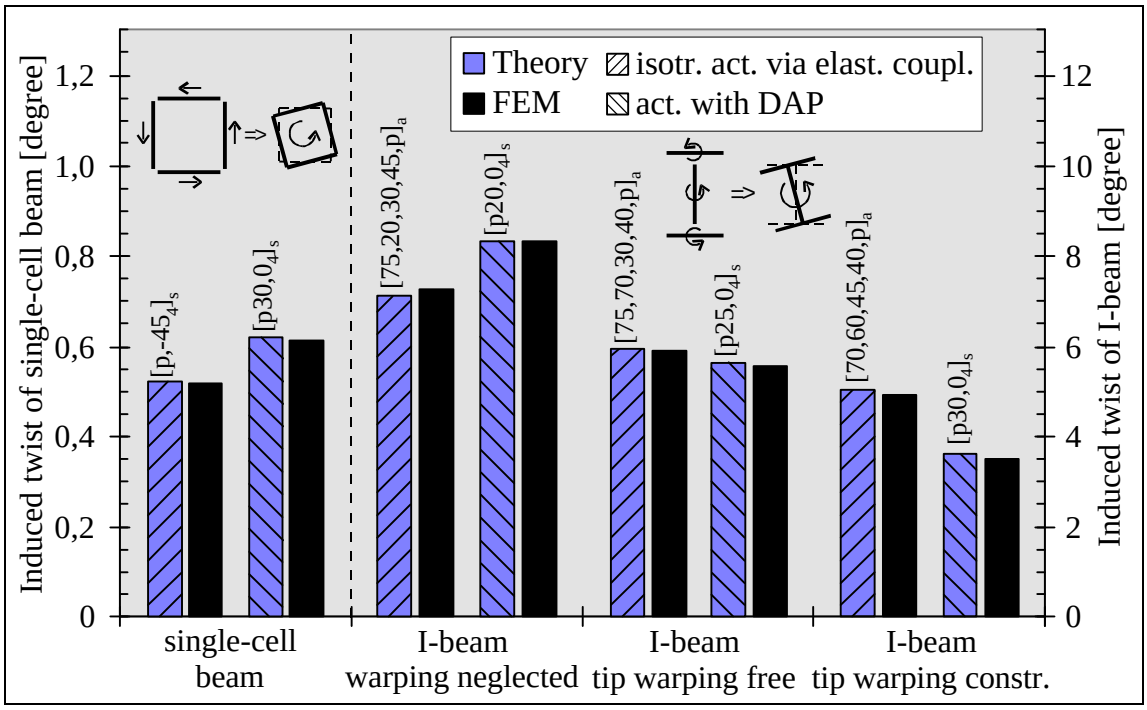

Fig. 7: Twist of smart tension-torsion-coupled thin-walled CFRP-beams of dimension 30mm by 30mm and length 1m

First thing to note is the influence of open or closed cross-section. The single-cell beams have only less than 1° twist but a significantly higher torsional stiffnesses. As expected the results from smart tension-torsion-coupled laminate analysis are only transferable to thin-walled beams in case of I-beam with neglected out-of-plane warping effects (i.e. pure St. Venant torsion). In all other cases there are different optimum lay-ups of the flanges. With higher constraining of warping the advantage of direct inducing twist with DAP gets lost, because warping stiffness has more influence on direct induced torsion than on tension-torsion-coupling. In case of single-cell beam there is no out-of-plane warping effects because of the quadratic cross-section. Here the circuit St. Venant shear flow influences the optimum lay-up.

The results of thin-walled beams with shear-actuators are not plotted in fig. 7. High deformation is achieved when shear actuation is used, which results into a similar increase regarding to the other kinds of actuation as outlined for smart laminates. Although only isotropic and to a lesser extent anisotropic piezoelectric actuators are state of the art, especially shear actuation promises a significant higher potential of inducing deformation in a composite structure combining with the anisotropy of the material.

SOME REMARKS ON OTHER FRP-MATERIALS

The presented analysis of smart composite laminates and thin-walled beams is confined to CFRP given in table 1 as substrate material. Investigations have been made with other FRP-materials, which allows following conclusions: Elastic couplings in anisotropic structures depends on the anisotropic ratio of Young‘s moduli E_1/E_2 and ratio of Young‘s modulus to shear-modulus E_1/G_{12}. By skillful combination of fibres with high modulus and an appropriate matrix-system significant increases of induced deformations actuated via elastic couplings are possible. In case of directly inducing deformations with DAP or by shear-actuation the corresponding stiffness has to be minimized to get maximum deformations. Here there are also a lot of possibilities of influence by several material combinations.

CONCLUSION

With considerations and analyses using CLT significant improvements of induced deformations of active laminates can be achieved. As could be shown, actuator type, position and orientation have not only influence on maximum actuation but also on stiffness-matrix and deformation-behavior of the whole active laminate in combination with isotropic, orthotropic or anisotropic substrates. Thus there is an interaction between actuators and substrate which leads to optimum active laminate design for generating shear-deformation and twist.

These results are important for the design of active thin-walled beams with closed or open cross-section under the goal of inducing maximum twist and leads to optimum design of the flanges and webs. For analysis of tension-torsion coupled thin-walled beams with integrated areal actuators in flanges and webs a modified linear beam theory basing on Vlasov and Chandra and Chopra was presented. The results show very good correlation with results from FEM-analysis. But additional effects coming from boundary conditions like constraining of out-of-plane warping lead to other optimum lay-up of flanges and webs than predicted in the analysis of smart laminates as was to be expected. At least conclusions can be drawn from the presented investigations for design of any kind of smart structures in static or dynamic case to achieve maximum influence of the actuators onto the structure. Because of the used analogy to thermal problems the results and the presented beam-theory can directly be transfered to anisotropic structures under thermal loads.

APPENDIX

The elements of stiffness matrix $[K^*]$ for an open-section I-beam with tension-torsion-coupled flanges of width *b* and web of height *h* (Index I) can be expressed following (1) and [5,6]:

$$K^*_{11_I} = \oint A^*_{11_I}\, ds = \left(A_{11_I} - \frac{A^2_{12_I}}{A_{22_I}} \right) (2b+h) \tag{9}$$

$$K^*_{15_I} = -2 \oint B^*_{13_I}\, ds = -2 \left(B_{13_I} - \frac{A_{12_I} B_{23_I}}{A_{22_I}} \right) (2b+h) \tag{10}$$

$$K^*_{55_I} = 4 \oint D^*_{33_I}\, ds = 4 \left(D_{33_I} - \frac{B^2_{23_I}}{A_{22_I}} \right) (2b+h) \tag{11}$$

$$K^*_{44_I} = \oint \omega^2 A^*_{11_I} - q^2 D^*_{11_I}\, ds = \frac{h^2 b^3}{24} \left(A_{11_I} - \frac{A^2_{12_I}}{A_{22_I}} \right) - \left(\frac{b^3}{6} + \frac{h^3}{12} \right) \left(D_{11_I} - \frac{D^2_{12_I}}{D_{22_I}} \right) \tag{12}$$

The corresponding modified active loads can be found in the same way as:

$$N^*_{\Lambda_I} = \oint n^*_{\Lambda x_I}\, ds = \left(n_{\Lambda x_I} - n_{\Lambda y_I} \frac{A_{12_I}}{A_{22_I}} \right) (2b+h) \tag{13}$$

$$T^*_{\Lambda_I} = -2 \oint m^*_{\Lambda xy_I}\, ds = -2 \left(m_{\Lambda xy_I} - n_{\Lambda y_I} \frac{B_{23_I}}{A_{22_I}} \right) (2b+h) \tag{14}$$

For the single-cell beam of width b and height h with tension-shear-coupled flanges (Index □) one gets with $F(s)$ as circuit St. Venant shear flow:

$$K^*_{11_\square} = \oint A^*_{11_\square}\, ds = \left(A_{11_\square} - \frac{A^2_{12_\square}}{A_{22_\square}} \right) (2b + 2h) \tag{15}$$

$$K^*_{15_\square} = \oint A^*_{13_\square} \frac{F(s)}{t}\, ds = \left(A_{13_\square} - \frac{A_{12_\square} A_{23_\square}}{A_{22_\square}} \right) 2bh \tag{16}$$

$$K^*_{55_\square} = \oint A^*_{33_\square} \frac{F(s)^2}{t^2} + 4\, D^*_{33_\square}\, ds = \left(A_{33_\square} - \frac{A^2_{23_\square}}{A_{22_\square}} \right) \frac{2b^2h^2}{b+h} + \left(D_{33_\square} - \frac{D^2_{23_\square}}{D_{22_\square}} \right) 8(b+h) \tag{17}$$

$$K^*_{44_\square} = \oint \omega^2 A^*_{11_\square} - q^2 D^*_{11_\square}\, ds = \frac{h^2 b^2 (b-h)^2}{24(b+h)} \left(A_{11_\square} - \frac{A^2_{12_\square}}{A_{22_\square}} \right) - \frac{b^3 + h^3}{6} \left(D_{11_\square} - \frac{D^2_{12_\square}}{D_{22_\square}} \right) \tag{18}$$

and

$$N^*_{\Lambda_\square} = \oint n^*_{\Lambda x_\square}\, ds = \left(n_{\Lambda x_\square} - n_{\Lambda y_\square} \frac{A_{12_\square}}{A_{22_\square}} \right) (2b + 2h) \tag{19}$$

$$T^*_{\Lambda_\square} = \oint n^*_{\Lambda xy_\square} \frac{F(s)}{t}\, ds = \left(n_{\Lambda xy_\square} - n_{\Lambda y_\square} \frac{A_{23_\square}}{A_{22_\square}} \right) 2bh \tag{20}$$

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