MOULDING ANALYSIS OF 3D WOVEN COMPOSITE PREFORMS: MAPPING ALGORITHMS

Shrikant B. Sharma, Prasad Potluri, Issac Porat

University of Manchester Institute of Science and Technology
PO Box 88, Manchester M60 1QD, United Kingdom

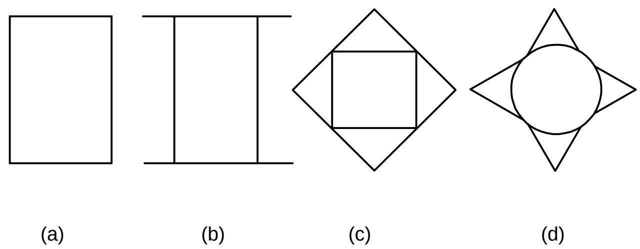
SUMMARY: Fibre alignment over the mould surface, when forming a composite material, affects the quality of the final product in terms of its structural homogeneity and mechanical properties. If the fibre-assembly is available in the form of a woven fabric, its yarns undergo an arrangement that can be predicted based on the shear deformation theory of a woven lamina. The present work tries to simulate the fitting of a woven fabric over an arbitrarily specified mould surface. Mapping algorithms have been developed which calculate the locations of the yarn crossover points, and hence the orientation of yarns. The algorithm has been enhanced so that the fabric covers most of the mould surface, if shear deformation permits it. Using this algorithm, one can also generate the 2-D outline of the woven lamina necessary to fit the surface.

KEYWORDS: moulding, woven composites, 3D, preform, fibre mapping, drapability, CAD.

INTRODUCTION

The work presented in this paper deals with the analysis of mouldability issues involved in the manufacturing of three-dimensional composite preforms, woven on conventional looms. The draping of woven fabrics has become a topic of interest to both the manufacturer of composite material preforms and to manufacturers using the materials to fabricate the components. An understanding of the draping characteristics of the fabric is required to shape the preform correctly, providing an allowance for the change in shape due to draping. This also provides predictive tools for determining the properties and quality of processed components.

The principal processes of fabric manufacture namely weaving, knitting and braiding have evolved primarily to produce 2D fabrics. However, these processes are also employed to produce 3D fabrics. Both conventional and new methods of preform construction can be used to produce a 2.5D or 3D shapes. A 2.5D shape is the one that has constant or uniformly varying properties along one of the dimensions. Most of the methods relying on conventional weaving machine to produce such shapes work on some variation of flattening method; an example is the work by Zhao *et al* [1] and Greenwood *et al* [2]. The structure is woven in the "flattened" state; it becomes 3-dimensional after it has been opened up. This kind of weaving method produces an essentially 2.5D component. Although attempts are being made to extend the method to weave tapered cross-sections, the present method restricts the woven cross-section to be straight along the warp direction. So far as the variety of shapes produced by this method are concerned, however, the possibilities are vast, as is clear from Fig. 1.



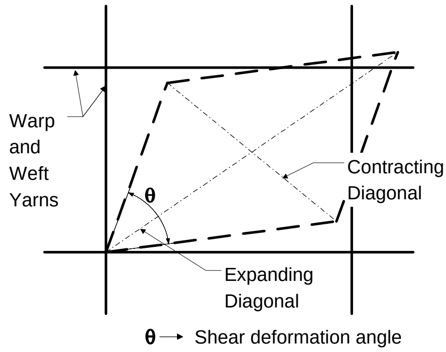


Fig. 1: Shapes produced by flattening method

Fig. 2: Shear deformation in a fabric cell

In order to produce complex shapes with double curvatures, like bent tubes, one needs to mould the woven preform to the desired shape. This paper presents a graphical simulation of the draping of a woven preform over the mould surface, with a view to highlight the parameters affecting the mouldability of the preforms, locate possible problems that might be encountered, and suggest ways to avoid them. The software developed for moulding analysis accepts the initial fabric geometry and the mould surface shape in a variety of possible ways, and simulates the fitting of the fabric over this surface. It also helps locate areas of excessive shear deformation and wrinkle formation.

Previous Work on Fibre Mapping (Draping)

Mack and Taylor [3] in 1956 derived differential equations for fitting fabrics to surfaces of revolution. The fitting equations were based on the shearing deformation of fabric where a square cell bounded by pairs of warp and weft yarns becomes a rhombus having unequal angles at adjacent corners, as shown in.

Mack and Taylor made a few assumptions about the geometry and kinematics of the draping process that have been used in subsequent research. Some of the most important assumptions are: yarn segments are assumed straight between pivot points, and the crossover points of warp and weft yarns act as pivoting points, i.e. no slippage occurs.

Subsequent to Mack and Taylor, several other researchers [4] [10] presented computational methods for fitting woven cloth to an analytically or numerically defined mould surface. Robertson *et al* [4] [10] developed a computational procedure for draping over a spherical or conical surface that involved finding the intersection points of three spheres. One sphere represents the surface being draped and the two other spheres represent all loci of the ends of a warp segment and a weft segment whose other ends lie on the surface at known points which coincide with the centres of the two spheres. Smiley and Pipes [6] applied Robertson's sphere intersection method of draping simulation to numerically defined surfaces of revolution. Haisey and Haller [7] developed a computational method for fitting woven fabrics to non-analytical surfaces using numerical analysis techniques.

Van West *et al* [8] [9] were the first to consider the fitting of woven cloth to a fairly arbitrary surface by modelling it as a bicubic Hermite patch. Their method required the specification of two sets of "constrained yarns" as three-dimensional curves on the surface over which fitting is carried out. Aono *et al* [10] presented a mapping algorithm that relaxes the restriction on the specification of "constrained yarns". These curves can be specified on the 2D lamina, and/or on the mould surface, in a variety of possible ways. They also presented a heuristic to calculate the contact points in the "unmapped" regions of the surface, where the normal sphere intersection method fails to predict the fitting.

GENERIC MAPPING ALGORITHM

Fabric Geometry

For the moulding problem, we represent the fabric as an assemblage of two sets of (warp and weft) yarns, initially orthogonal to each other. The weave pattern is assumed to be plain, although the behaviour of the fabric drape will be more or less applicable to other weave patterns as well. Fibre undulation due to yarn crimp is also ignored.

A yarn is specified by its diameter. Spacing between successive yarns is a property of the yarn assembly. Thus spacing between two consecutive warp yarns (hereafter called warp-pitch) is an attribute of the warp yarn, and so on. The most important property of the fabric is its shearing ability, which is specified in terms of the maximum and minimum angle attainable by the diagonals. The parameters used in the analysis are shown in Fig. 3.

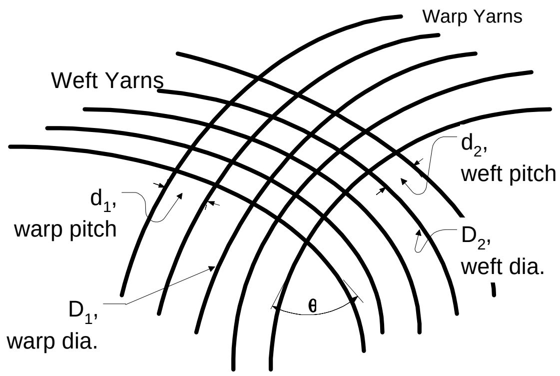


Fig. 3: Terms involved in fabric geometry

Mould Surface Geometry

The mould surface may be specified as:

- a set of connected parametric surface patches, or
- in terms of the cross-section of the 2.5D preform geometry, and a sweep path which characterises the deformation applied along the length of the woven preform.

A composite 3D surface [11] is modelled by assembling a series of topological patches. A reparametrising scheme is presented which represents the connected surface set as a superset of all the patches, and the mapping algorithm is applied to this set on the whole. The equation is finally reduced to the form:

$$P(u,v) = \{x\} = \{x(u,v)\}, \qquad u_{\min} \le u \le u_{\max}, \quad v_{\min} \le v \le v_{\max} \tag{1}$$

In the latter case, Frenet frame or similar sweeping technique [12] of surface modelling is used to generate the surface. Each of the curves can be specified as a series of connected segments; the resulting surface is then a composite surface. After sweeping the cross-section curve over the path curve, equation of the surface is obtained in the same form as in Eqn 1.

Constrained Yarn Specification

To achieve a unique draped configuration constraints must be imposed that uniquely determines all yarn paths. The constrained yarn paths are specified by co-ordinates u_c and v_c, such that $\mathbf{P}(u_c, v_c)$ locates the point of intersection of the two constrained yarn paths. The two orthogonal curves passing through this point (hereafter referred to as the starting point) specify the constrained yarn paths on the mould surface which are given by the equations u_c = constant and v_c = constant. For the 2.5D types of preforms under study the lines of "corners" along the cross-section and the path curves form the most likely paths along which the fabricator will align two sets of yarns. For most shapes, It also turns out to be the configuration producing low shear deformation. As an illustration, if an L-shaped cross-section curve is swept along an L-shaped path, the constrained yarns would most likely be the curves running along the corners of the L-sections (See Fig. 4).

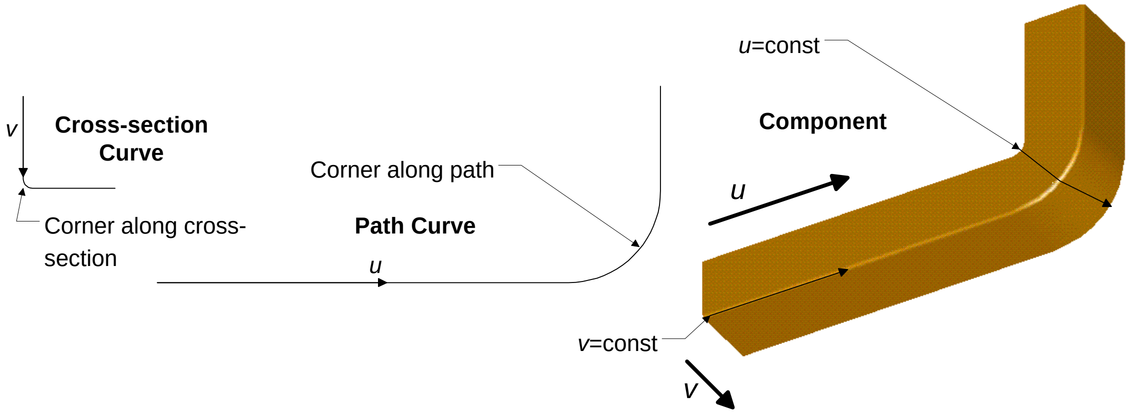


Fig. 4 Constrained yarns: possible locations along "corners" of the mould surface

Mapping Constrained Yarn Points

The starting constrained point $\mathbf{P}(u_c, v_c)$ is obtained using Eqn 1. In a 2D array of points, its value is stored in $\mathbf{P}_{0,0}$. Once $\mathbf{P}_{0,0}$ is known, other points on the constrained yarns can be calculated by "extending" them in the *u*- or *v*- direction. Extending in the *u*-direction generates $\mathbf{P}_{i,0}$ points while extending in *v*-direction generates the $\mathbf{P}_{0,j}$ points. Note that the *u*-constrained yarn (the constrained yarn in *u*-direction) could be a warp or a weft yarn. We denote d_u to be the spacing of yarns in the *u*-direction, i.e. distance between two *u*-yarns, if the constrained warp yarn runs along the *u*-axis. Similarly, d_v denotes the spacing of yarns in the *v*-direction.

Point $\mathbf{P}_{i+1,0}$ can be evaluated using the fact that the distance between it and point $\mathbf{P}_{i,0}$ is equal to d_u, and that both the points have the same *u*-coordinate on the surface. The distance between the two points is given by:

$$d_u = \sqrt{\left(P_{i+1,0} - P_{i,0}\right) \bullet \left(P_{i+1,0} - P_{i,0}\right)} \quad (2)$$

The *v*-coordinate of the point $\mathbf{P}_{i+1,0}$, i.e. $v_{i+1,0}$, is the unknown in the above equation. Because the yarn is being extended in the positive *u*-direction, $v_{i+1,0} > v_{I,0}$. One can easily solve Eqn 2 using Newton-Raphson or other similar numerical solution techniques to determine $v_{I+1,0}$.

Similar equations could be used for determining the points on the constrained *v*-yarns. Once all the constrained points have been located, the mould surface is divided into a maximum of four parts; each part will hereafter be referred to as a quadrant, as illustrated in Fig. 5.

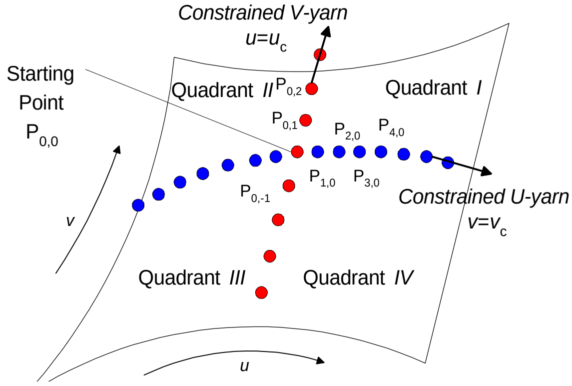


Fig. 5: Constrained points and surface quadrants

Mapping Unconstrained Points

Mapping begins in the first quadrant, starting from $P_{0,0}$. The mapping sequence is chosen to minimise the total computation time. The unconstrained crossover points adjacent to the constrained ones are calculated by solving equations of intersection of two spheres whose centres are the two constrained points and whose radii are given by the warp and weft spacing respectively. Subsequent crossover points are found by the same procedure but treating the previously calculated points as constrained points.

The mathematical model is developed first for quadrant I and is then extended to other quadrants. Lets say points $P_{i,j}$, $P_{i+1,j}$ and $P_{i,j+1}$ are known. Our aim is to locate the crossover point $P_{i+1,j+1}$ which is also on the mould surface. The distance of this point from the point $P_{i,j+1}$ is d_u and that from the point $P_{i+1,j}$ is d_v. The sphere-intersection equations are formulated as:

$$d_u = \sqrt{\left(\mathrm{P}_{i+1,j+1} - \mathrm{P}_{i,j+1}\right) \bullet \left(\mathrm{P}_{i+1,j+1} - \mathrm{P}_{i,j+1}\right)} \tag{3}$$

$$d_v = \sqrt{\left(\mathrm{P}_{i+1,j+1} - \mathrm{P}_{i+1,j}\right) \bullet \left(\mathrm{P}_{i+1,j+1} - \mathrm{P}_{i+1,j}\right)} \tag{4}$$

where $\mathbf{P}_{i,j} = \mathbf{P}(u_{i,j}, v_{i,j})$, and so on. Note that the two equations have two unknowns: $u_{i+1,j+1}$ and $v_{i+1,j+1}$. This set of non-linear equation could be solved using Newton-Raphson or similar numerical solution technique.

For locating points in other quadrants, similar approach is used. For example, in quadrant II, known points would be $\mathbf{P}_{i,j}$, $\mathbf{P}_{i-1,j}$, and $\mathbf{P}_{i,j+1}$ while the unknown point would be $\mathbf{P}_{i-1,j-1}$.

Choosing the Correct Solution

Eqn 3 and 4 might result in more than one solutions; one obvious one to be discarded is the reference point $\mathbf{P}_{i,j}$ itself. The following guidelines are used to get the best results.

- The most effective starting guess-values used for the solution are those assuming that the two sets of *u*- and *v*-segments involved in the sphere intersection problem form a parallelogram. **I**t's easy to determine the guess values for the new point's co-ordinate:

$$\begin{aligned} u_{i+1,j+1} &= u_{i,j+1} + (u_{i+1,j} - u_{i,j}) \\ v_{i+1,j+1} &= v_{i+1,j} + (v_{i,j+1} - v_{i,j}) \end{aligned} \tag{5}$$

- There maybe some invalid solutions other than that not coinciding with the reference point $\mathbf{P}_{i,j}$. These are the solutions which suggest the fabric would "fold back" at the crossover points, so that one of the *u*-segments would intersect with the previous *u*-segments on the UV plane, as shown in Fig.6 and Fig. 7.

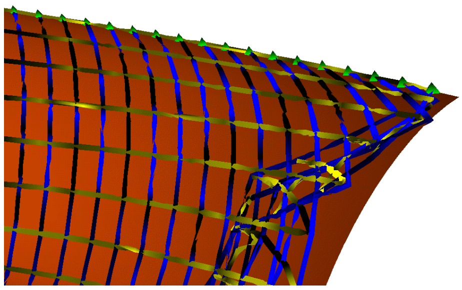


Fig. 6: Fabric folding back

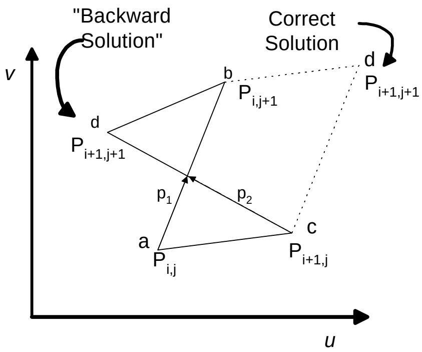


Fig. 7: v-segments intersecting

We refer to such solutions as "backward" solutions, which should be ignored. One easy way to test if a solution is "backward" is to check the intersection point of the new *u*-segment with the previous one, and that of the new *v*-segment with the previous one. If the intersection point is within the previous segments, then it is a backward solution. For example, the previous *v*-segment would be formed by (Ref. Fig. 7) the segment **ab**. Our aim is to find if the new *v*-segment **cd** intersects it or not. Let p_1 denote the parametrized distance of the intersection point from point a, and p_2 be its parametrized distance from point c. If the values of p_1 **and** p_2 are within the range {0, 1}, it means that the intersection is within the segment range, and it is a backward solution which should be ignored.
- Among all the valid results obtained, the solution which is farthest from the reference point $\mathbf{P}_{i,j}$ is the one finally chosen.

If the surface is not very irregular, mapping sequence would not be important so far as the final output is concerned. However, in order to implement the mapping algorithm for the "unmapped" region, the following sequence is recommended: $\mathbf{P}_{i,1}$, $\mathbf{P}_{1,j}$, $\mathbf{P}_{i,2}$, $\mathbf{P}_{2,j}$, … for the first quadrant (*i* increasing, *j* increasing), then $\mathbf{P}_{i,1}$, $\mathbf{P}_{-1,j}$, $\mathbf{P}_{i,2}$, $\mathbf{P}_{-2,j}$, … for the second quadrant (*i* decreasing, *j* increasing), and so on.

EXTENDED MAPPING ALGORITHM

The generic mapping algorithm presented above is good enough for most cases. However, it needs fine-tuning for special cases discussed below.

Multiply Connected Surface Patches

If the mould surface comprises of more than one patch, connected end to end, then care must be taken to continue the yarns from end of one surface to the beginning of the next surface patch. To keep the mapping process simple and general, we propose the idea of a "Solid".

A solid is made up of one or more surface patches, each of which in turn is made up of four boundary curves. A surface is connected to another when they share a common boundary. The structure of the solid can be established based on the topology of the surfaces, i.e. ways in which they are connected. To illustrate this, consider a cross-section curve which comprises of more than one segment $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, …$\mathbf{C}_{n-1}$ connected end-to-end. This curve is swept over a path curve, which itself could comprise of segments $\mathbf{T}_0$, $\mathbf{T}_1$, $\mathbf{T}_2$, …$\mathbf{T}_{m-1}$ connected end-to-end. As before, *u* denotes the position along the path (trajectory), and *v* denotes the position on the cross-section curve. Sweeping curve $\mathbf{C}_0$ over $\mathbf{T}_0$ produces surface patch $S_{0,0}$, say, that over $\mathbf{T}_1$ produces $S_{1,0}$, and so on. Similarly sweeping $\mathbf{C}_1$ over the path curve would produce $S_{0,1}$, $S_{1,1}$, etc. At the end we would have a set of $(m)*(n)$ surface patches connected as shown in Fig. 8.

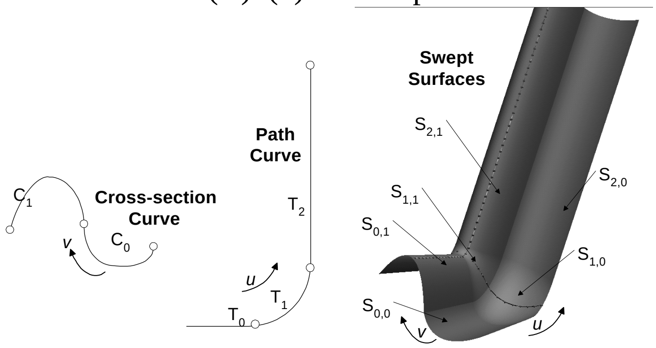


Fig. 8: Topology of multiple swept surfaces ("Solid")

One could imagine a "Solid" to be a super-set of all the surfaces, with its u-axis along the u-axes of all the surfaces contained within it, and so on. If all the patches are parametrised from 0 to 1 (both u and v directions), the parametric range of the solid would be: $0 \le u \le$ m and $0 \le v \le$ n. All the discussions that applies to a surface patch would be applicable to a solid too.

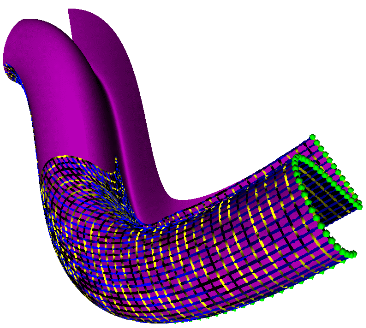
Predicting the Best Initial Constrained Yarn Location

As discussed before, it was observed that for the 2.5D types of preforms under study, constraining the yarns along the "corner-lines" on the mould surfaces gives the best mapping, i.e. minimum wrinkling. These are the lines along which the cross-section curve, or the path curve has a sharp bend. Mathematically, these are the set of orthogonal curves on the mould surface patch along which the curvature is maximum, or the radius of curvature is minimum.

So to find the best map, one has to find the locations of maximum curvature on the cross-section and the path curves. For example, curvature of the path curve $\mathbf{T}$(u) is given by:

$$\kappa = \frac{\left\| \mathrm{T}^{u} \times \mathrm{T}^{uu} \right\|}{\left\| \mathrm{T}^{u} \right\|^{3}} \tag{6}$$

where $\mathbf{T}^u$ is the tangent and $\mathbf{T}^{uu}$ is the second derivative vector. Using standard numerical optimisation routines, one can determine the parameter u for which the curvature is maximum. Fig. 9 shows an example of an unsuccessful fit. Same component, when mapped with the preferred constrained yarn locations, results in a fit shown in Fig 10.



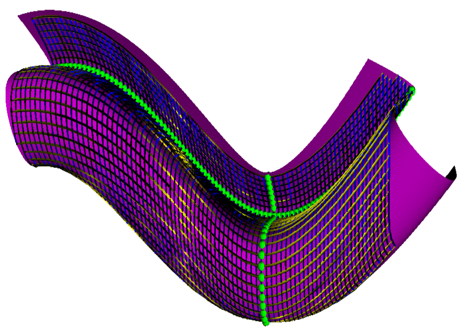


Fig. 9: Example of an unsuccessful fit

Fig. 10: A successful (but incomplete) mapping

Mapping the Unmapped Region

Even if mapping is successful, it may not be possible to map the entire surface region, due to the limitations of the generic mapping algorithm. The generic mapping algorithm needs three points of a rhombus known in order to locate the other point. If the surface geometry is such that all the yarns are drawn near the constrained yarns (Fig. 10), then we may not have sufficient number of points near the edges of the surface to successfully map the points. In this case, even though the fabric in reality would be able to cover the entire solid region (if shear deformation is within permissible limits), we will not be able to determine what orientations the fibres near the edges of the solid would take.

The following modified mapping sequence would try to map points onto every region on the fabric surface, provided the shear deformation is not exceeding the limit.

1. Map all the Points $\mathbf{P}_{i,1}$ first. (scanning in *u*-direction)
2. Extend the yarn on which the previous points were mapped, in roughly the same direction (*u*-direction). This would add a few more $\mathbf{P}_{i,1}$ points until the end of the surface is reached.
3. Map all the Points $\mathbf{P}_{1,j}$ now. (scanning in *v*-direction)
4. Extend the yarn on which the previous points were mapped, in roughly the same direction (*v*-direction). This would add a few more $\mathbf{P}_{1,j}$ points until the end of the surface is reached.
5. Repeat steps 1 and 2 for all the subsequent yarns in the current quadrant.
6. Repeat all the above steps for all the quadrants.

The exact algorithm used to extend the yarns in steps 2 and 4 is important. There are only two neighbouring points known is this case, the end points on the current yarn and the previous yarn. For illustration we refer to only step 2 (Fig. 11), where a yarn is being extended in the *u*-direction. As new points are calculated using the sphere intersection model, it is possible to calculate the shear deformation angle corresponding to them. It was observed that the shear deformation on a yarn over the points mapped sequentially follows a very clear pattern, and that it is not linear. The non-linearity could only be of the order 2 or 3, since the surface is most likely to be cubic. This suggests that the angle could be extrapolated, and the next point located using this. Note that this would only be able to predict one point, since the shear deformation angle for the next point can not be determined. Therefore, one could simply extend the yarn beyond this point in the same direction on the UV plane, as the previous segment. The new algorithm, when applied to the component shown Fig. 9 results in a complete fit, as shown in Fig. 12.

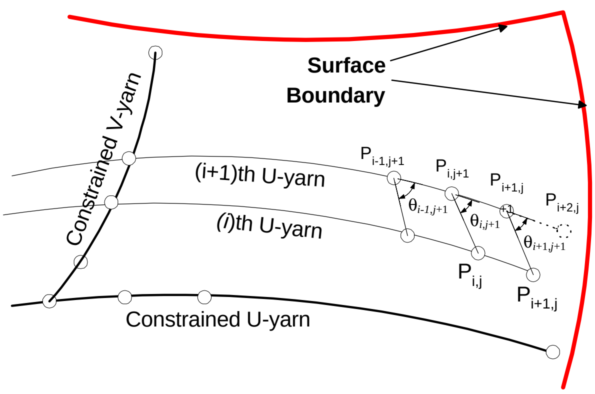


Fig. 11: Only two neighbouring points known

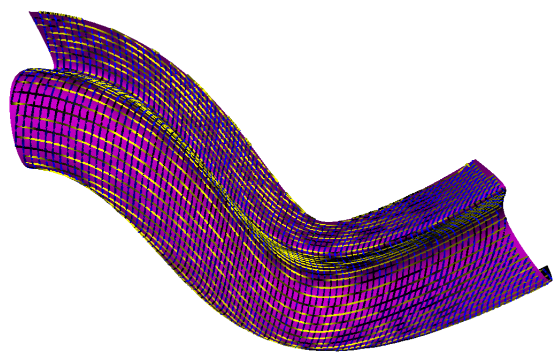


Fig. 12: Successful and complete fit

Predicting the Initial Preform Layout

All the fabric cells surrounded by warp and weft yarns are rectangular at the beginning, they deform into rhombus shape only when placed over an irregular surface. Using this fact, one needs to simply find the indices of all the points on all the yarns and represent them on a rectangular grid, the abscissa unit is represented in terms of d_u, and the vertical axis in terms of d_v. The resulting grid is nothing but the outline of the fabric area. Fig. 13 shows the initial layout of the fabric needed to cover the mould surface completely.

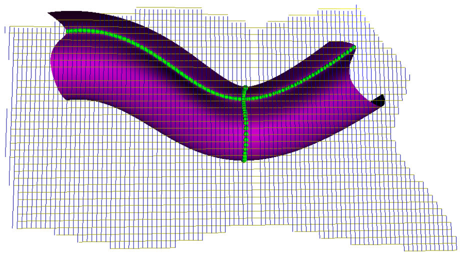
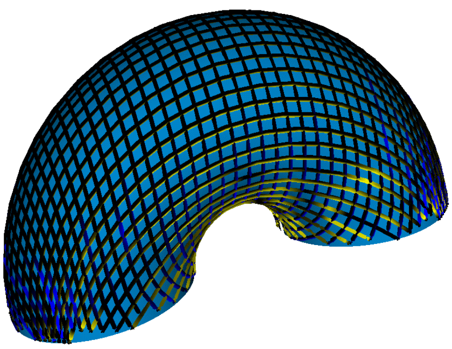


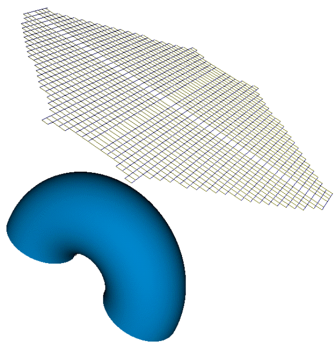
Fig. 13: Initial Preform Layout

CLOSED SECTIONS

By closed surface we mean a preform which has a cross-section made up of a closed curve. Moulding analysis of a closed surface is not trivial; one has to modify the mapping algorithm such that the yarn continuity is maintained at the adjoining edge. A first approximation is presented to the solution by trying to 'wrap' an open fabric along a symmetrically bent tube, and using the extended mapping algorithm to predict the fabric outline. The solution suggests that the woven shape will not be a straight tube, instead a profiled section (See Fig. 14).



a. Mapping



b. Developed preform shape

Fig. 14: Mapping Algorithm applied to a closed section

The problem of mapping a straight tubular fabric over a bent tube is more complex. Research is underway to develop a model to predict the fibre distribution in such a case.

CONCLUSIONS

An algorithm to predict the fibre alignment of a woven assembly over the mould surface has been presented and successfully implemented. The algorithm has been particularly suited to the three-dimensional composite preforms woven by the flattening method. It can predict the fitting near the edges of the mould surface, where the normal mapping algorithm fails due to lack of sufficient known points. The algorithm has also been applied on closed sections.

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