COST VERSUS RELIABILITY IN REINFORCED HSC COLUMN DESIGN

Sofia M. C. Diniz[1]

[1] *Department of Structural Engineering, Federal University of Minas Gerais, Av. do Contorno 842, 2º Andar, Belo Horizonte, MG 30110-060 Brazil*

SUMMARY: High-strength concrete (HSC) can largely extend the advantages of reinforced concrete (RC) technology. In fact, HSC has found an increasing utilization, especially in the lower columns of high-rise buildings. From a deterministic viewpoint, it has been shown that the most economical column corresponds to the use of a minimum amount of longitudinal steel and the highest available concrete compressive strength. However, the most economical solution does not necessarily represent a reliable one. In this study, a decision analysis process for the case of axially loaded RC columns is presented. The weighted objective approach is employed; two main objectives are pursued: minimize costs and maximize column reliability. Probabilistic methods are used to define column reliability with respect to ultimate strength. Several combinations of concrete compressive strengths and longitudinal steel ratios are taken as the possible alternatives.

KEYWORDS: reinforced concrete, high-strength concrete, columns, random variables, probabilistic methods, reliability, cost, decision process.

RESEARCH SIGNIFICANCE

Reinforced concrete (RC) combines the advantages of concrete and steel: the relatively low cost, good weather and fire resistance, good compressive strength, and excellent formability of concrete and the high tensile strength and much greater ductility and toughness of steel. In the case of columns, the use of high-strength concrete (HSC) can largely extend the advantages of RC technology. In fact, in the last twenty years, HSC has found an increasing utilization, especially in the columns of high-rise buildings. In this case, the advantages of HSC are easily realized: the increased cost of the unit price of the concrete itself may be overcome by reduced column sections as well as other benefits such as more rentable floor space, less cracking, higher stiffness, higher durability, etc[1].

It is a well known fact that the structural designer has to cope with two conflicting requirements: cost and reliability. In the designer's mind, structural safety is usually equated to code compliance. In this sense, the cheapest structural element designed according to a given code would provide the best solution. However, this may not be true in the case of HSC columns[2]:

1) HSC is a relatively new material and its use has preceded a full understanding of its mechanical properties and structural behavior;
2) from a deterministic viewpoint, it has been shown that the most economical column corresponds to the use of a minimum amount of longitudinal steel and the highest available concrete compressive strength[3]. On the other hand, higher amounts of longitudinal steel have a beneficial effect on the resulting column reliability;
3) recent investigations of the in-situ behavior of HSC columns have shown that these columns display relatively smaller strengths as compared to their normal-strength concrete (NSC) counterparts[4, 5, 6, 7];
4) much of the information obtained to date regarding this material has not been reflected in the codes. As a result, using current recommendations for HSC columns does not necessarily mean that the same level of safety as for NSC columns is obtained.

From the above discussion it is clear that not only costs, but also reliability should be addressed in the case of RC column design process. In this study, a decision analysis process[8] for the case of axially loaded reinforced concrete columns is presented. Two main objectives are pursued: minimize cost and maximize reliability. Probabilistic methods are used to define column reliability. Concrete compressive strength, yield strength of longitudinal steel, cross-sectional dimensions, model error, dead load, and live load are assumed as the basic random variables. Columns made of different combinations of concrete compressive strengths f'_c (in the range of low- to high- strengths) and longitudinal steel ratios are taken as the possible alternatives.

DESIGN CONSIDERATIONS FOR AXIALLY LOADED RC COLUMNS

Present design practice, in calculating the nominal strength of an axially loaded member, is to assume a direct addition law summing the strength of the concrete and that of the steel. The usual assumption is made that steel and concrete strains are identical at any load stage[1]. The nominal axial strength of a reinforced concrete column is given by the expression:

$$P = 0.85 \,.\, f'_c \,.\left(A_c - A_{st}\right) + f_y \,.\, A_{st}$$

(1)

in which f'_c is the specified concrete compressive strength, f_y is the yield strength of the longitudinal steel, A_c is the area of concrete section, A_{st} is the area of steel. In this expression, the concrete area displaced by the longitudinal reinforcement was taken into account. The factor 0.85 accounts for observed differences between the strength of concrete in columns and the control specimens of the same mix.

For design purposes, the design strength of an axially loaded tied RC column is reduced further by two multipliers and is given by:

$$P_n = 0.80 \,.\, \phi \left[0.85 \,.\, f'_c \,.\left(A_c - A_{st}\right) + f_y \,.\, A_{st}\right]$$

(2)

where ϕ is the strength reduction factor (0.70) prescribed by the ACI Code[9] and 0.80 is a factor that takes into account the effect of accidental eccentricities of loading not considered in the analysis.

In the case of the computation of a HSC column capacity, some care shall be exercised with respect to Eq.(1). First, the ratio column to cylinder strength (k_3) is assumed as constant,

irrespective to the concrete compressive strength. Although it has been reported that this factor can also be applicable to HSC[1], there is experimental evidence that in the case of HSC columns, this factor is somewhat smaller than for NSC columns[4, 5, 6, 7]. For instance, Collins et al.[4] proposed the following expression for k_3 :

$$k_3 = 0.6 + \frac{10}{f'_c} \qquad \text{but } k_3 \leq 0.85$$

(3)

Ibrahim and MacGregor[10] also found that k_3 decreases as the concrete compressive strength increases. Their proposed equation is:

$$k_3 = 0.85 - \frac{f'_c}{800} \qquad \text{but} \quad k_3 \geq 0.725$$

(4)

Second, in the case of confined columns, same stresses are assumed at both the concrete cover and confined core according to Eq.(1). However, in the case of a confined column:

- lateral confinement does increase the strength of the confined core;
- at loads close to the ultimate different stresses may exist in the concrete cover and the confined core;
- the maximum contribution of the cover and confined core may not occur at the same strain value.

Therefore, an accurate computation of the axial resistance P of a RC column may be obtained through the following expression[11, 12]:

$$P = P_{COV} + P_{CORE} + P_{STEEL}$$

(5)

where P_{COV}, P_{CORE}, and P_{STEEL} are the amounts of load shared by the concrete cover, confined core, and longitudinal steel, respectively. However, previous studies[11, 12, 13] on both tied and spirally reinforced columns have shown that in the case of HSC columns, while the resulting column ductility is largely affected by the amount of transverse reinforcement, this parameter has a negligible impact on the ultimate column strength.

DESIGNED COLUMNS

In the present study, the performances of various axially loaded RC columns are compared. These columns were designed according to the ACI Code[9] requirements for short columns, and they are subjected to the same maximum factored load (dead load plus live load). Fig. 1 shows the cross-section configuration. The columns are reinforced by eight longitudinal bars of Grade 60 steel. This tie configuration was chosen due to its better performance than crossties[14]. Table 1 summarizes the data for the twenty-nine designed columns. In this table, ρ is the longitudinal steel ratio; b and h are the lateral dimensions of the column cross-section.

For each design, different amounts of concrete and steel (both longitudinal and transversal) will result and consequently different costs will be attained in each case. Table 1 also shows material costs (steel and concrete) of each designed column. The following costs were assumed (in US dollars):

- reinforcing steel: 760/ ton in place[3]
- 20.7-MPa concrete: 47/ yd^3 in place
- 27.6-MPa concrete: 54/ yd^3 in place
- 34.5-MPa concrete: 61/ yd^3 in place
- 41.4-MPa concrete: 68/ yd^3 in place
- 48.3-MPa concrete: 80/ yd^3 in place[3]
- 55.2-MPa concrete: 83/ yd^3 in place
- 62.1-MPa concrete: 85/ yd^3 in place[3]
- 69.0-MPa concrete: 94/ yd^3 in place

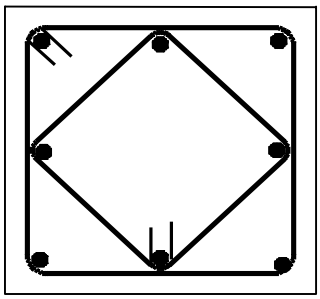

Fig. 1- Column cross-section

Regarding costs, it can be observed from Table 1 that:

- for a given f'_c, cost increases as the amount of reinforcement increases;
- for a given steel ratio, cost decreases as f'_c increases;
- as expected, the minimum cost is obtained in column # 4, i.e., the column with the highest concrete compressive strength and the minimum amount of longitudinal steel.

Table 1: Designed columns and corresponding material costs.

Column #	**f'_c (MPa)**	**ρ**	**b, h (cm)**	**Steel cost ($)**	**Concrete cost ($)**	**Total cost ($)**
1	69.0	0.046	50.0	491	90	581
2		0.030	52.3	350	100	450
3		0.023	53.1	280	103	383
4		0.010	54.9	149	112	261
5	62.1	0.040	52.8	494	91	586
6		0.027	54.9	352	99	451
7		0.021	55.9	280	104	384
8		0.012	57.4	185	110	295
9	55.2	0.037	55.6	496	99	595
10		0.024	58.2	355	109	464
11		0.019	59.2	282	114	396
12		0.011	60.7	187	121	308
13	48.3	0.076	52.1	855	80	935
14		0.033	59.4	499	109	608
15		0.021	62.0	357	120	477
16		0.013	64.3	230	130	360
17	41.4	0.066	55.9	858	79	937
18		0.030	64.0	493	108	601
19		0.018	67.1	361	120	481
20		0.011	69.3	232	129	361
21	34.5	0.056	61.0	856	85	941
22		0.024	69.9	499	116	615
23		0.015	73.2	365	128	493
24		0.012	74.7	289	134	423
25	27.6	0.045	67.8	860	95	955
26		0.019	78.0	504	128	632
27		0.012	81.8	371	143	514

28	20.7	0.034	77.7	865	109	974
29		0.014	89.9	511	149	660

RELIABILITY ANALYSIS

Due to the variability of the cross-section dimensions as well as material strengths, different values of the column reliability with respect to the ultimate strength will result for each designed column.

The reliability of a reinforced concrete column depends on several random variables, the basic variables $\boldsymbol{X}_i$. For each set of values of the basic variables it must be possible to state whether or not the column has failed. In order to define the performance of the column, a performance function $g(\boldsymbol{X})$ is used, where $\boldsymbol{X} = (X_1, \ldots, X_n)$ is a vector of basic variables. The limiting performance requirement may be defined as $g(\boldsymbol{X})=0$, which is the limit-state of the system. Therefore $g(\boldsymbol{X}) > 0$ is the safe state and $g(\boldsymbol{X}) < 0$ is the failure state[8].

The following expression is used to define the limit-state in the present study:

$$g(\mathbf{X}) = m\left\{k_3\ f_c\left[(b+b_d)(h+h_d) - A_{st}\right] + f_y\ A_{st}\right\} - D - L = 0$$

(6)

where m is the model error; k_3 is the ratio column to cylinder strength; f_c is the concrete compressive strength; b and h are the specified lateral dimensions of the column; b_d and h_d are the deviations from the specified values of the lateral dimensions of the column; A_{st} is the area of longitudinal steel; f_y is the yield strength of longitudinal steel; D and L are, respectively, the dead load and the live load acting on the column.

In the present study, the following variables are assumed as random:

- model error (X_1);
- concrete compressive strength (X_2);
- deviations in the lateral dimensions of the column (X_3, X_4) ;
- yield strength of longitudinal steel (X_5);
- dead load (X_6), and;
- live load (X_7).

The ratio column to cylinder strength, k_3, and the area of longitudinal steel, A_{st}, are assumed as deterministic. As stated previously, the coefficient k_3 is usually taken as 0.85, irrespective to the concrete compressive strength, but there is experimental evidence that this coefficient, rather than constant, decreases as the concrete compressive strength increases. In the present study, k_3 is given by the expression proposed by Collins et al.[4] (Eq.(3)).

The statistics of the basic variables related to column strength are summarized in Table 2. These statistics were compiled from the available data in the literature, or, as in the case of the concrete compressive strength estimated from the information contained in Diniz and Frangopol[15] and Anderson[16].

The statistics of the basic variables related to the column loading are summarized in Table 3. These statistics were obtained using the procedures and data given in *Diniz and Frangopol*[15]. Due to the larger variability of the live load (COV=0.25 as compared to COV=0.10 for the dead load), column reliability increases with increasing mean dead to mean live load ratio (μ_D / μ_L)[2]. However, since all columns in the present study are subjected to the same loads, they will

be equally affected by different load ratios. Therefore only one load ratio, i.e. $\mu_D / \mu_L = 1$, will be analyzed herein.

Table 2: Statistics of the basic variables related to column strength.

Variable	Mean value	Standard deviation	COV	Type of distribution	Reference
f'_c= 20.7 MPa	27.0	4.73	0.175	Lognormal	assumed
f'_c= 27.6 MPa	36.0	6.30	0.175	Lognormal	assumed
f'_c= 34.5 MPa	45.0	7.88	0.175	Lognormal	assumed
f'_c= 41.4 MPa	52.7	8.43	0.160	Lognormal	assumed
f'_c= 48.3 MPa	59.4	8.62	0.145	Lognormal	assumed
f'_c= 55.2 MPa	68.8	8.69	0.130	Lognormal	assumed
f'_c= 62.1 MPa	73.9	8.87	0.120	Lognormal	assumed
f'_c= 69.0 MPa	80.9	8.90	0.110	Lognormal	assumed
f_y (MPa)	460	38.2	0.083	Beta	17
b_d, h_d (cm)	+ 0.15	0.635		Normal	18
model error	1.0	0.11	0.11	Normal	18

Table 3: Statistics of the basic variables related to column loading.

Variable	Mean value	Standard deviation	COV	Type of distribution	Reference
(a) Dead Load					
D (kN), $\mu_D/\mu_L = 1$	3472	347.2	0.10	Normal	15
(b) Live Load					
L (kN), $\mu_D/\mu_L = 1$	3472	868	0.25	Extreme Type I	15

RELIABILITY RESULTS

In the present study the Advanced First Order Reliability Method (AFORM)[8] has been used to compute the reliability index β and the probability of failure P_f of each column. The resulting values of β and P_f are shown in Table 4.

From the results in Table 4 it can be seen that:

- for a given f'_c, reliability increases as the steel ratio increases;
- for a given steel ratio, reliability decreases as f'_c increases. This fact is due to the relatively smaller in-situ HSC strengths as given by the coefficient k_3. The reduced k_3 displayed by HSC has overcome the advantages of lesser strength variability that results from a more strict quality control in the production of this material;

- the most reliable column is # 25;
- the least reliable column is # 4, i.e. the column with the highest concrete compressive strength and minimum amount of longitudinal steel.

Table 4: Reliability index β and probability of failure P_f corresponding to the designed columns.

Column #	f'_c	ρ	β	P_f
1	69.0	0.046	4.64	1.72×10^{-6}
2		0.030	4.60	2.08×10^{-6}
3		0.023	4.56	2.55×10^{-6}
4		0.010	4.47	3.86×10^{-6}
5	62.1	0.040	4.72	1.18×10^{-6}
6		0.027	4.64	1.75×10^{-6}
7		0.021	4.62	1.93×10^{-6}
8		0.012	4.57	2.41×10^{-6}
9	55.2	0.037	4.76	9.62×10^{-7}
10		0.024	4.72	1.20×10^{-6}
11		0.019	4.69	1.37×10^{-6}
12		0.011	4.6321	1.81×10^{-6}
13	48.3	0.076	4.92	4.25×10^{-7}
14		0.033	4.83	6.90×10^{-7}
15		0.021	4.76	9.57×10^{-7}
16		0.013	4.73	1.14×10^{-6}
17	41.4	0.066	5.02	2.60×10^{-7}
18		0.030	4.94	3.89×10^{-7}
19		0.018	4.90	4.75×10^{-7}
20		0.011	4.85	6.12×10^{-7}
21	34.5	0.056	5.04	2.40×10^{-7}
22		0.024	4.94	3.93×10^{-7}
23		0.015	4.89	5.17×10^{-7}
24		0.012	4.87	5.61×10^{-7}
25	27.6	0.045	5.04	2.31×10^{-7}
26		0.019	4.94	3.91×10^{-7}
27		0.012	4.89	5.03×10^{-7}
28	20.7	0.034	5.04	2.38×10^{-7}
29		0.014	4.95	3.75×10^{-7}

DECISION PROCESS

In the design process, a compromise must be made between cost and reliability. As the results in Tables 1 and 4 show, the most reliable design is not the least expensive and vice-versa. Therefore, the Weighted Objective Decision Analysis (WODA)[8] is used in the present study to evaluate the overall utility of each design. Two objectives were considered: (1) minimize costs, and; (2) maximize reliability. In order to do so, the following steps are followed:

- the costs are listed in increasing order. A value of 1 is assigned to the least expensive alternative and 0 is assigned to the most expensive. The cost utilities of the other alternatives are obtained by linear interpolation between 0 and 1;

- the reliability indexes are listed in decreasing order. A value of 1 is assigned to the most reliable alternative and 0 is assigned to the least reliable. The reliability utilities of the other alternatives are obtained by linear interpolation between 0 and 1;
- relative weights are assigned to each objective. In the present study, the same importance is given to cost and reliability, i.e., a unit weight is assigned to both objectives.

Table 5 summarizes the development and the results of the WODA approach.

Table 5: Column utility

Column #	Cost rank	Cost utility	Reliability rank	Reliability utility	Total utility
1	17	0.55	22	0.30	0.85
2	10	0.73	26	0.23	0.96
3	6	0.83	28	0.15	0.98
4	1	1	29	0	1.0
5	18	0.54	19	0.44	0.98
6	11	0.73	23	0.29	1.02
7	7	0.83	25	0.26	1.09
8	2	0.95	27	0.18	1.13
9	19	0.53	17	0.51	1.04
10	12	0.72	20	0.43	1.15
11	8	0.81	21	0.38	1.19
12	3	0.93	24	0.28	1.21
13	25	0.05	9	0.79	0.84
14	21	0.51	15	0.62	1.13
15	13	0.70	16	0.51	1.21
16	4	0.86	18	0.45	1.31
17	26	0.05	4	0.96	1.01
18	20	0.52	6	0.82	1.34
19	14	0.69	10	0.75	1.44
20	5	0.86	14	0.67	1.53
21	27	0.05	2	1	1.05
22	22	0.50	8	0.82	1.32
23	15	0.67	12	0.73	1.40
24	9	0.77	13	0.70	1.47
25	28	0.03	1	1	1.03
26	23	0.48	7	0.82	1.30
27	16	0.65	11	0.73	1.38
28	29	0	3	0.99	0.99
29	24	0.44	5	0.84	1.28

From the results shown in Table 5 it can be concluded that:

- at a given f'_c, the total utility increases as the amount of longitudinal decreases;
- at similar longitudinal steel ratios, the total utility decreases as f'_c increases;
- the alternative that maximizes the total utility is column # 20, with f'_c = 41.4 MPa and minimum longitudinal steel ratio.

Finally, it should be realized that due to its large variability, costs related to improved rentable floor area were not included in the present study. Also, due to the difficulties in the assessment

of the costs related to the improved durability of HSC, this has not been included in this analysis. However, if these costs were taken into account the overall utility of HSC columns would increase.

SUMMARY AND CONCLUSIONS

In the present study, the WODA approach has been used in order to select the best alternative among several RC columns. A compromise between cost and reliability has been searched; twenty-nine columns have been designed and the corresponding costs and reliabilities have been computed. In applying the WODA approach, the same importance was given to the objectives "maximize reliability" and "minimize costs".

Important conclusions have been drawn from the results reported herein. From the results shown in Tables 4 and 5 it can be concluded that:

- for a given f'_c, reliability increases as the steel ratio increases;
- for a given steel ratio, reliability decreases as f'_c increases;
- the most reliable column is # 25;
- the least reliable column is # 4, i.e., the column with the highest concrete compressive strength and minimum amount of longitudinal steel.
- for a given f'_c, cost increases as the amount of reinforcement increases;
- for a given steel ratio, cost decreases as f'_c increases;
- as expected, the minimum cost is obtained in column # 4, i.e., the column with the highest concrete compressive strength and the minimum amount of longitudinal steel.
- at a given f'_c, the total utility increases as the amount of longitudinal decreases;
- at similar longitudinal steel ratios, the total utility decreases as f'_c increases;
- the alternative that maximizes the total utility is column # 20, with f'_c = 41.4 MPa and minimum longitudinal steel ratio;
- both concrete compressive strength and longitudinal steel ratio play an important role in the resulting column reliability;
- higher steel ratios have a beneficial effect on the column reliability, while higher concrete compressive strengths resulted in smaller column reliabilities;
- the column with the highest concrete compressive strength and minimum steel ratio is the most economical solution, but also the least reliable one;
- the WODA approach has proved to be a simple and important tool in the decision making process.

REFERENCES

1. ACI Committee 363, "State-of-the-Art Report on High-Strength Concrete", *ACI Journal*, Vol. 81, No. 4, 1984, pp. 364-411.

2. Diniz, S.M.C. and Frangopol, D.M., "Reliability Assessment of High-Strength Concrete Columns", *Journal of Engineering Mechanics*, ASCE, Vol. 124, No. 5, 1998, pp.529-536.

3. Moreno, J. and Zils, J., "Optimization of High Rise Concrete Buildings", *ACI SP-97*, American Concrete Institute, 1985, pp. 25-92.

4. Collins, M.P., Mitchell, D., and MacGregor, J.G., "Structural Design Considerations for High-Strength Concrete", *Concrete International*, Vol. 15, No. 5, 1993, pp. 27-34.

5. Pessiki, S. and Pieroni, A., "Axial Load Behavior of Large-Scale Spirally-Reinforced High-Strength Concrete Columns", *ACI Structural Journal*, Vol. 94, No. 3, 1997, pp. 304-314.

6. Diniz, S.M.C., "Discussion on Axial Load Behavior of Large-Scale Spirally-Reinforced High-Strength Concrete Columns", by S. Pessiki and A. Pieroni, *ACI Structural Journal*, Vol. 95, No. 4, 1998, pp. 456-457.

7. Mak, S.L., Attard, M.M., Ho, D.W.S. and Darvall, P., "Effective In-Situ Strength of High-Strength Concrete Columns", *Australian Civil Engineering Transactions*, Vol. 35, No. 2, 1993, pp. 87-94.

8. Ang, A.H. and Tang, W.H., *Probability Concepts in Engineering Planning and Design*, Vol. II, 1990, 562 p.

9. *Building Code Requirements For Reinforced Concrete And Commentary*, American Concrete Institute, 1995.

10. Ibrahim, H.H.H. and MacGregor, J.G., "Modification of the ACI Rectangular Stress Block for High-Strength Concrete", *ACI Structural Journal*, Vol. 94, No. 1, 1997, pp.40-48.

11. Diniz, S.M.C., "Probabilistic Assessment of the Strength of Tied HSC Columns", *Third International Conference on Computational Stochastic Mechanics*, Island of Santorini, Greece, 1998 (in print).

12. Diniz, S.M.C., "Probabilistic Assessment of the Strength of Spirally Reinforced HSC Columns", *International Symposium on High-Performance and Reactive Powder Concretes*, Aïtcin, P. C., and Delagrave, Y., eds, Sherbrooke, Vol. 1, Canada, 1998, pp. 191-210.

13. Diniz, S.M.C. and Frangopol, D.M. , "Strength and Ductility Simulation of High-Strength Concrete Columns", *Journal of Structural Engineering*, ASCE, Vol. 123, No. 10, 1997, pp. 1365-1374.

14. Sakai, K. and Sheikh, S., "What Do We Know About Confinement in Reinforced Concrete Columns (A Critical Review of Previous Work and Code Provisions)", *ACI Journal*, Vol. 86, No. 2, 1989, pp. 192-207.

15. Diniz, S.M.C. and Frangopol, D.M., "Reliability Bases for High-Strength Concrete Columns", *Journal of Structural Engineering*, ASCE, Vol. 123, No. 10, 1997, pp. 1375-7381.

16. Anderson, F.D., "Statistical Controls for High-Strength Concrete", *ACI SP-87*, American Concrete Institute, 1985, pp. 71-82.

17. Mirza, S.A. and J.G. MacGregor, "Variability of Mechanical Properties of Reinforcing Bars", *Journal of Structural Division*, ASCE, Vol. 105, No. 5, 1979, pp. 921-937.

18. Mirza, S.A. and MacGregor, J.G., "Variations in Dimensions of Reinforced Concrete Members", *Journal of Structural Division*, ASCE, Vol. 105, No. 4, 1979, pp. 751-766.