THE BINARY MODEL – A COMPUTATIONAL APPROACH TO TEXTILE COMPOSITES

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SUMMARY: This paper describes the development of a computer code called the Binary Model for modeling textile composites. The model has the features that it needs no assumption of periodicity in the textile structure, can deal with significant volumes of a structure (typically ~ 10^3 – 10^4 mm^3), treats arbitrary external loads including loads with large spatial gradients, invokes arbitrary nonlinear constitutive laws, and, in its most recent version, computes thermal strains and temperature distributions as well as stress distributions. Here the formulation of the model is reviewed and some of its capabilities illustrated by computations of the effects of random disorder in the textile structure on stress distribution in the elastic regime and simulations of the progression to ultimate failure in a textile composite in which some unusual failure phenomena are known to occur.

KEYWORDS: textiles, computational models, nonlinearity, failure, structural modeling

INTRODUCTION

One of the most challenging and interesting problems in textile composites is calculating in a tractably efficient scheme the distribution of loads among the constituent fiber tows. Because of the geometrical complexity of the tow arrangements, the structure is generally nonperiodic; or, if it is periodic, the unit cell (repeating unit) typically contains tens or hundreds of tow segments [1]. It is not feasible to represent each of these tow segments by a large number of finite elements to ensure accuracy of local stresses in all their detail and then solve for whole system. Therefore, conventional finite element gridding methods are not applicable. On the other hand, many of the failure mechanisms in textile composites involve the failure of single tows, so the physics of failure cannot be faithfully represented faithfully if stresses are averaged over distances larger than a tow diameter; i.e., the material cannot be homogenized over gauge lengths exceeding the tow diameter.

Models of the interaction of observed local failure mechanisms must be based on the most efficient finite element formulation possible that distinguishes individual tows while representing their three-dimensional pattern of interlacing correctly. One such model, called the Binary Model, has been developed for work on polymer and ceramic composites. The Binary Model is a finite element formulation in which the geometry of the 3D textile architecture is represented in one-to-one correspondence, while the element size is chosen to be sufficiently coarse to enable experimentally significant volumes of a 3D textile to be simulated [2,3]. By careful definition of the elements' constitutive properties, the mechanics of deformation can be made approximately independent of the choice of element size.

The Binary Model assigns a string of one-dimensional elements to each tow, which represent the tow's axial stiffness. The tow elements are embedded in solid, so-called effective medium elements, which represent matrix dominated composite properties, including transverse stiffness, shear stiffness, and Poisson's effect. Figure 1 shows tow elements representing stuffers, fillers, and warp weavers in a through-the-thickness angle interlock weave. The tow elements are located along a path corresponding to the axis of the tow they represent in the actual composite. A single stuffer or filler element extends a distance equal to the centre-to-centre separation of neighbouring tows. The warp weavers are idealized as following saw-tooth paths in the model depicted, with a single element extending through the thickness.

As shown in Fig. 1, an effective medium element also usually extends in any direction from the centre of one tow to the centre of one of its neighbours. It therefore represents a volume of material containing fragments of more than one tow, usually with different orientations. In interlock weaves, effective medium elements are conveniently defined as eight-noded cuboidal elements containing eight Gaussian quadrature points and whose vertices coincide in the elastic regime with the nodes of either stuffer or filler tow elements.

In its most general formulation, the Binary Model can deal with nonlinearity due to distributed damage, such as microcracking and plasticity, as well as tow rupture, shear load transfer around tow breaks, and tow pullout. These are all essential mechanisms in the ultimate tensile failure of interlock weaves. The model is often used to execute Monte Carlo simulations of specimens or structures, with the strength and spatial distribution of a large number of flaws assigned according to various trial distribution laws. Such simulations offer considerable insight into the statistical nature of damage development in textile composites.

THE ELASTIC REGIME

A short segment of a tow in a textile composite is effectively a unidirectional composite. Therefore, constitutive laws for tow and effective medium elements in the elastic regime can be defined in terms of the properties expected for a unidirectional composite [3], which can be calculated a priori from the properties of the constituent fibers and matrix using micromechanical models available in the literature. In assigning volumes to tow segments and fiber volume fractions in this process, the most important constraint is that the total count of fibers in the entire composite and their orientations should be correct. The assignments should therefore usually involve an experimental measurement of the total fiber volume fraction as a check on weaver's specifications, which are often an inaccurate representation of the processed composite [3].

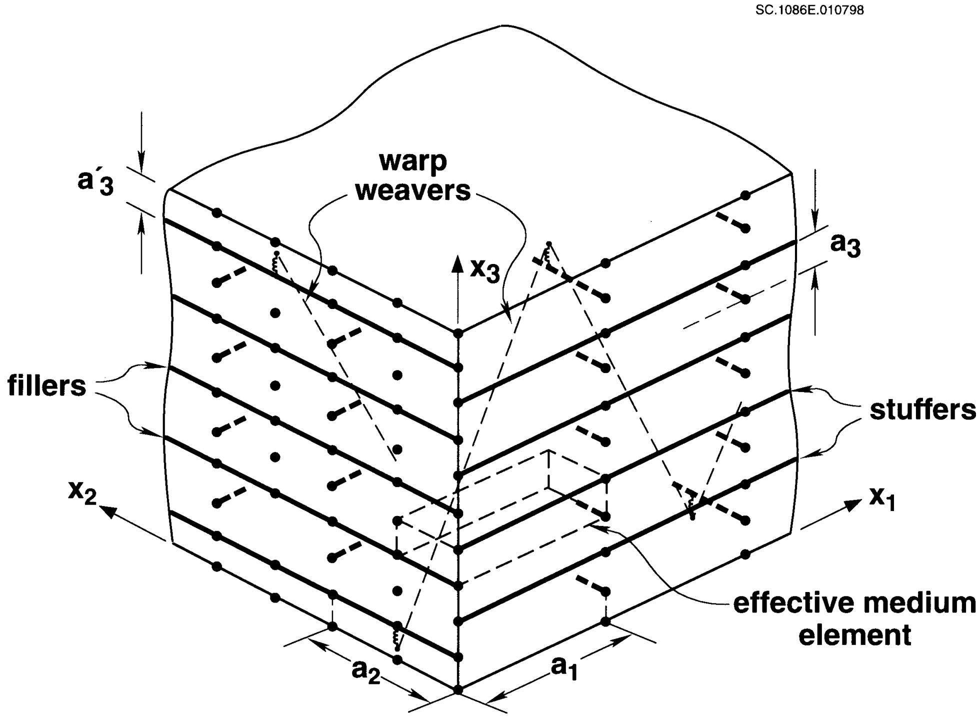

Fig. 1: Small part of a Binary Model element array for a simulation of a through-the-thickness angle interlock weave. All tow elements are shown but only one representative effective medium element.

Since effective medium elements usually extend from the centre of one tow to the centres of its neighbors in the textile composite (e.g., Fig. 1), each effective medium element represents fragments of different tows with generally different orientations. Nevertheless, since the effective medium elements represent only those components of elasticity that are matrix dominated (shear and transverse stiffness and local Poisson's effects), their properties may be assigned by approximate rules without compromising predictions for the textile composite. For example, with (x, y, z) a local coordinate system aligned so that the x-axis is parallel to the fiber direction in a unidirectional composite segment and (x_1, x_2, x_3) a global coordinate system defined as shown in Fig. 1, the engineering elastic constants of effective medium elements can be defined for the interlock weave of Fig. 1 by

$$E_1^{(m)} = E_2^{(m)} = E_3^{(m)} = E_y^{(UD)}$$

$$G_{12}^{(m)} = G_{xy}^{(UD)}; \quad G_{23}^{(m)} = \frac{\left(p_f G_{xy}^{(UD)} + p_s G_{yz}^{(UD)}\right)}{p_s + p_f}; \quad G_{31}^{(m)} = \frac{\left(p_s G_{xy}^{(UD)} + p_f G_{yz}^{(UD)}\right)}{p_s + p_f}$$

$$\nu_{12}^{(m)} = \nu_{xy}^{(UD)}; \quad \nu_{23}^{(m)} = \frac{\left(p_f \nu_{xy}^{(UD)} + p_s \nu_{yx}^{(UD)}\right)}{p_s + p_f}; \quad \nu_{13}^{(m)} = \frac{\left(p_s \nu_{xy}^{(UD)} + p_f \nu_{yx}^{(UD)}\right)}{p_s + p_f} \tag{1}$$

where p_s and p_f are the proportions of the effective medium element comprised of stuffer and filler tows and the superscripts "m" and "UD" denote properties of the effective medium element and unidirectional composite respectively. The component of stiffness for the UD composite chosen to define the contribution of a stuffer or filler to a component of elasticity for the effective medium element depends on the orientation of the stuffer or filler fragment with respect to the global coordinate system.[1] In angle interlock composites, the warp weaver tows occupy a relatively small fraction of the composite volume and may be neglected in defining the effective medium elements. The spring constants for stuffer and filler tow elements are prescribed by

$$k_{\mathrm{s}} = D_{\mathrm{s}}\left(E_{\mathrm{x}}^{(\mathrm{UD})} - E_{\mathrm{y}}^{(\mathrm{UD})}\right);\;\; k_{\mathrm{f}} = D_{\mathrm{f}}\left(E_{\mathrm{x}}^{(\mathrm{UD})} - E_{\mathrm{y}}^{(\mathrm{UD})}\right);\;\; k_{\mathrm{w}} = D_{\mathrm{w}}\left(E_{\mathrm{x}}^{(\mathrm{UD})} - E_{\mathrm{y}}^{(\mathrm{UD})}\right) \quad (2)$$

where D_{s}, D_{f}, and D_{w} are the effective cross-sectional areas of the stuffer, filler and warp weaver tows, computed so as to preserve total fiber volume counts. Since the effective medium elements fill all space while the tow elements are one-dimensional line elements (zero volume), the term $E_{\mathrm{y}}^{(\mathrm{UD})}$ must be subtracted to avoid double counting of stiffness.

In the example of the angle interlock weave depicted in Fig. 1, in-plane composite moduli and Poisson's ratios will be dominated by the axial extension of fibers in the stuffers and fillers and therefore by the properties of the tow elements. Thus the definition of in-plane shear moduli and Poisson's ratios for the effective medium elements cannot be too critical to overall in-plane properties. In a well-designed composite, a significant fraction of all fibers will be aligned with any anticipated large load and therefore the Binary Model is assured to give reasonable estimates of the composite response to such loads. However, other loads, which may be minor but still important as design limits, such as through-thickness loads in the interlock composite of Fig. 1, may not be aligned with large numbers of fibers. Details in the assignment of the effective medium element properties are then more important.

Extensive comparisons have been made of the predictions of the Binary Model for interlock weaves with experimental measurements of stiffness components and the predictions of a simple, analytical model based on assuming isostrain conditions throughout the composite [4]. The Binary Model agrees well with the experimental data in all elasticity components. It also agrees well with the isostrain model for the in-plane stiffness components, which is to be expected since these correspond to aligned loading, where near isostrain conditions will hold to a good approximation. But significant differences with the isostrain model are found for the through-thickness modulus and other stiffness components.

If the tows in the textile composite are assumed in the models to be ideally straight (as depicted in Fig. 1), then there are also consistent discrepancies with experiment due to the fact that tows in the real composites are wavy and otherwise irregular. Irregularity is an inevitable feature of textile composites, due to the rigours of textile manufacturing processes. The effects of irregularity can easily be studied using the Binary Model.

Large scale irregularity in the positioning of tows can be replicated in the Binary Model by offsetting tow element nodes. The offsets can be chosen in a Monte Carlo simulation according to any prescribed distributions. The effects of small scale irregularities (over scales less than the tow element length) can be incorporated by modifying the constitutive laws for tow elements. Some illustrations of the effects of large-scale irregularities in the elastic regime are shown in Fig. 2. This figure shows statistical inferences presented as cumulative probability distributions from large numbers of Monte Carlo simulations of specimens of

[1] Eq. (1) corresponds to assuming that isostrain conditions hold within a single effective medium element. For shear, one might prefer to assume isostress and Eq. (1) would be replaced by capacitance type laws (proportions of inverses).

dimensions approximately 10x 15 x 50 mm. In the simulations, the positions of the nodes of stuffer elements were offset in the direction x_3 to mimic the waviness observed in the through-thickness direction in angle interlock composites. The nodal offsets were chosen at random so that the stuffer elements deviated in alignment from the direction x_1 by an angle ξ that followed a normal distribution with mean zero and deviance σ_ξ. Figure 2a shows how the axial stress in tow elements was distributed for different values of σ_ξ. There is some variance in the axial stress even when $\sigma_\xi = 0$ (ideal geometry), because the warp weaver tows break the symmetry of the composite. But much greater variance exists when irregularity is present typical of that observed in interlock weaves (σ_ξ up to approximately 5° [4]).

Information is also available about stress variations in the effective medium elements, which can be interpreted as representing stress components in particular tows in the composite. For example, the shear stress component σ_{13} evaluated at the Gaussian quadrature points in the effective medium elements that surround a particular stuffer tow element and are closest to the stuffer tow element reveals the axial shear stress in the corresponding stuffer segment in the composite. Figure 2b shows cumulative probability distributions for this axial shear stress component for the simulations in which stuffer nodes were offset to mimic irregularity. Once again there is variance even when $\sigma_\xi = 0$ (ideal geometry), due to the effect of warp weavers, but far higher variance ensues from irregularity, as tow elements become more or less aligned with the load axis. The axial shear stress in a tow segment is especially important in compressive failure, since it triggers kink band formation, which is the dominant compressive failure mechanism [5]. Figure 2b demonstrates that initial tow misalignment dominates the determination of compressive strength, rather than load-sharing interactions between tows of different type (represented by the results for $\sigma_\xi = 0$), in angle interlock weaves.

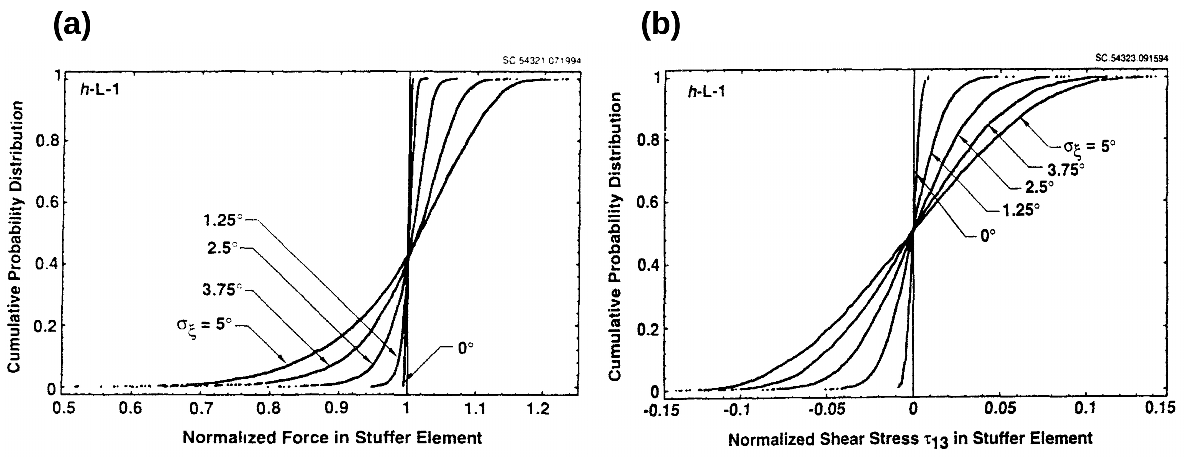

Fig. 2: Distributions of local stresses in an angle interlock weave in which stuffer segments are randomly misaligned (from [3]). (a) The axial stress and (b) the axial shear stress in stuffer segments.

SIMULATIONS OF ULTIMATE FAILURE IN POLYMER COMPOSITES

Under tensile loads, 3D interlock weaves fail by the rupture of the aligned, primary load-bearing tows. The ultimate strengths are satisfactory from the point of view of engineering applications, with 1 GPa easily attained with fairly balanced proportions of stuffers and fillers. But the most remarkable property is the work of fracture in tension, which has been measured in the range 0.4 – 1.1 MJm^{-2} [6]. This is considerably higher than values for the toughest metal alloys (< 200 kJm^{-2}) or tape laminates (< 150 kJm^{-2}) [7]. Such large values of work of fracture point to many applications for 3D interlock weaves where high damage tolerance is demanded, including armour and explosion containment.

The high work of fracture is associated with the ability of specimens to bear loads near the peak load, ~ 1 GPa, at strains of up to 3 – 4%, which significantly exceed the nominal failure strain of the tows (≈ 1.5%) [6]. Destructive examination of specimens at strains exceeding 1.5% reveals that all or at least most of the primary load bearing tows are ruptured, even though the composite stress at the time of interrupting the test was ~ 1 GPa. Thus some unusually effective mechanism exists for transferring load from one end of a specimen to the other past sites of tow rupture. The mechanism of load transfer was conjectured in prior work to involve the locking up of sliding, load-bearing tows against one another. The effectiveness of the lockup could be enhanced by the contact of asperities on tows under the constraining influence of the warp weaver tows [6].

Along with lockup, the spatial distribution of the sites of tow rupture must also have an important role. If, for example, all load-bearing tows rupture on a single plane, then the specimen must fail catastrophically at modest strain. Spatially distributed tow breaks will in contrast favour ductility and high work of fracture.

The Binary Model can be generalized to deal with nonlinearity due to distributed damage, such as microcracking and plasticity, as well as tow rupture, shear load transfer around tow breaks, and tow pullout [8]. These are the essential mechanisms in the ultimate tensile failure of interlock weaves. The generalized model can be used to execute Monte Carlo simulations of the entire gauge sections of the specimens that were previously tested. Within each Monte Carlo simulation, the strengths and locations of flaws are assigned according to various trial distribution laws. The simulation then predicts the full test life of the specimen.

Sliding and load transfer effects around a broken stuffer are simulated in the Binary Model by allowing the stuffer tow elements near a break to displace relative to the surrounding effective medium nodes and applying opposing friction forces to the displaced nodes (Fig. 3). When the broken tow slides in the real composite, its irregular features push the composite apart in the through-thickness direction. This effect is reproduced in the model by introducing a dilation strain, ε_s, in the through-thickness direction that is proportional to the computed sliding displacement, u_1, of the i^{th} stuffer element:

$$\varepsilon_3^{(\mathrm{i})} = \eta u_1^{(\mathrm{i})} \tag{3}$$

where η is a geometrical factor proportional to the misalignment of the stuffer segment. The constitutive law for the neighbouring effective medium elements is then modified to introduce ε_s as a stress-free eigenstrain:

$$\sigma_{\mathrm{ij}}^{(\mathrm{m})} = C_{\mathrm{ijkl}}^{(\mathrm{m})} (\varepsilon_{\mathrm{kl}}^{(\mathrm{m})} - \varepsilon_{\mathrm{s}}^{(\mathrm{i})} \delta_{\mathrm{k3}} \delta_{\mathrm{l3}}) \tag{4}$$

where $\sigma^{(\mathrm{m})}$, $\varepsilon^{(\mathrm{m})}$, and $C^{(\mathrm{m})}$ are the stress, strain, and stiffness tensors for the effective medium element, the last defined consistently with Eq. (1). In the next step of the simulation, the Binary Model computes the local stress component, σ_3, in the through-thickness direction in the effective medium elements surrounding the sliding stuffer element. Because the warp weavers constrain dilation in the through-thickness direction (until they break), the computed stress will usually be compressive, with a magnitude that increases with the sliding displacement. Friction forces will imply a stress differential, $\Delta\sigma_1$, between the two ends of a sliding stuffer element, which is introduced into the model as a force, Q_s, imposed at one end of the stuffer element and to the corresponding effective medium element (Fig. 3). Assuming Coulomb friction, the friction force will have the magnitude

$$Q_s^{(i)} = r\mu\left|\sigma_3^{(i)}\right| \tag{5}$$

where μ is a friction coefficient and r is a geometrical factor that can be so defined as to make the outcome independent of the choice of the stuffer element size [2,8].

Simulated stress-strain curves and other fracture characteristics have been reported in detail elsewhere for various distributions of the strength and location of flaws in stuffers [8]. Generally stuffer rupture events occur over a range of applied strains, which increases in breadth when flaws are more broadly distributed in strength. At some critical strain, every stuffer is broken and the slip zones emanating from the rupture sites all cross a single plane normal to the load axis. At this point, further global load increase is impossible: the rupture sites define the ultimate failure surface and there is typically a large drop in the applied load, the so-called primary load drop, as seen in experiments. Only small loads remain, corresponding to the pullout of stuffers in the absence of the constraining influence of the warp weavers, which are always broken following the primary load drop.

If the stuffer flaws are distributed in space according to a Poisson process, then the large strains measured in some experiments after stuffer rupture but before the primary load drop cannot be replicated, whatever the distribution of flaws in strength. Neither can large strains before primary load drop be achieved if flaws are associated with particular sites in the interlock architecture (such as stuffer segments adjacent to the turning points of warp weavers) if these sites are also fairly uniformly distributed throughout the composite.

However, if flaws are located preferentially on two or more planes normal to the load direction, then a failure sequence arises that is very similar to that observed experimentally. Stuffers tend to fail alternately first on one plane, then on the other, with no more than three or four successive failure events occurring on the same plane. Long slip zones and strong lockup effects develop, so that even after every stuffer has failed, the total load remains near the ultimate strength. A typical stress-strain curve for such a simulation is shown in Fig. 4a. The primary load drop follows the last point for which the load has been calculated.

In the simulation, the primary load drop occurs when the warp weavers reach their assigned failure strains. Thereupon, constraint is lost, the clamping stress around sliding stuffers falls to a small value, and the friction forces become much smaller. Experimental evidence supports this depiction of the controlling role of the warp weavers [6]. At small composite strains, prior to stuffer rupture, the composite is observed to contract in the through-thickness direction. After stuffer rupture, in contrast, it begins to swell and soon exhibits a negative Poisson's effect. In specimens sectioned before the primary load drop, the warp weavers are always found to be intact, whereas following the primary load drop, they have always failed [6]. The evolution of Poisson's ratio, ν_{13}, through the simulation of Fig. 4a is shown in Fig. 4b. It corresponds well with experiment.

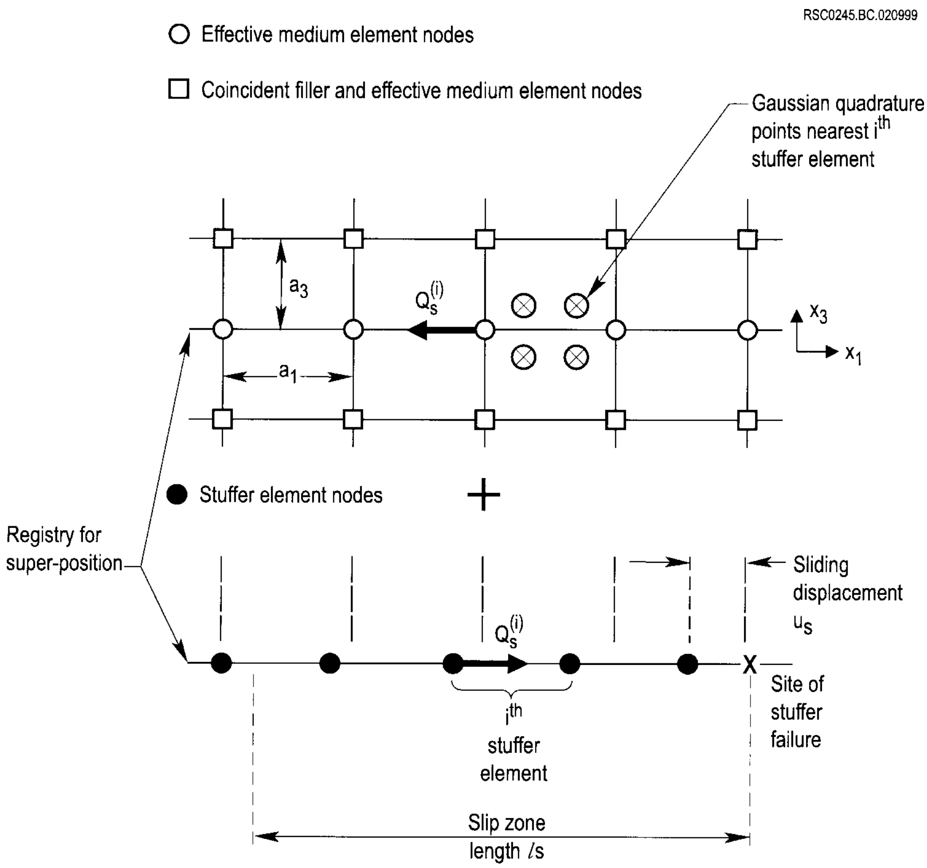

Fig. 3: Schematic showing the separation of stuffer and effective medium nodes when a stuffer slides following rupture. The effective medium and stuffer elements are drawn separately but are superimposed in the model. The stuffer elements are sliding to the left away from the rupture. The sliding displacement disappears at the left end of the slip zone. Friction forces couple the displaced stuffer and effective medium nodes.

While falling short of a rigorous proof, the Binary Model simulations quantify the roles of various material and geometrical factors in determining the work of fracture of the 3D interlock weaves. The highest values of work of fracture arise when high loads endure significantly beyond the failure strain of individual tows. This phenomenon has been shown to require an unusual spatial distribution of flaws in stuffers, as well as a strong lockup effect during the sliding of ruptured stuffers through the surrounding composite. The required spatial distribution of flaws is that planes of flaws should exist separated by 10 – 20 mm. The mechanism for the creation of such planes during the manufacture of a textile composite is unknown, but their existence is certainly feasible. For example, variations in tensioning or in the beating up of weft yarns during the weaving process can lead to bands of shear or buckling defects in the dry fiber preform. Furthermore, post-mortem studies of the specimens tested in [6] suggest that flaws populated planes or near-planes. For designing materials with the highest possible work of fracture, a systematic method of introducing flaws of known strength at preferred locations would be very useful.

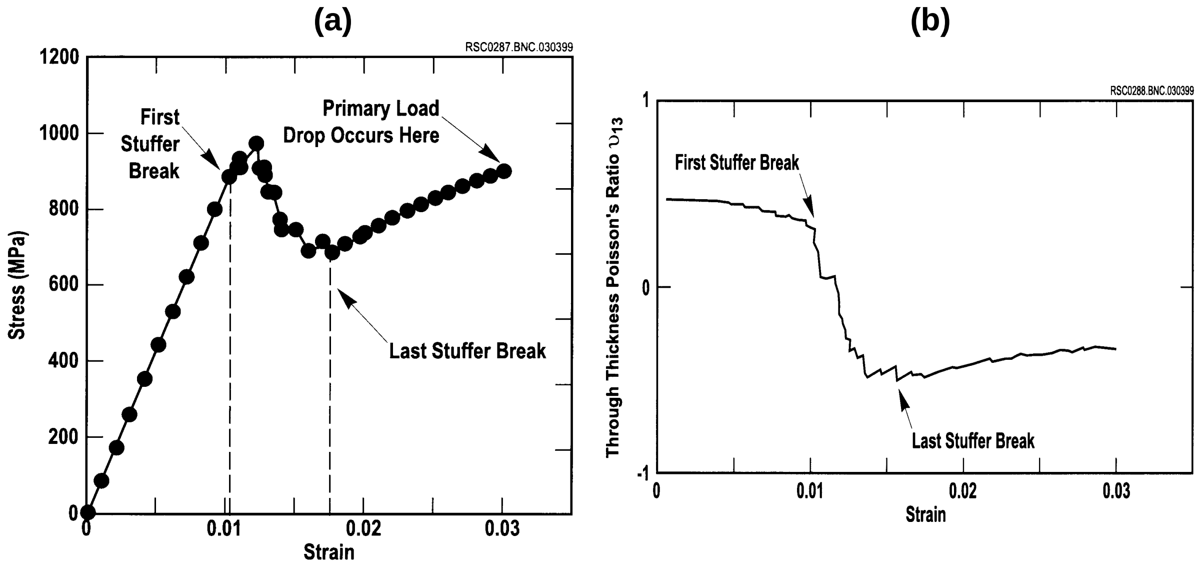

Fig. 4: A simulation of the tensile rupture of an angle interlock composite, showing the effects of distributed flaws and a frictional lockup mechanism. (a) Tensile stress-strain curve up to the point of warp weaver failure. (b) Poisson's effect in the through-thickness direction.

Regardless of the spatial distribution of flaws, the three-dimensional nature of the reinforcement in angle interlock weaves has a profound influence on the work of fracture through the lockup mechanism or enhanced friction. Even when high loads extend only slightly beyond the failure strain of individual tows, usefully large values of the work of fracture are obtained, which are attributable to high friction prior to the primary load drop and lesser but still substantial pullout loads following it.

CONCLUDING REMARKS

The Binary Model has evolved through a number of research tasks carried out over the last six years into a powerful research tool. Grid generation routines are now available to set up complex textile architectures, including integrally formed structures created by weaving or other textile processes. The model appears currently to be unique in its ability to treat complex structures in which load distributions are nonuniform and either no periodicity exists or the repeating unit is very large.

Significant current effort exists in developing constitutive laws and model formulations to simulate ceramic composite structures that operate at high temperature. Constitutive laws are now available to deal with nonlinearity due to matrix microcracking, thermal strains, and damage modified thermal conductivity to solve coupled heat transfer and stress problems. The preferred approach to certifying the accuracy of the model in such complex material/structure problems is to validate results by direct comparison with full field measurements, for example of displacements and temperature fields.

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