ORIENTATION PROCESS DURING EXTRUSION OF REACTIVE POWDER CONCRETE REINFORCED WITH SHORT METAL FIBERS

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SUMMARY: This paper addresses the problem of orienting short metal fibers (1cm length, aspect ratio 50) in a reactive powder concrete during an extrusion process dedicated to manufacture reinforced tubes. It is shown that in choosing an appropriate geometry for the die it is possible to orient the fibers in an ortho-radial direction which is the most efficient if one whishes to reinforce the tube mechanical properties for an internal pressure loading. This result is obtained with both a theoretical and an experimental approach. In a theoretical framework we carried out a numerical modeling with a constitutive behavior deduced from the anisotropic suspensions theory. This calculus anabled us to imagine the shape of the die. Tubes are then extruded and the fibers orientation is measured through optical microscopy.

KEYWORDS: Reactive powder concrete, short fibers, extrusion, numerical modelling, experiments, orientation prediction.

INTRODUCTION

The Reactive Powder Concrete has high performances (from 150 to 200 MPa in compression) The main reasons for this are (i) its reduced porosity and (ii) the strength of the interactions between its constituents. However, in order to avoid a fragile fracture, it has been shown that well oriented short metallic fibers can enhance significantly its tension resistance and its ductility. One of the goal of a forming process for these materials is therefore to control the fibers orientation in order to enhance the mechanical properties in the right directions. This paper whishes to show for an example that it is possible to achieve this control through a correct design of the tools. The choice of the forming process is here tube extrusion. We will show in the following that (i) it is possible to orient the fibers along the circumference of the tube and (ii) the optimization parameters are both the shape of the die and the surface conditions (friction, sliding or sticking). First section is devoted to the equations, second section describes the numerical procedure that have been implemented to solve them, third section gives some numerical results which are compared to experiments.

EQUATIONS

Material

We consider a reactive powder concrete at a paste state. The general idea that yields to this concrete formulation is (i) to adjust the concrete granulometry in a polydisperse way so that the volumic fraction of the grains in the mixture is large (ii) to put enough water (but not to much) so that the chemical hydration reaction can occur but certainly not more than necessary to saturate the reaction. In this way in one hand the possible fracture initiations that could appear in the pores are minimized and in an other hand the cohesion of the material is optimized. The different constituents of our concrete was :

- Sand
- Cement
- Quartz
- Silicium oxyde slurries
- Water
- Ad hoc adjuvents
- Short metallic fibers (a few percent in volume)

A mechanical modeling deduced from suspension theory through polymer processing

There is a wide literature on the mechanical modeling of short fibers reinforced thermoplastics [1]. The equations are written in the framework of suspensions of non spherical particules immersed in a newtonian fluid. The reality for thermoplastic matrix composites is very different (for example because of the visco-elasticity of the matrix or because of the high volume fraction of fibers) but it turns out that the orientation of fibers is satisfactory described with this approach. Following what is done for thermoplastics, we will therefore consider here that the fiber filled concrete at a paste state can be described as a suspension for which the fluid would be the concrete paste and the particules the metallic chopped fibers. This model does pretend to predict very accurately the stresses but intend to give a first approximation to predict the fibers orientation.

Balance equations

If one neglects inertia terms, if the materials constituants are assumed to be incompressible and if $\underline{\underline{\boldsymbol{\sigma}}}$ denotes the stress tensor and $\underline{\mathbf{v}}$ the velocity vector, the balance of mass masse and momentum can be written:

$$\mathrm{Div}\,(\underline{\mathbf{v}}) = 0 \qquad \text{and} \qquad \mathrm{Div}\,(\underline{\underline{\boldsymbol{\sigma}}}) = 0 \tag{1}$$

Constitutive relation

Let P denote the pressure, μ the isotropic viscosity, N_p the particule number (which caracterizes the capability of the material to become anisotropic). Let $\underline{\underline{\mathbf{Id}}}$ be the identity tensor, and $\underline{\underline{\mathbf{a}}}$ the orientation tensor. $\underline{\underline{a}}$ is a symmetric second order tensor ($a_{ij} = \langle p_i\ p_j \rangle$ where $\langle\ \rangle$ denotes an ensemble averaging at a given point, and $\underline{p}$ is the fiber axis direction). The physical meaning of this orientation tensor is that its eigenvalues quantify the probability for a fiber to be oriented along the associated eigenvector. The constitutive equation is given by:

$$\underline{\underline{\sigma}} = - P\, \underline{\underline{\mathbf{Id}}} + 2\mu \{\underline{\underline{\mathbf{d}}} + 2\, N_p\, Tr(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}})\, \underline{\underline{\mathbf{a}}}\} \qquad (2)$$

Eqn. 2 is a generalization of the newtonian behavior to fluid anisotropic materials: the stress tensor is a linear but non scalar function of the strain rate tensor. It means that if one knows the fibers orientation (that is $\underline{\underline{\mathbf{a}}}$), and the kinematics, one knows the stress tensor.

Orientation equation

In constrast with solid composite material analysis, the fibers orientation is not a datum here because the flow itself tends to orient the fibers. This phenomenon is accounted for through the orientation equation

$$d\underline{\underline{\mathbf{a}}}/dt = \mathbf{Grad}(\underline{\mathbf{v}})\, \underline{\underline{\mathbf{a}}} + \underline{\underline{\mathbf{a}}}\, (\mathbf{Grad}(\underline{\mathbf{v}}))^T - 2\, \mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}})\, \underline{\underline{\mathbf{a}}} \qquad (3)$$

For a general discussion of this modeling applied to composite materials, see for example [2] and the numerous associated references.

NUMERICAL SIMULATION

A fixed point strategy

We search for a numerical solution of a problem with the traction null on Γ_1 and with the velocity $\underline{\mathbf{v}} = \underline{\mathbf{v}}_{\mathbf{g}}$ on Γ_2. The boundary Γ_2 is subdivided into Γ_- and Γ_0. Through Γ_- the material is introduced into the domain and we impose an orientation tensor $\underline{\underline{\mathbf{a}}}^{\mathbf{0}}$ and a velocity vector ($\underline{\mathbf{v}}_{\mathbf{g}} \neq \underline{\mathbf{0}}$). The fluid leaves the domain though Γ_1 without any orientation condition prescribed. On the boundary $\Gamma_0 = \Gamma_2 - \Gamma_-$ we assume $\underline{\mathbf{v}}_{\mathbf{g}} = \underline{\mathbf{0}}$ (sticking condition) or both $\underline{\mathbf{v}}_{\mathbf{g}}\, \underline{\mathbf{n}} = 0$ and $(\underline{\underline{\sigma}}\, \underline{\mathbf{n}})\, \underline{\mathbf{t}} = 0$ (slip conditions), and no orientation condition is required.

The problem is defined as: *Find* ($\underline{\mathbf{v}}$, $\underline{\underline{\sigma}}$, $\underline{\underline{\mathbf{a}}}$) *satisfying simultaneously kinematic and static admissibility (Eqn. 4 and 5) as well as the constitutive relation and orientation equation (Eqn. 2, 3 with inflow boundary condition Eqn. 6)*

- Kinematic admissibility:

$$\underline{\mathbf{v}} = \underline{\mathbf{v}}_{\mathbf{g}} \text{ on } \Gamma_-$$

$$\underline{\mathbf{v}} = \underline{\mathbf{0}} \text{ (sticking condition) or } \underline{\mathbf{v}}\, \underline{\mathbf{n}} = 0 \text{ (slip condition) on } \Gamma_0$$

$$\mathbf{Div}\, \underline{\mathbf{v}} = \underline{\mathbf{0}} \text{ in } \Omega \qquad (4)$$

- Static admissibility:

$$\underline{\underline{\sigma}}\, \underline{\mathbf{n}} = \underline{\mathbf{0}} \text{ on } \Gamma_1$$

$$(\underline{\underline{\sigma}}\, \underline{\mathbf{n}})\, \underline{\mathbf{t}} = 0 \text{ on } \Gamma_0 \text{ (if a slip condition is assumed)}$$

$$\mathbf{Div}\, \underline{\underline{\sigma}} = \underline{0} \text{ in } \Omega \qquad (5)$$

- Inflow boundary condition

$$\underline{\underline{\mathbf{a}}} = \underline{\underline{\mathbf{a}}}^{\mathbf{0}} \text{ on } \Gamma_- \qquad (6)$$

For the solution of the coupled problem we introduce a fixed point algorithm and solve successively the following two steps until convergence:

Problem 1. In a first step, if the orientation tensor $\underline{\underline{\mathbf{a}}}$ is known, the problem to be solved is expressed as a constrained minimization problem:

Find $\underline{\mathbf{v}} \in U=\{\underline{\mathbf{v}}, \text{Div } \underline{\mathbf{v}} = 0, \ \underline{\mathbf{v}} = \underline{\mathbf{v}}_{\mathbf{g}} \text{ on } \Gamma_2\}$ *so that* $\mathbf{J}(\underline{\mathbf{v}})$ *reaches a minimum*, with :

$$\mathbf{J}(\underline{\mathbf{v}}) = \int_\Omega \mu \, \mathbf{Tr}(\underline{\underline{\mathbf{d}}}^2) + N_P \, (\mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}))^2 \, d\Omega$$

Problem 2 : In a second step, if the velocity field is known, the steady state orientation problem results a convection-type problem, which can be expressed as:

Find $\underline{\underline{\mathbf{a}}}$, *with* $\underline{\underline{\mathbf{a}}} = \underline{\underline{\mathbf{a}}}^{\mathbf{T}}$ *and* $\mathbf{Tr}(\underline{\underline{\mathbf{a}}}) = 1$, *verifying*

$$(\underline{\mathbf{v}} \, \mathbf{Grad})\underline{\underline{\mathbf{a}}} = \mathbf{Grad}(\underline{\mathbf{v}}) \, \underline{\underline{\mathbf{a}}} + \underline{\underline{\mathbf{a}}} \, (\mathbf{Grad}(\underline{\mathbf{v}}))^T - 2 \, \mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}) \, \underline{\underline{\mathbf{a}}}$$

$$\underline{\underline{\mathbf{a}}} = \underline{\underline{\mathbf{a}}}^{\mathbf{0}} \text{ on } \Gamma_-$$

The initialization of the algorithm is carried out with the newtonian solution (N_P=0). The efficiency of this fixed point scheme can be appreciably enhanced in using the following strategy : we calculate firstly the coupled solution for a low N_P value and then increment N_P step by step. The initialization of the fixed point algorithm at each step is carried out with the coupled solution of the previous one.

Velocity solver

Problem 1 can also be written:

Find $\underline{\mathbf{v}} \in U=\{\underline{\mathbf{v}}, \text{Div } \underline{\mathbf{v}} = 0, \ \underline{\mathbf{v}} = \underline{\mathbf{v}}_{\mathbf{g}} \text{ on } \Gamma_2\}$ *so that*

$$\int_\Omega \left(2\mu \ (\mathbf{Tr}(\underline{\underline{\mathbf{d}}}\, \underline{\underline{\mathbf{d}}}^{*}) + N_P \, \mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}) \, \mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}^{*})\right) - \mathbf{P} \, \mathbf{Div} \, \underline{\mathbf{v}}^{*} \ d\Omega = 0$$

$$\forall \ \underline{\mathbf{v}}^{*} \in U^{*} = \{\underline{\mathbf{v}}^*, \ \underline{\mathbf{v}}^* = \underline{\mathbf{0}} \text{ on } \Gamma_2\}$$

The discretization of this problem is carried out by the finite element method. In order to satisfy the Babuska Brezi's condition, the velocity is interpolated by P2 triangles and the pressure by consistent P1 triangles. The problem is solved with an augmented Lagrangian method.

Steady state orientation solver

The velocity field is assumed to be a given datum and we wish to solve the orientation equation (Eqn. 3 and 6). Even for a given velocity field, this equation is non-linear due to the term $\mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}) \, \underline{\underline{\mathbf{a}}}$. This equation is hyperbolic and its characteristics are determine by:

$$d\underline{\mathbf{x}} = \underline{\mathbf{v}} \, dt$$

which are the streamlines for the steady state regimes.

The method of characteristics.

If we define the tensor $\underline{\underline{\mathbf{c}}}$ by :

$$\underline{\underline{\mathbf{c}}} = \text{Grad}(\underline{\mathbf{v}}) \, \underline{\underline{\mathbf{a}}} + \underline{\underline{\mathbf{a}}} \, (\mathbf{Grad}(\underline{\mathbf{v}}))^T - 2 \, \mathbf{Tr}(\underline{\underline{\mathbf{a}}}\, \underline{\underline{\mathbf{d}}}) \, \underline{\underline{\mathbf{a}}}$$

then, applying the method of the characteristics [4] to the orientation equation, we will obtain

$$d\underline{\underline{\mathbf{a}}} = \underline{\underline{\mathbf{c}}} \, dt$$

and taking into account the inflow boundary condition (Eqn. 6), we will be able to compute ($\underline{\mathbf{x}}(t)$, $\underline{\mathbf{a}}(t)$). A fourth order Runge-Kutta scheme with step control has been used in the integrations. At each point where we want to know the orientation, the characteristic must be reconstructed until it reaches Γ_-, from where we must follow the flow in integrating the orientation equation from the boundary condition until coming back to the starting point.

RESULTS AND DISCUSSION

Computations

It can be shown from Eqn. 3 (see [3] for example), that if the flow is purely elongational, the fibers tends to align in the direction of maximal stretch. We will compute therefore the orientation state resulting from a divergent flow (Fig. 1). This flow is divergent since, because of the axisymmetry of the die and because of the material incompressibility, the norm of the velocity field decreases along a trajectory. In other words, the maximum stretch direction is $\underline{e}_\theta$, which is the direction of maximum load if the tube is submitted to an internal pressure. Fig. 2 shows the contour plot for the component $a_{\theta\theta}$. The more $a_{\theta\theta}$ is close to one, the more the fibers are aligned along this direction. It is clear on this figure that the orientation is approximately good except near the boundary of the tool where because of a sticking contact, the shear strains dominate the elongational effects. In order to enhance the orientation along $\underline{e}_\theta$, and also because the sticking condition is uncertain with a paste [5], we carried out a second computation for which the boundary condition at the surface of the die is a perfect sliding (Fig. 3). The wall effects of the sticking contact condition is not any more visible.

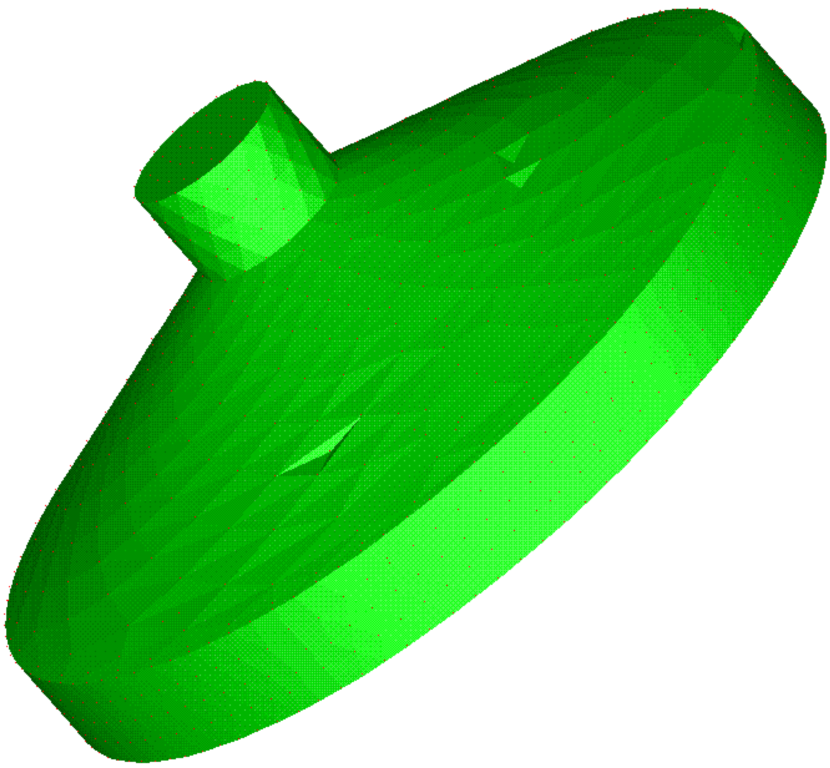

Figure 1 – Shape of the inner part of the die

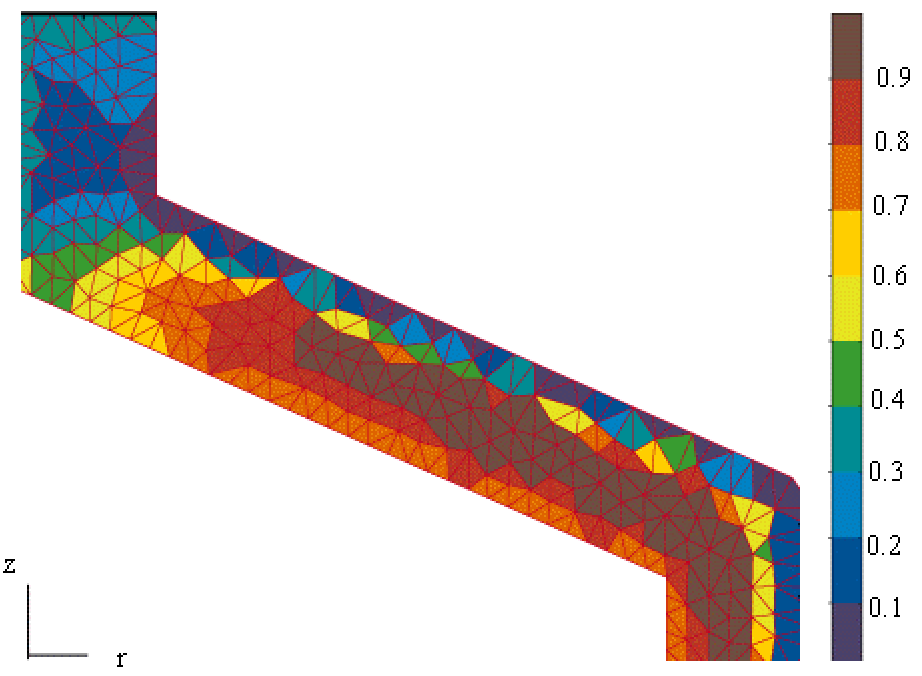

Figure 2 – Contour plot of $a_{\theta\theta}$ with a sticking contact of concrete on the die

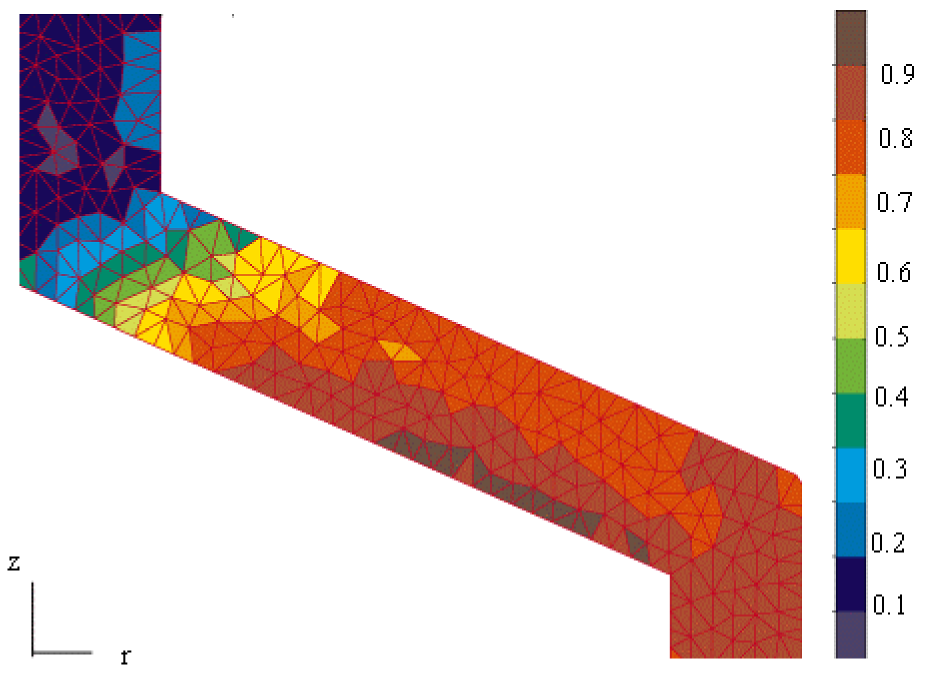

Figure 3 – Contour plot of $a_{\theta\theta}$ with a sliding contact of concrete on the die

Experiments

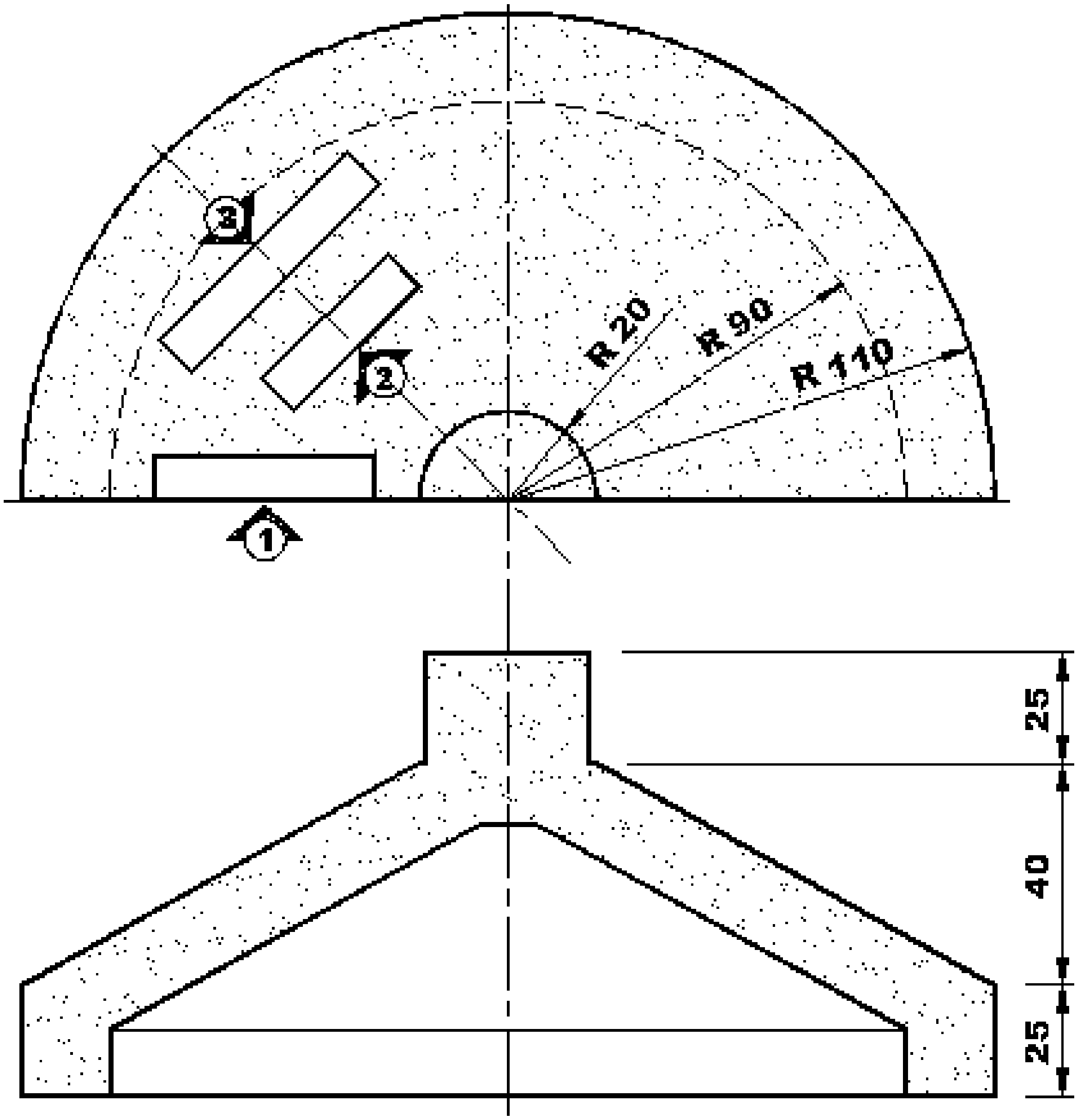

Figure 4 – Experimental device

The experiments have been carried out on the device depicted in Fig. 4. In order to avoid creeping problems for the tube (which was not our purpose in this study), we only carried out experiments that were stopped when the die was full of concrete. The concrete could then solidify inside the die so that after one day, it was possible to demold it and to analyse after cutting and polishing some samples taken inside the material as shown on Fig. 5. The fibers have a cylindrical shape, so that after cutting by a plane the resulting intersection is an ellipse which turn out to be a circle only if the analysis section is exactly perpendicular to the fiber.

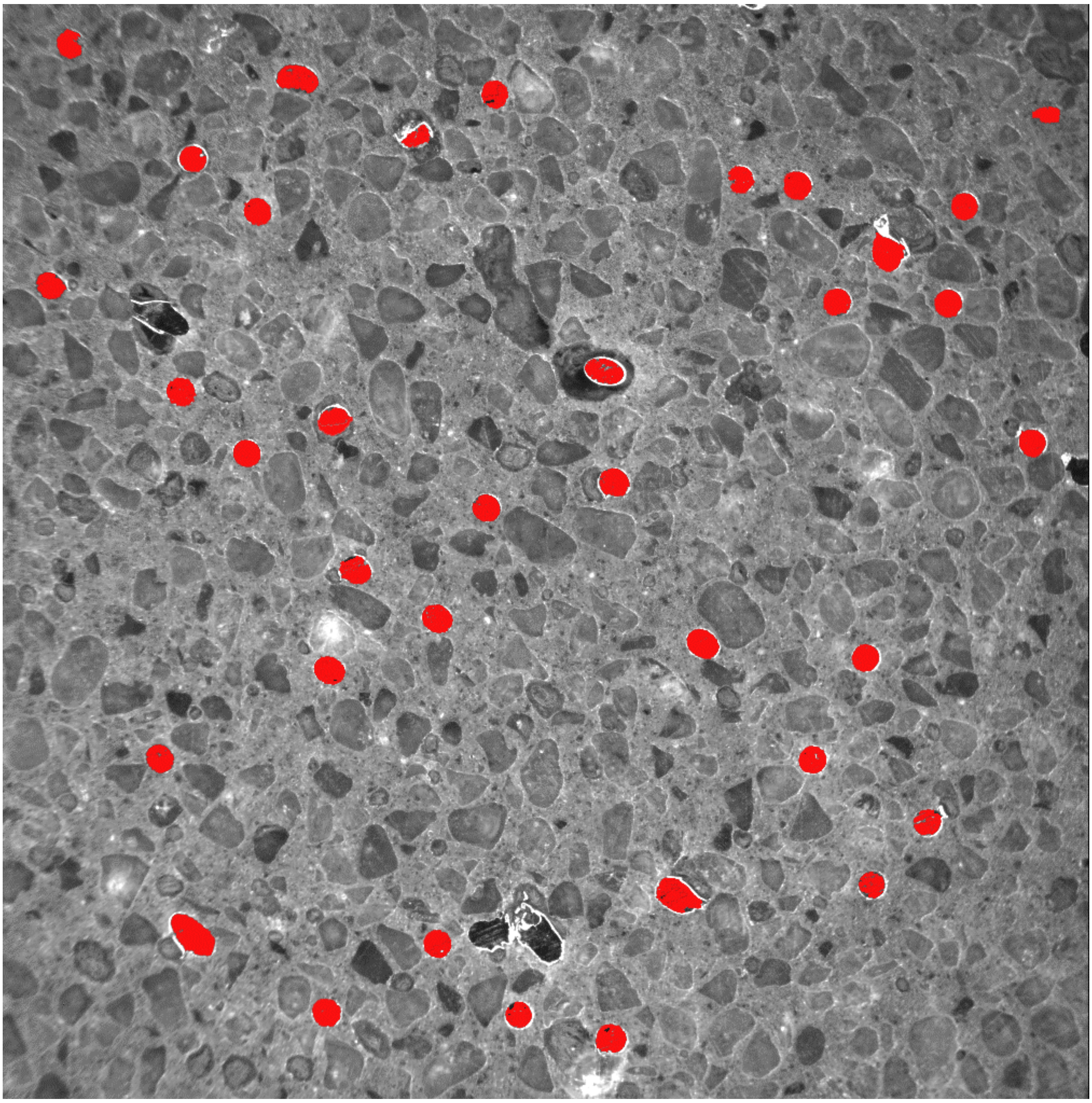

Figure 6 – Micrography corresponding to sample 1
The fibers (in red) are normal the plane

Fig. 6 shows such an intersection (sample 1 of Fig. 5), for which the normal to the plane is $\underline{e}_{\theta}$. Fig. 7 shows an other section (at 45° from the first one). In order to enhance the contrast, we have colored artificially the fibers intersection in red to differentiate them from the concrete granules. Fig. 6 shows perfect circles, whereas Fig. 7 shows ellipses whose length are in good agreement with the fibers aspect ratio. These experimental result valid in very good way the previsions from the computations.

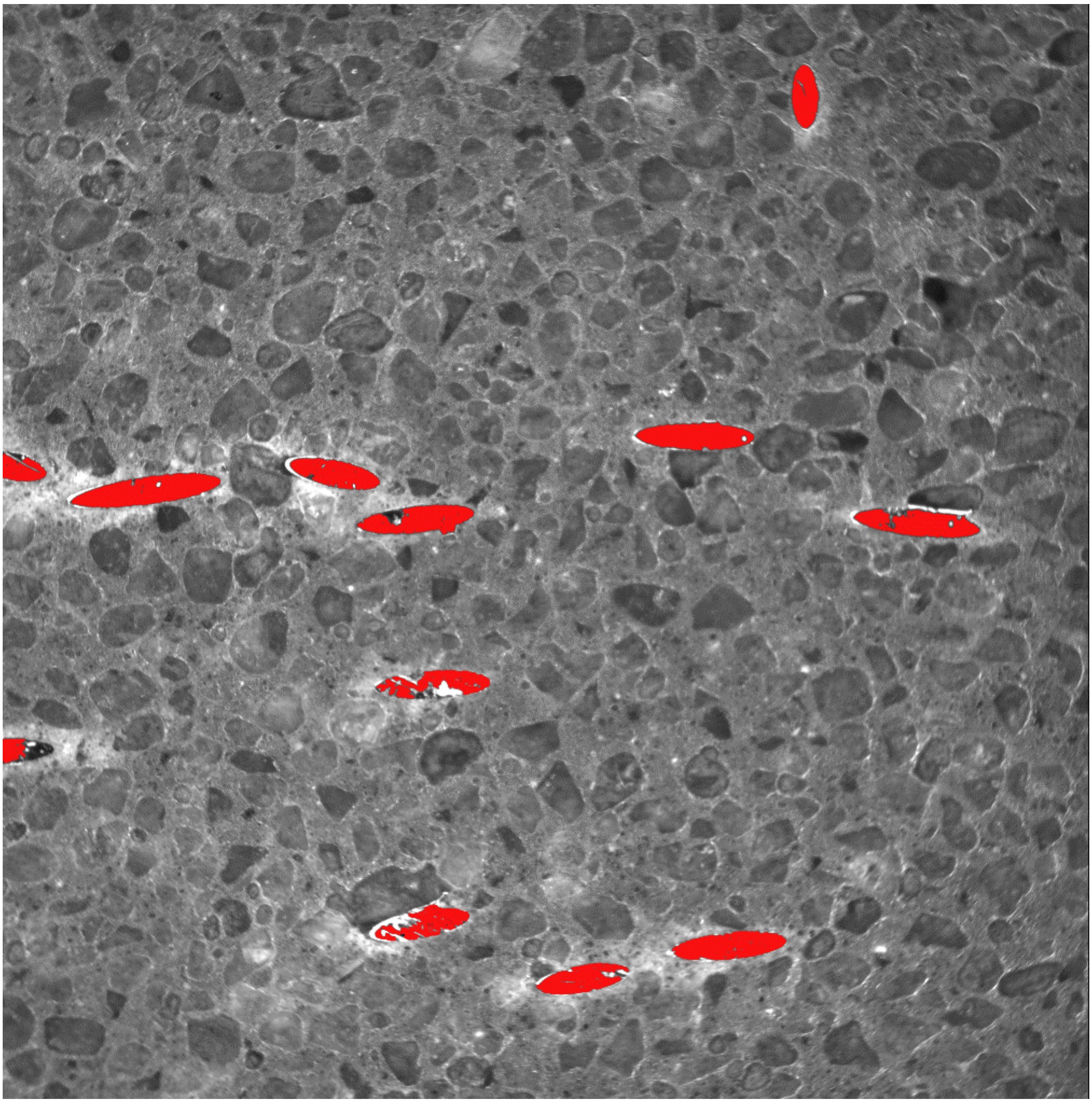

Figure 7 – Micrography corresponding to sample 2
The fibers (in red) are not normal to the plane

CONCLUSION

We have shown in this paper, both a theoretical calculation and its experimental validation. This gives rise to a possible design for fiber reinforced concrete tubes. This result is new for concrete material but confirms some results already mentioned for thermoplastics matrix composites [6]. The agreement between calculation and experiments are surprisingly good in comparison to the accuracy of the constitutive relation (Eqn. 2) applied to concrete pastes. This result is certainly due to the fact that the kinematics is strongly governed by the incompressibility condition in extrusion. So that, even if the constitutive relation is inaccurate, the fluid velocity, which give the orientation through Eqn. 3, is quite good. This quality of the orientation results does not mean that the stresses which could be deduced from this computation are correct. A better understand of the constitutive relation is certainly a matter of future works.

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