Design of Multiaxial Warp Knitted Fabrics for Required Tensile Properties in Composite Applications

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Summary: Multiaxial Warp Knitted (MWK) fabrics play an important role in the field of industrial applications due to their desired mechanical properties. Since the beginning of 1990’s, MWK fabrics have been widely used for structural composites to produce aerospace components, marine parts and automotive frames and become the preferred 3D textile composite preform [1]. Although some reports can be found on the production technology, general structure and properties of MWK fabrics as well as their composites[2-6], there is not so far any model dealing with the design of mechanical properties in terms of fabric structure parameters. This greatly restrains the application of this kind of fabrics. In this paper, a uniaxial tensile model is established, and a formula for calculating tensile modulus of the fabric in any direction is presented. With this model, the tensile properties of MWK fabrics can be designed to meet the particular requirements for composites applications.

Keywords: Multiaxial Warp Knitted Fabrics, Tensile properties, Fabric design parameters, 3D composite preform.

INTRODUCTION

As one of the 3D composite preforms, multiaxial warp knitted (MWK) fabrics are attracting more and more interest due to their low cost, high production efficiency, structural integrity, flexibility in design and the improved through-the-thickness strength for structural composites such as aerospace quality components, marine parts and automotive frames. As shown in Fig.1, a MWK fabric normally consists of five yarn systems, 4 for inserting yarns, namely, warp(0°), weft(90°), bias(±θ₀) yarns, which are held together by the fifth system—the stitching system through the thickness of the fabric.

![Fig. 1 A MWK fabric with 4 inserting yarn systems (Tricot stitch)](image-url)
There are many possibilities to design a MWK in terms of fabric density, yarn tensile properties and bias angle $\theta_0$ (can vary between $30^\circ$ and $90^\circ$). However, there are few reports to link quantitatively fabric tensile properties with fabric design parameters. This greatly restrains the application and designability of this kind of fabrics. In this paper, a uniaxial tensile model is established, and a formula for calculating tensile modulus of the fabric in any direction is presented. By using this model, a fabric can be designed for the required tensile properties in composite applications.

MODELLING OF UNIAXIAL TENSILE BEHAVIOUR

Before we can design fabric tensile properties, it is necessary to establish a model to describe its tensile behaviour in terms of fabric structure. Under the assumption that the MWK fabric is in a small planar strain state and the yarns in each inserting system remain straight and parallel to one another both before and after deformation, modeling of tensile stress-strain relationship is based on the simplified fabric structure in Fig.2 where the coarse solid lines represent inserting yarns in the deformed state, and the fine ones are those in the undeformed state.

According to Fig.2, it is easy to obtain the following equation:

$$\varepsilon = \frac{1}{2} \frac{\Delta L + \frac{1}{2} \Delta L}{L_0} = \frac{\Delta L}{L_0}$$

where $\Delta L$-fabric elongation; $L_0$ orginal fabric length. In order to get the relationship between $\varepsilon$ (fabric strain) and $\varepsilon_i$ (yarn strain, i=1~4), here we take the warp inserting yarn as an example, as shown in Fig.3.

**Fig.2 The uniaxial tensile deformation of a MWK fabric**

**Fig.3 The tensile deformation of a single warp inserting yarn**
The relationship between $\varepsilon$ and $\varepsilon_1$ can be obtained from plane-strain transformation (from off-axis to on-axis) in elasticity[7]. The formula for transformation is as below(refer to the coordinates in Fig.(4)):

$$
\begin{bmatrix}
\varepsilon_m \\
\varepsilon_n \\
\frac{1}{2} \gamma_{mn}
\end{bmatrix}
= \begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\frac{1}{2} \gamma_{xy}
\end{bmatrix}
$$

(2)

where:

$$
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
$$

For low strain uniaxial tensile deformation, Eqn.[2] can be simplified into Eqn.[3], in which the Poisson's ratio and shear strain are ignored both off-axis and on-axis and only the tensile strain along the yarn's axis is particularly emphasized.

$$
\begin{bmatrix}
\varepsilon_m \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
0 \\
0
\end{bmatrix}
$$

(3)

We can have:

$$
\varepsilon_i = \varepsilon \cos^2 \theta_i \quad (i=1\sim4)
$$

(4)

When a MWK fabric is subjected to a uniaxial stretch, the forces exerting on a single inserting yarn can be illustrated in Fig.4.

![Fig.4 The forces exerting on a single inserting yarn](image)

If $F_{isR} = (F_{is} + f_{is}) / \cos \theta_i$, $n_i = n_i \cos \theta_i$ (i=1~4), and a single inserting yarn's load-elongation relationship can be represented by $F_{is} = f_i(\varepsilon_i)$ (i=1~4), from Fig.3 and Fig.4, Equ.5 can be obtained:

$$
F = \frac{1}{W_0} \sum_{i=1}^{4} n_i \left[ f_i(\varepsilon_i) + f_{is}(\varepsilon_i) \right] (W_0 - L_0 \tan \theta_i) + F_L(\varepsilon)
$$

(5)

In Eqn.(5), $n_i$ – fabric density (/cm), $W_0$, $L_0$ for fabric specimen width and length, $f_i(\varepsilon)$ and $F_L(\varepsilon)$ for the inserting yarns' tensile stress-strain relationship and the stitching system's behavior respectively. On the right hand side of Eqn.(5), all the terms can be determined from simple tests except for $f_{is}(\varepsilon)$ and $F_L(\varepsilon)$. When the fabric is subjected to small strain, the terms $f_{is}(\varepsilon)$ and $F_L(\varepsilon)$ may be neglected, then we have:

$$
F = \sum_{i=1}^{4} n_i \left[ f_i(\varepsilon_i) + f_{is}(\varepsilon_i) \right]
$$

(6)
DESIGN OF TENSILE MODULUS OF MWK FABRICS

When Eqn (6) is used, fabric tensile modulus can be predicted by using fabric structural parameters and yarns uniaxial tensile stress-strain relationship. The following are some examples to show the ways applying the model established above.

If the four inserting yarn systems have exactly the same density (n) and tensile modulus, then Eqn.[6] can be rewritten as Eqn.[7]:

\[ F = nE_i \sum_{i=1}^{4} \varepsilon_i \]  

Substitute Eqn. (4) in Eqn.(7), Eqn.(8) can be obtained:

\[ F = nE_L \varepsilon \sum_{i=1}^{4} \cos^2 \theta_i \]  

From Eqn.(8), we can conclude Eqn.(9) as below:

\[ E = nE_L \sum_{i=1}^{4} \cos^2 \theta_i \]  

Let \( \frac{dE}{d\theta} = 0 \), we can get \( \theta \)'s for \( E_{\text{max}} \) (maximum) and \( E_{\text{min}} \) (minimum) just as below:

\[ \sin 2\theta \cos 2\theta_0 = 0 \]  

\[ E = \begin{cases} 
E_{\text{max}}, \text{when } \theta = \frac{\pi}{6}, & \frac{\pi}{6} < \theta_0 < \frac{\pi}{4} \\
\frac{\pi}{2}, & \frac{\pi}{4} < \theta_0 < \frac{\pi}{2} \\
E_{\text{min}}, \text{when } \theta = \frac{\pi}{2}, & \frac{\pi}{2} < \theta_0 < \frac{\pi}{4} \\
\frac{\pi}{2}, & \frac{\pi}{4} < \theta_0 < \frac{\pi}{4} 
\end{cases} \]  

\[ E = 2nE_L \left( \theta_0 = \frac{\pi}{4} \right) \]  

According to Eqn.(12), we can conclude that the fabric is nearly isotropical when \( \theta_0 = \frac{\pi}{4} \). Three polar diagrams for Eqn.(9) are given in Fig.5.

\[ (a) \theta_0 = 38^\circ \times 3.1415927/180^\circ \]
Fig. 5 The tensile moduli in different directions of a MWK fabric

The above figures show that the tensile moduli along different directions with \( \theta_0 = 38^\circ \), \( \theta_0 = 45^\circ \) and \( \theta_0 = 75^\circ \) for bias yarns in fabric design. Other parameters such as fabric density, different yarn tensile properties can be used to predict fabric tensile properties. The model is of great value in the design and engineering application of MWK fabrics for required tensile properties.
CONCLUSIONS

As far as MWK fabrics are concerned, there are many possibilities to design a MWK in terms of fabric density, yarn tensile properties and bias angle $\theta_0$ (can vary between $30^\circ$ and $90^\circ$). However, there are few reports to link quantitatively fabric tensile properties with fabric design parameters. This paper presents a simplified method dealing with deformation using macroscopic structure of MWK fabrics, from which a uniaxial tensile model is established and a formula for calculating tensile moduli in any direction is worked out when neglecting the effect of frictional force and the stitching system. MWK fabrics have low cost, high production efficiency, structural integrity, flexibility in design and the improved through-the-thickness strength for structural composites such as aerospace quality components, marine parts and automotive frames. The model for uniaxial stress-strain relationship and the formula for tensile modulus are of great value in the design and engineering application of MWK fabrics.

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REFERENCES