A FINITE ELEMENT MODEL FOR SIZE EFFECT AND HETEROGENEITY IN CONCRETE STRUCTURES

Roque Luiz Pitangueira[1] and Raul Rosas e Silva[2]

[1]*Department of Structural Engineering -Federal University of Minas Gerais - UFMG*
Av. do Contorno 842 - Belo Horizonte MG Brazil 30110-060 - roque@dees.ufmg.br
[2]*Department of Civil Engineering - Pontifical Catholic University of Rio de Janeiro –PUC-Rio*
Rua Marquês de São Vicente 225 - Rio de Janeiro RJ Brazil 22453-900 - raul@civ.puc-rio.br

SUMMARY: A finite element model for nonlinear analysis of structures prone to brittle collapse, which occurs with strain localization on softening material is presented. The model introduces the heterogeneity in simplified manner to allow for size effect to be considered. The size of a basic cell and the randomness of its components are the required information for the simulation of equilibrium paths. The elements can contain a random mixture of the material components, in proportion to their volume ratio in the mixture. It is shown that the experimental results of size effect in three-point bending and Brazilian split-cylinder tests are satisfactorily verified by the proposed model, in a computationally efficient and simple manner.

KEYWORDS: concrete, size effect, heterogeneity, finite elements.

INTRODUCTION

The representation of the mechanical behavior of concrete structures benefits much from developments in the area of constitutive modeling. However, the heterogeneity of material and the differences of response due to size effect should also be considered in a rational model for realistic prediction.

Models of crack propagation in concrete can be discrete or smeared. The smeared crack approach considers the cracks as regions of damaged material in the finite element. It appears to be a more convenient tool for the approximate modeling of the propagation of cracking patters occurring in concrete structures. Using this approach, the heterogeneity of material can be accounted for in a simple manner, by assigning different properties to each integration point. However, the local constitutive law (of an integration point) cannot be considered as a simple stress-strain relation of one of the concrete constituents. That law should be a combination of the constitutive (local) and structural (global) responses to allow for size effect to be considered.

In this paper, a finite element model including both heterogeneity and size effects is presented. The concrete is considered as a statistical combination of constituent materials with different properties (aggregate, mortar and interface material as in Figure 1). The finite element integration point response is based on the random occurrence of the solid phases in the structural volume as well as on the assumed differences of structural response due to the size effect.

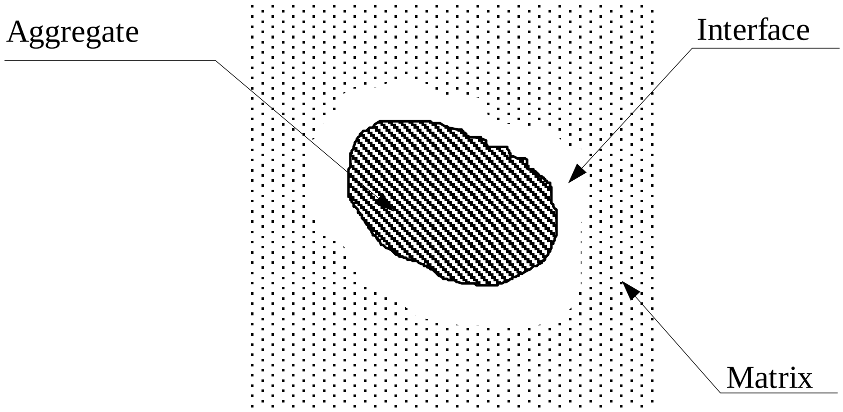

Figure 1 – Solid structure of concrete adopted in the model for heterogeneity and size effect

MODEL FOR HETEROGENEITY

The mechanical properties of normal strength concrete are related to the low strength of the interface region between the mortar and aggregate. Solid concrete can be represented as a three-phase material: aggregate, mortar and interface material (Figure 1).

In the usual models, the constitutive relation is the same for all integration points, in the start of the analysis. With deformation, this relation is modified to account for damage in tension and compression. In the present study, the initial properties of each Gauss point are associated to the components of concrete, according to a random number generator and a constant probability corresponding to the volume fraction of each constituent in the mixture.

Using all integration points of the finite element mesh to define a statistical space, a random number is generated to each point. With that number and assuming a uniform probability model, one of the three phase of the heterogeneity mixture is chosen to represent the point, based on the volume fraction of each phase (Figures. 2 and 3).

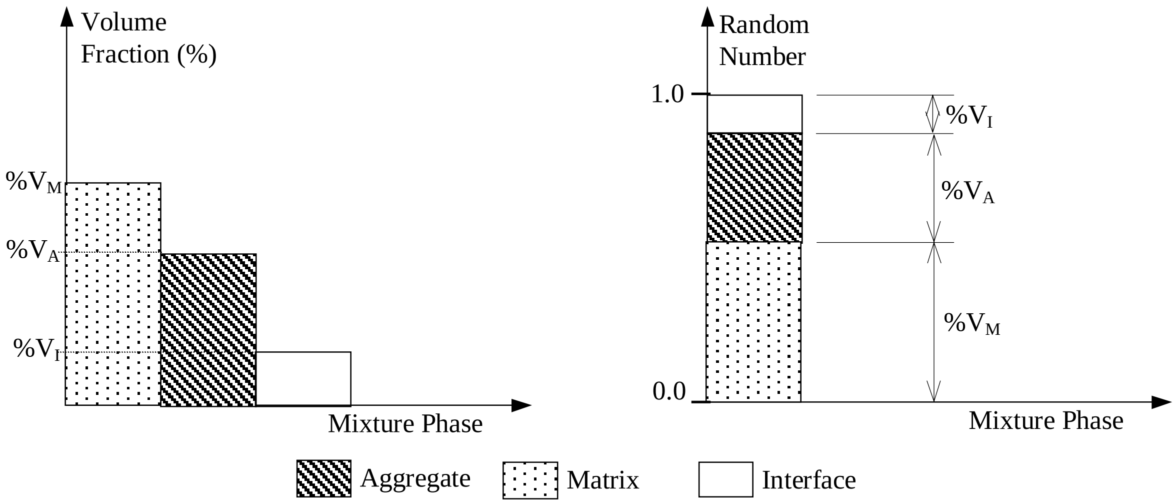

Figure 2 – Heterogeneity simulation with constant probability.

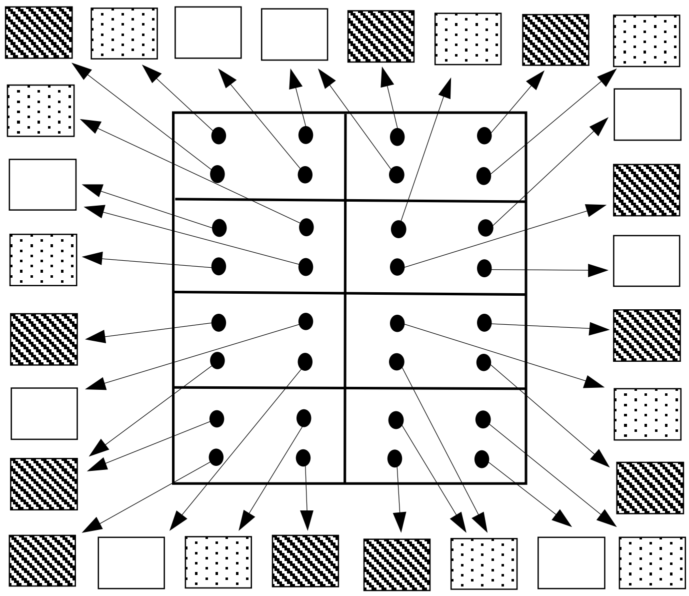

Figure 3 - Solid concrete phases to each integration point.

Constitutive behavior of each phase can be described by a simple stress-strain relation. Experimental results of the parameters used to describe that stress-strain relation are available. Using a bilinear relationship to describe compression and tensile behavior such parameters are: elastic modulus of the ascending branch (E_0), elastic modulus of the descending branch (E_2), axial compression strength (f_c), axial tensile strength (f_t) and fracture energy per volume unit (g_f).

MODEL FOR SIZE EFFECT

To allow for size effect to be investigated, the differences of constitutive response due to structural size should be considered. This is taken into account by defining an index of heterogeneity (IH) as the ratio between the volume of a basic cell (V_{CB}), representing the material inhomogeneities and the finite element volume (V_{PI}), representing the integration point.

$$\mathbf{IH} = \frac{\mathbf{V_{CB}}}{\mathbf{V_{PI}}}. \tag{1}$$

Using the heterogeneity index, the integration point constitutive law can be obtained by combining the solid phase (aggregate, mortar or interface) properties of the heterogeneity model with the properties of the basic cell representing the mixture law. That combination allows to consider higher or lower heterogeneity corresponding to smaller or larger structural size.

For a property P the above mentioned combination can be written as

$$\mathbf{P_{PI}} = \mathbf{IH} \cdot \mathbf{P_{LC}} + (\mathbf{1} - \mathbf{IH}) \cdot \mathbf{P_{LE}}, \tag{2}$$

where P_{PI} represents the value of the constitutive property P at integration point PI, P_{LC} represents such property locally measured (local constitutive law) and P_{LE} represents the value of the property in the structural level.

Such a combination contemplates the size effect since a property P measured in the structural and constitutive level has very different values. The heterogeneity randomness is also represented because the property P_{LC} can be associated to any mixture constituent phase. Figure 4 illustrates the idea.

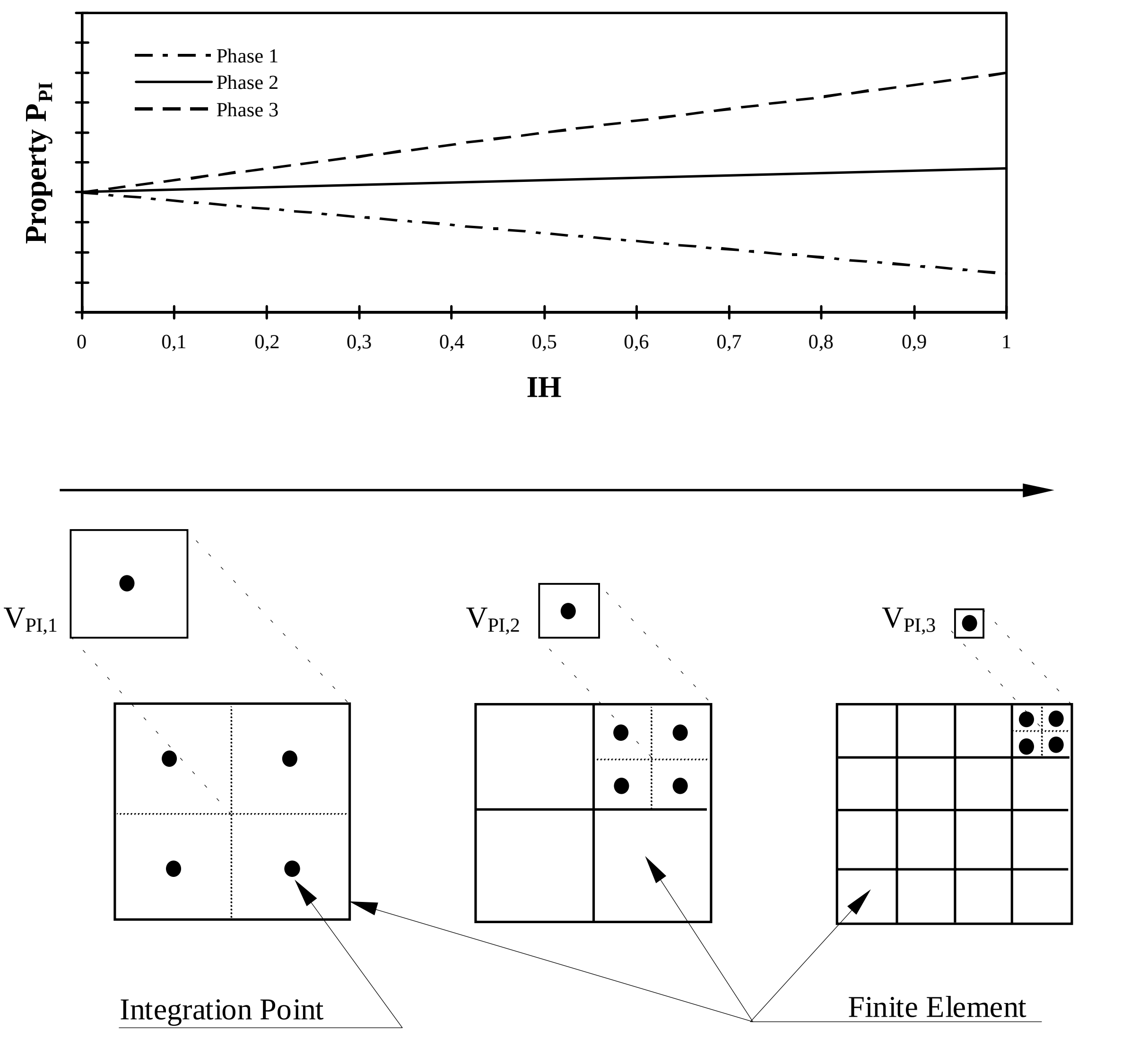

Figure 4 - Proposed model to size effect.

SIMULATION RESULTS

Brazilian Split-Cylinder tests

The Brazilian split-cylinder test has been modeled for different structural size, changing the heterogeneity index.

To represent the local behavior of each phase (aggregate, matrix and interface), bilinear relationships are adopted, with the mechanical properties shown in Table 1.

Table 1 – Mechanical properties used in the model.

Constituent	f_c (MPa)	f_t (MPa)	ν	E_0 (MPa)	E_2 (MPa)	g_f (MPa)
Matrix	48.0	3.4	0.2	3.2e4	-3.2e3	1.81e-3
Aggregate	80.0	16	0.2	1.0e5	-1.0e4	1.28e-2
Interface	13.0	2.0	0.2	1.7e4	-1.7e3	1.18e-3
Mixture	40.0	3.8	0.2	4.4e4	-4.4e3	1.64e-3

The properties of matrix corresponds to the mortar of Bazant and Pffeifer [1]. The aggregate phase is represented as the basalt of Monteiro [2] and the Interface phase is simulated with the test results of Monteiro [2]. The concrete mixture adopted has the proportion of 30% of aggregate, 55% of matrix and 15% of interface. The mixture properties are also shown in Table 1. On the basis of the above mentioned test results, the bilinear constitutive law parameters of table are obtained [3].

Figure 5 shows a mesh of four-node quadrilateral elements used in the analysis of a cylinder of thickness of 100 mm, with a initial central crack. The diameter was taken as 150 mm as the base dimension of the size effect analysis. Four integration points in each element and plane stress conditions are adopted.

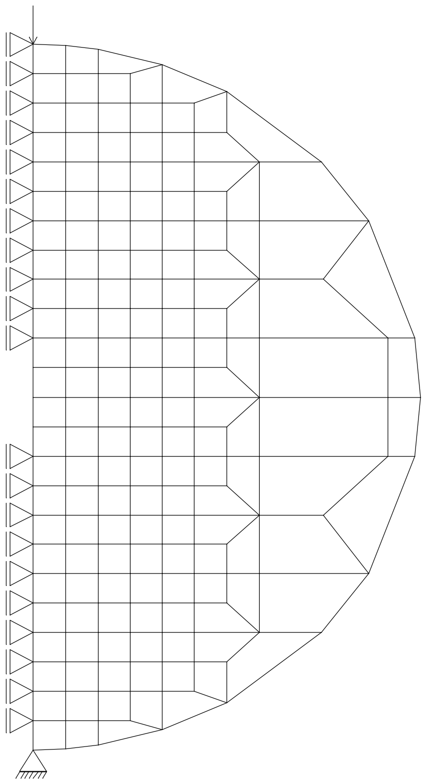

Figure 5 – Finite element mesh used in Brazilian split-cylinder tests model

The simulated equilibrium paths, obtained with the generalized displacement control method [4], reference load of -30 kN, tolerance of 10^{-4} and different heterogeneity index (IH) are shown in Figure 6.

As shown in the Figure 6, the splitting tensile strength decreases with the diameter (or, in that case, with the decrease of the heterogeneity). Experimental evidence indicating such behavior was obtained by different researchers ([5], [6] and [7]).

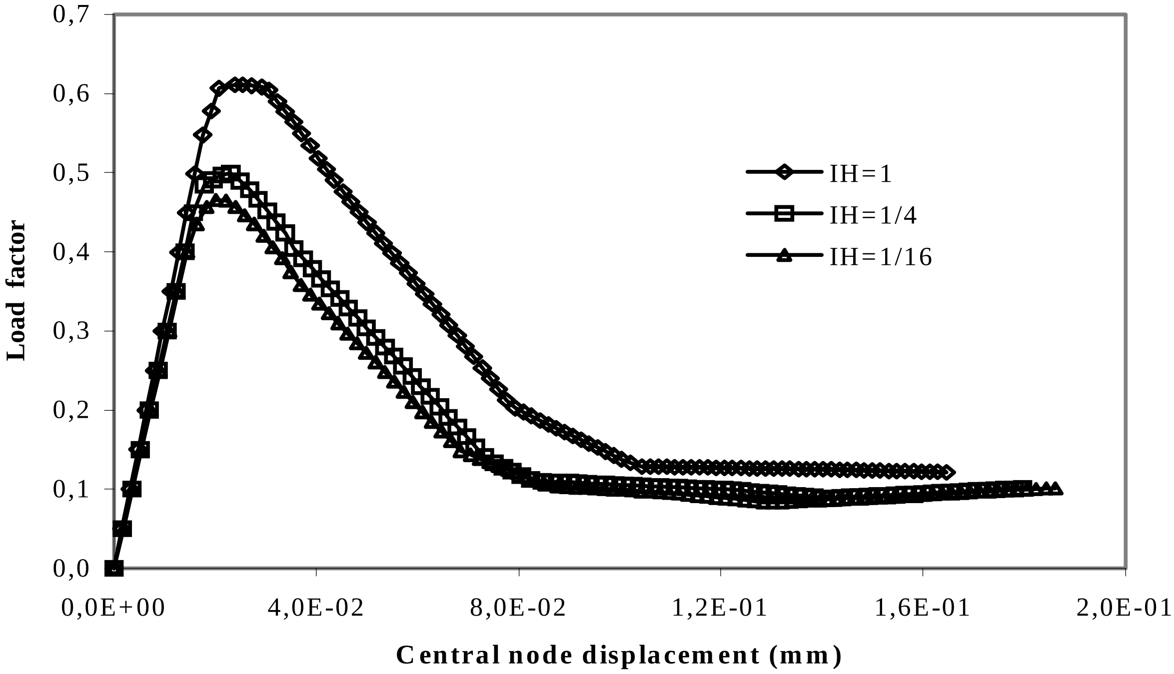

Figure 6 – Equilibrium paths of Brazilian split-cylinder tests

Three-Point Bending tests

A concrete beam of dimensions 90x36x36 mm with a notch of dimensions 2.5x6.0 mm is taken as reference for the size effect analysis in three-point bending test.

A mesh of four-node quadrilateral elements used in the analysis is shown in Figure 7. Four integration points in each element and plane stress conditions are adopted.

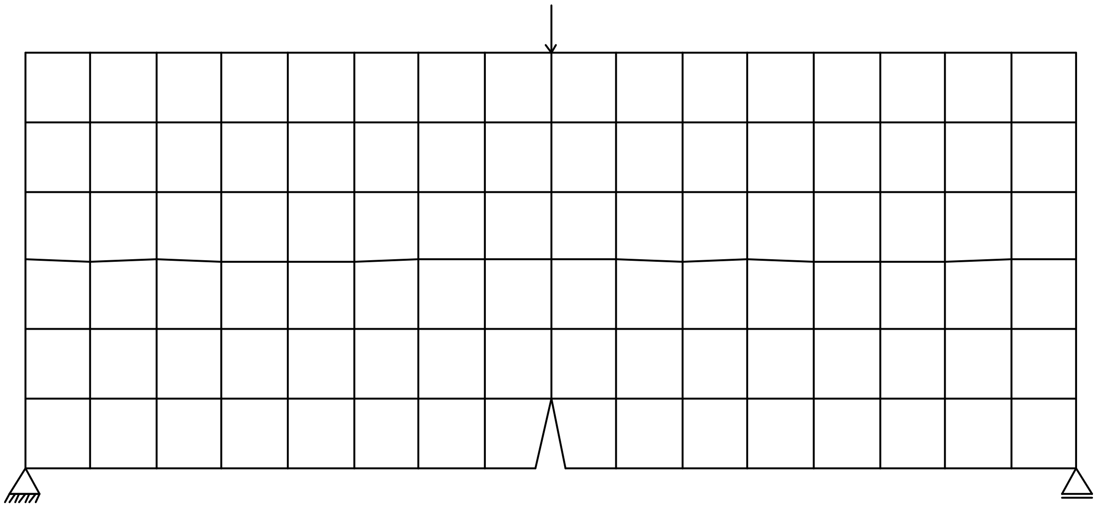

Figure 7 – Finite element mesh used in three-point bending tests model.

The mixture proportions as well as the mechanical properties to the bilinear relationships for the constituents and the mixture are the same of the previous example (Brazilian split-cylinder test).

By taking the smallest finite element to represent the maximum degree of heterogeneity or the smallest beam (IH ≈ 1.0 or $V_{CB} \approx V_{PI}$), the variation of the basic cell volume (V_{CB}) allows for size effect to be analised for different heterogeneity indexes (IH) or different structural sizes.

The equilibrium paths obtained with the generalized displacement control method [4], reference load of –10 kN and tolerance of 10^{-4} are shown in Figure 8.

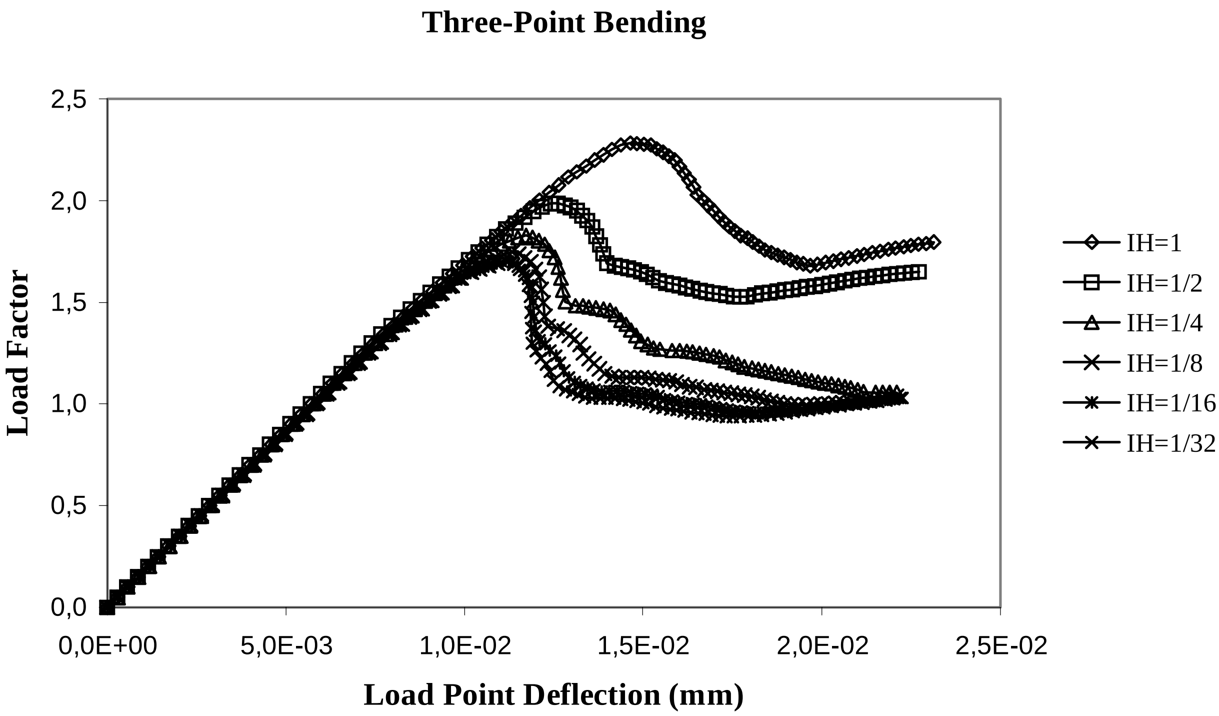

Figure 8 – Load Factor versus Load Point Deflection curves.

Two different situations are observed in the descending branch of the equilibrium paths obtained (Figure 8). A fragile behavior is observed in the case of larger structures (more homogeneous structures with IH tending to 0.0). On the other hand a more ductile behavior is observed in the case of smaller structures, (more heterogeneous with IH tending to 1.0).

The influence of size in the structure strength is also observed. As shown in the Figure 8, the load factor decrease with the decrease of the heterogeneity index, indicating small strength in the case of larger structures.

This evidence shows a good agreement with experimental results of various researchers ([1], [8], [9] and [10]).

CONCLUDING REMARKS

The proposed model allows for better understanding of size effect phenomena in concrete structures. Simulations of the material heterogeneity and associated size effect in a computationally efficient and simple manner show good agreement between experimental and proposed model results.

REFERENCES

1. Bazant, Z. P. and Pfeiffer, P. A. (1987) - “Determination of Fracture Energy from Size Effect and Brittleness Number”, ACI Materials Journal, Vol. 84, Nov-Dec, pp. 463-480.

2. Monteiro, P. J. M. (1985) - “Microstructure of Concrete and its Influence on the Mechanical Properties”, Ph.D Thesis, Department of Civil Engineering, University of California, Berkeley.

3. Pitangueira, R. L. S (1998) - “Mechanics of Concrete Structures considering Size Effects and Heterogeneitry”, (in Portuguese), Ph.D. Thesis, Department of Civil Engineering, PUC-Rio.

4. Yang, Y. B. and Kuo, S. R. (1994) - “Theory & Analysis of Nonlinear Framed Structures”, Prentice Hall, Singapore.

5. Sabnis, G. M. and Mirza, S. M. (1979) - Size Effects in Model Concrete”, Journal of Structural Division (ASCE), Vol. 106, No. ST6, pp.1007-1020.

6. Chen, W. F. and Yuan, R. L. (1980) - “Tensile Strength of Concrete: Double-Punch Tests”, Journal of Structural Division (ASCE), Vol. 106, No. ST8, pp.1673-1693.

7. Ross, C. A., Thompson, P. Y. and Tadesco, J. W. (1989) - “Split-Hopkinson Pressure-Bar Tests on Concrete and Mortar in Tension and Compression”, ACI Materials Journal, Vol. 86, Sep-Oct, pp. 475-481.

8. Gettu, R., Bazant, Z. P. e Karr, M. E. (1990) - “Fracture Properties and Brittleness of High-Strength Concrete”, ACI Materials Journal, Vol. 87, Nov-Dec, pp. 608-618.

9. Perdikaris, P. C. e Romeo, A. (1995) - “Size Effect on fracture Energy of Concrete and Stability Issues in Three-Point Bending Fracture Toughness Testing”, ACI Materials Journal, Vol. 92, Sep-Oct, pp. 483-496.

10. Chang, T. P. e Shieh, M. M. (1996) - “Fracture Properties of Lightweight Concrete”, Cement and Concrete Research, Vol. 26, No. 2, pp. 181-188.