OPTIMAL DESIGN OF VISCOELASTIC COMPOSITES WITH PERIODIC MICROSTRUCTURES

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SUMMARY: Optimal design of viscoelastic composites with prescribed or optimal stiffness/damping characteristics is presented. The effective complex moduli in frequency domain are obtained by applying the homogenization method in frequency domain with Correspondence Principle. To find a viscoelastic composite with given constituent material properties, an inverse homogenization problem is formulated as a topology optimization problem using the homogenization method. An artificial material model is presented in order to construct a topology optimization problem as a density distribution problem. The sensitivity with respect to density is formulated. Design variables, objective function and design constraints including material symmetry and geometric symmetry are defined to construct the optimization problem, as a microstructure design problem by an inverse homogenization approach. Numerical design examples are presented with discussions on the optimal design of microstructures for viscoelastic composites.

KEYWORDS: viscoelastic, damping, inverse homogenization, optimization.

INTRODUCTION

Viscoelastic composites have been widely applied for the purpose of reducing noise and vibration as well as for the purpose of increasing stiffness to weight ratio in structures. These applications are mainly due to the fact that viscoelastic composites have desirable damping characteristics and design flexibility, i.e., tradeoffs between damping and stiffness. Polymer composites, rubber-toughened composites, and engineering plastics are the typical examples.
From the designer’s viewpoint, it would be very useful if there are materials whose service performances are optimal in the given environments. The service performances may be the damping characteristics in the case of noise and vibration controls. However, it is practically impossible to invent new materials with optimal properties for each new design. Instead, it is more practical to construct and use composite materials, which have good service performances in the given environments, by using the already existing materials.

The inverse homogenization approach emerged for design of materials with prescribed material properties as a special application of the homogenization method and the topology optimization [1]. It has been applied to the microstructure design of elastic materials with extreme material properties such as Poisson's ratio close to –1 and 0.5 [2]. It has been extended to the microstructure design of thermoelastic composites, which have unusual or extreme material properties such as negative thermal expansion coefficient or zero thermal expansion coefficient [3]. It has been suggested that stiffness and damping characteristics of viscoelastic composites may be improved by changing the topology of the microstructures of the constituent materials [4]. In the present work, an inverse homogenization approach for obtaining a microstructure of two-phase viscoelastic composites is presented to achieve desirable stiffness and damping characteristics of the viscoelastic composites.

HOMOGENIZATION IN VISCOELASTICITY

The homogenization method has been applied to the estimation of the effective material properties of composite materials with periodic microstructures [5]. For general viscoelastic composites, the homogenization process is formulated and a systematic way of obtaining the effective relaxation moduli in time domain is presented [4]. The effective relaxation moduli in time domain could be obtained from the effective relaxation moduli in Laplace transform domain by inverse Laplace transforms. Then the effective complex moduli in frequency domain could be readily obtained using simple formuli. Damping characteristics of viscoelastic composites are directly related to the complex moduli of the composites. The values of complex moduli are strongly dependent on frequency.

Since the present work deals with the microstructure design problem in which damping characteristics are of major concerns, the effective relaxation moduli in time domain need not to be computed. Instead, the effective complex moduli in frequency domain can be directly obtained by applying the homogenization method in frequency domain with Correspondence Principle. The homogenization problem in viscoelasticity can be easily formulated in complex domain by the same procedures as the homogenization in elasticity with the use of Correspondence Principle. In general, in the homogenization process, two problems, the local and the global problem, are obtained respectively. By solving the local problem, the homogenized moduli are obtained. The local problem in complex domain is as follows.

$$\int_Y G_{ijpq}(y,\omega)\frac{\partial \chi_p^{kl}}{\partial y_q}\frac{\partial v_i}{\partial y_j}dY = \int_Y G_{ijkl}(y,\omega)\frac{\partial v_i}{\partial y_j}dY, \quad \forall v \in V_Y \tag{1}$$

The homogenized complex modulus is obtained as follows.

$$G_{ijkl}^h(\omega) = \int_Y G_{pqrs}(\omega)(\delta_{kp}\delta_{lq} - \frac{\partial \chi_p^{kl}}{\partial y_q})(\delta_{ir}\delta_{js} - \frac{\partial \chi_r^{ij}}{\partial y_s})dY \tag{2}$$

where χ_p^{kl} is the solution of the local problem (1). By solving the local problem (1) and using (2), the effective complex moduli at a given frequency are obtained. The most important reason for using the complex modulus in frequency domain, is to facilitate the sensitivity analysis, which is an indispensable element for optimization. As will be discussed in the later section, the sensitivities of the object function are readily obtained from the effective moduli in frequency domain.

INVERSE HOMOGENIZATION PROBLEM

Design variables and artificial two-phase material model

An inverse homogenization problem is formulated as a topology optimization problem in which the distribution of two viscoelastic material phases is to be found so as to optimize the stiffness and damping characteristics of the viscoelastic composites. At this stage, an artificial two-phase material model [3] is introduced with a concept of density. An artificial material is defined for each density ρ with the following properties.

$$G_{ijkl}(\omega) = \rho G^{(1)}_{ijkl}(\omega) + (1-\rho) G^{(2)}_{ijkl}(\omega) \tag{3}$$

where $G^{(1)}_{ijkl}$ and $G^{(2)}_{ijkl}$ are the complex moduli of phase 1 and phase 2 material, respectively. The density, ρ, which is usually introduced in topology optimization problems, means the volume fraction of phase 1 material in the artificial two-phase material. ρ =1 means that it is pure phase 1 material and ρ =0 means that it is pure phase 2 material. Design domain, which is a periodic base cell, is discretized with N finite elements. The density is assumed to be constant in each element but it can be varied from one element to another. The density in each element becomes the design variable of the optimization problem. With the setting, the inverse homogenization problem is defined as the problem of finding the optimal densities in the elements so as to optimize the objective function, which will be defined later. It also should be noted that intermediate value of density is allowed for artificial material model. Artificial material with intermediate density can be interpreted as a mixture of two material phases.

Sensitivity analysis

Sensitivity analysis is required for iterative design modifications in design optimization process. In the process, it is assumed that the change of strain field in each element due to the change of density in an element is not large. This assumption is justified since a large number of elements are needed for the topology optimization. Under this assumption, approximate sensitivities of the effective complex moduli with respect to density in an element can be obtained from equation (2) as follows.

$$\frac{\partial G^h_{ijkl}(\omega)}{\partial \rho} = \int_Y \frac{\partial G_{pqrs}(y,\omega)}{\partial \rho} (\delta_{kp}\delta_{lq} - \frac{\partial \chi^{kl}_p}{\partial y_q})(\delta_{ir}\delta_{js} - \frac{\partial \chi^{ij}_r}{\partial y_s}) dY \tag{4}$$

Also, from Equation (3) the following simple relation holds for the artificial two-phase material model.

$$\frac{\partial G_{ijkl}(\rho,\omega)}{\partial \rho} = G^{(1)}_{ijkl}(\omega) - G^{(2)}_{ijkl}(\omega) \tag{5}$$

Objective function

The objective function in this work is values of the components of the complex moduli, or any combination of the components of the complex moduli at given frequencies. So the following objective function is defined for design optimization.

$$f = \sum_{i=1}^{ndv} w_i (g_i)^{\eta_i} + w_{penalty} [\sum_{i=1}^{ndv} \rho_i (1 - \rho_i)]^{\eta_{penalty}} \quad (6)$$

where g_i is one of the complex modulus, that is, storage modulus, the loss modulus or the loss tangent and where ρ_i is the density in an element and w_i, η_i, $w_{penalty}$, and $\eta_{penalty}$, the weighting factors. The first term in (6) is for stiffness and/or damping characteristics for optimization. The second term is the penalty to suppress the intermediate values of density in elements.

Design constraints

Available design constraints in this work can be total volume fraction of phase 1 material, effective complex moduli, geometric symmetries and material symmetries. Volume constraint is defined as follows.

$$V_{\min} \le \frac{1}{|Y|} \int_Y \rho \, dY \le V_{\max} \quad (7)$$

where Y is the domain of the base cell and $V_{\min}$ and $V_{\max}$, the bounds of volume fraction. Constraints on the effective complex moduli can be the bounds of the storage modulus, loss modulus and loss tangent of the visocoelastic composites. The constraints are enforced as numerical values as described in the following numerical examples. The geometric symmetry constraint can be applied directly by equating densities of the symmetric elements in a base cell. Orthotropic viscoelastic composite materials can be designed by enforcing one geometric symmetry constraint into elements in a base cell. It is possible to design viscoelastic composite materials with material symmetry. The condition for square symmetry of a viscoelastic composite material is as follows.

$$G_{1111} = G_{2222} \quad (8)$$

The conditions for transverse isotropy of viscoelastic composites are as follows.

$$G_{1111} = G_{2222} \;\; and \;\; G_{1111} = G_{1122} + 2G_{1212} \quad (9)$$

Optimization problem statements

With the definitions for design variables, objective functions and design constraints, an inverse homogenization problem for optimal microstructure design can be stated as a topology optimization problem as follows:

Find the density distribution in a base cell so as to
Maximize damping or stiffness at given frequencies
Subject to constraints on volume fraction
and/or constraints on damping or stiffness at given frequencies
and/or material symmetry
and/or geometric symmetry

For the optimization algorithm, the sequential linear programming (SLP) is used. In SLP, the optimization problem is linearized around the current design point in each iteration and the next design is found by the linear programming. In this work, the method of feasible directions is used as the constrained linear programming method. The reason for using SLP is its robustness since the inverse homogenization problem has numerous local minima and is not a well-behaved problem. For the implementation of the optimization algorithm, DOT (Design Optimization Tool) version 4.00 is used [6].

NUMERICAL EXAMPLES AND DISCUSSIONS

Introduction

In this section, numerical examples for the microstructure design to improve the stiffness and damping characteristics of viscoelastic composites are presented. The design domain is discretized with 12 by 12 square finite elements in a unit base cell. The design optimization starts with an initial density distribution of the two-phase material. The initial density in each element is randomly given. Initial density distribution should not be uniform since the design changes can not occur with uniform density distribution in all elements. In the following figures of microstructures in a base cell, as shown in Fig. 1, elements in black color represent material 1 and elements in white color represent material 2. Elements in gray color represent the material of intermediate property.

Example 1

As a first example, the microstructure design of the viscoelastic composite composed of an elastic material phase and a viscoelastic material phase is considered. Although the material properties are non-dimensional, their relative magnitudes are taken from the approximate material properties of the typical glass for the elastic material and the typical epoxy for the viscoelastic material. For the viscoelastic material, the viscoelasticity is modeled by the standard linear solid [7] and the relaxation time is artificially assumed. The material properties are as follows.

$$E = 70\,\text{GPa}, \qquad \nu = 0.22 \qquad \textit{for phase 1 material} \tag{10}$$

$$E(t) = 1 + 2.5e^{-t}\ \text{GPa}, \quad \nu = 0.35 \qquad \textit{for phase 2 material} \tag{11}$$

Suppose we are to design microstructures of a viscoelastic composite which is composed of the above two materials with 50% volume fraction for each material. This composite is to be used in the vibrating structures and the operating frequency is artificially assumed to be 0.5 as the relaxation time is assumed.

A simple way for constructing microstructures is to use a square unit cell with a circular inclusion. Let us define two kinds of composites with this configuration of the microstructure for later references. For the composite 1, the elastic material is used for the circular inclusion and the viscoelastic material for the matrix. For the composite 2, the roles of the two materials are interchanged. Table 1 shows the effective complex moduli of these two composites at the operating frequency. In many engineering applications, certain degrees of lower bounds are required for both stiffness and damping in composites. For example, to reduce vibrations effectively by use of a damper, stiffness of a damper should not be too low. Suppose that we need a composite whose stiffness, $G_{11}=G_{22}$, should be greater than 15 and the corresponding loss tangent should be as large as possible at any given frequency. With a simple microstructure configuration such as the composite 1 and the composite 2, we cannot obtain the satisfactory design. Even if we increase the volume fraction of the elastic inclusion in the composite 1 up to 70%, G_{11}=G_{22} have a value of only 10.16. In that case, the diameter of the circular inclusion becomes 0.944 if the unit cell size is 1. If we use the composite 2 to get the sufficient stiffness, then the damping characteristics becomes worse.

The first design example has been selected with the above observations in mind. The objective of this example is to find a microstructure which yields as large damping as possible while the desired stiffness of the microstructure is to be achieved at an operating frequency, ω. The exact description of the objective functions and the constraints are as follows.

Objective function: $\tan\delta_{11}(\omega) = \tan\delta_{22}(\omega),\ where\ \ \omega = 0.5$

Constraints: $G'_{11}(\omega) = G'_{22}(\omega)\ \ \geq 15.0$

$\mathrm{v}_f = 0.5\quad for\ \ each\ \ material$

Geometric symmetry about X-axis and Y-axis.

Where $\tan\delta_{11}(\omega)$ means loss tangent in 1-1 (x) direction, $G'_{11}(\omega)$ means the storage modulus in 1-1(x) direction and v_f represents the volume fraction of material 1. X-axis and Y-axis mean the horizontal centerline and the vertical centerline of the base cell, respectively. Also, the penalty term has been added as in (6), to reduce the regions of intermediate density. Fig. 1 shows a unit cell with the optimal microstructure, for which the solution converged after 148 function calls in design optimization routines. Fig. 1 also shows the same microstructures in a coarse 3 by 3 cell for easier understanding of the designed microstructure. The effective complex moduli are listed in Table 1.

Table 1: Effective complex moduli of example 1

	Frequency ω	Storage Modulus $G'_{11}(\omega)$	Loss Modulus $G''_{11}(\omega)$	Loss Tangent $\tan\ \delta_{11}(\omega)$
Material 1	0.5	73.56	0.00	0.000
Material 2	0.5	1.71	1.14	0.667
Composite 1	0.5	4.58	2.83	0.618
Composite 2	0.5	24.84	1.10	0.044
Optimal Composite	0.5	15.01	2.76	0.184

It can be concluded that the newly designed microstructure shows the successful tradeoff between stiffness and damping. Some interpretations are possible for the designed microstructure. The elastic phase materials are nearly connected so as to provide sufficient stiffness. At the same time, the elastic phase materials form mechanism-like structures in some region and lumps of elastic and viscoelastic materials exist around the linkage. This mechanism-like structures in the elastic phase materials result in sufficient deformations in the viscoelastic phase materials.

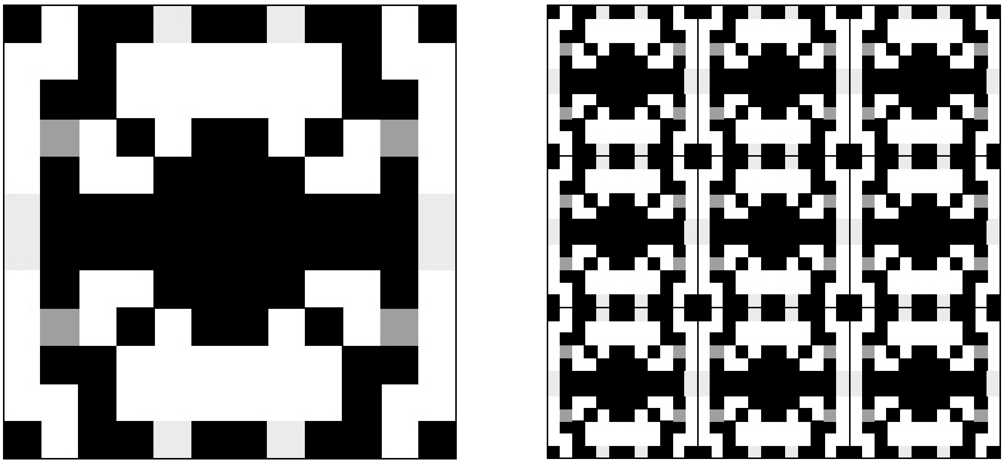

Fig. 1: Microstructures and periodic microstructures for maximum damping in example 1

These two facts show the tradeoff between stiffness and damping in the composites and show that the design gives as large damping as possible while maintaining the stiffness at the desired level. The mechanism-like structures are also found in the microstructures with extreme properties such as negative Poisson's ratio and it has been pointed out that the mechanism-like structures have a crucial role in such extreme microstructures [2]. At this point, it should be mentioned that the inverse homogenization problems have such numerical difficulties as mesh dependency and numerous local minima mainly because of the characteristics of their loading and boundary conditions. In fact, the loading and the boundary conditions in a unit cell are very uniform, especially at the initial stages of the optimization process, compared to the typical structural optimization problems. The more uniform the loading and the boundary conditions are, the more the local minima are present. Also, mesh dependency of the convergence can be increased because of the existence of the numerous local minima. In other words, the objective function surface or the response surface is very complex for the inverse homogenization problems. As a result, the optimization processes tend to proceed towards a local minimum and the optimization steps need large number of iterations in such cases. Some techniques such as employing low-pass filters have been suggested to escape from these numerical problems. Thus, it is recommended that the initial density distributions be nicely chosen, which can be done either by the designer's intuitions or by the already existing nearly optimal solutions [3]. If these are not available, the next choice is the trial and error method. Although the designed solutions, i.e. the microstructures, can be different with different initial guesses, the common characteristics of the optimal microstructures are the same as described before. Also, the values of the objective functions for the different optimal microstructures give little difference in the numerical values.

Example 2

As a second example, the microstructure design of the viscoelastic composite composed of an elastic material and a viscoelastic material is considered. The material properties are as follows.

$$E = 70\,\text{GPa}, \qquad \nu = 0.22 \qquad \textit{for phase 1 material} \tag{12}$$

$$E(t) = 1 + 2.5e^{-t}\ \text{GPa}, \quad \nu = 0.35 \qquad \textit{for phase 2 material} \tag{13}$$

Suppose we are to design a microstructure of a viscoelastic composite, which is composed of the above two materials. This composite is to be used in the vibrating structure and the operating frequency is assumed to be 0.5 same as in example 1. The structure experiences higher vibration levels in one direction. For example, the payloads mount structure for installing electronic devices in a launch vehicle, experiences higher vibration levels in longitudinal direction than in lateral direction. So we want to design a vibration damper made of viscoelastic composite materials, which should have a good damping characteristics in that direction as an anti-vibration damper. To reduce vibrations effectively by use of a damper, stiffness of a damper should not be too low. Suppose that we need a composite whose stiffness, G_{11}=G_{22}, should be greater than 15 and the corresponding loss tangent should be as large as possible at a given frequency in a specified direction. With such a simple microstructure configuration as the composite 1 and the composite 2 in example 1, we cannot obtain the satisfactory design as stated in example 1. The second design example has been selected with the above observations in mind. The objective of this example is to find a microstructure that yields as large damping as possible in a specified direction while the desired stiffness of the microstructure is maintained. The exact description of the objective functions and the constraints are as follows.

Objective function: $\tan \delta_{11}(\omega), \textit{where } \omega = 0.5$

Constraints : $G'_{11}(\omega) = G'_{22}(\omega) \geq 15.0$

Geometric symmetry about X-axis.

The constraint on volume fraction is not given in this example. Fig. 2 shows a unit cell with the resulting optimal microstructure. Fig. 2 also shows the same microstructures in a 3 by 3 cell for easier understanding of the designed microstructure. The effective complex moduli are listed in Table 2. It can be concluded that the newly designed microstructure shows the successful tradeoff between stiffness and damping. Some interpretations are possible for the designed microstructure as in example 1. The elastic phase materials are lumped together so as to provide sufficient stiffness as in example 1. At the same time, the elastic phase material forms mechanism-like structures and lumps of elastic material surround the viscoelastic materials. The viscoelastic phase materials form a wave-like fashion along the Y-axis in the periodic microstructures as shown in Fig. 2. This microstructure with wave-like viscoelastic phase materials and lumps of elastic phase materials result in sufficient deformations in the viscoelastic phase materials in the specified direction.

Table 2: Effective complex moduli of example 2

	Frequency ω	Storage Modulus $G'_{11}(\omega)$	Loss Modulus $G''_{11}(\omega)$	Loss Tangent $\tan \delta_{11}(\omega)$
Material 1	0.5	73.56	0.00	0.000
Material 2	0.5	1.71	1.14	0.667
Composite 1	0.5	4.58	2.83	0.618
Composite 2	0.5	24.84	1.10	0.044
Optimal Composite	0.5	15.02	6.58	0.4381

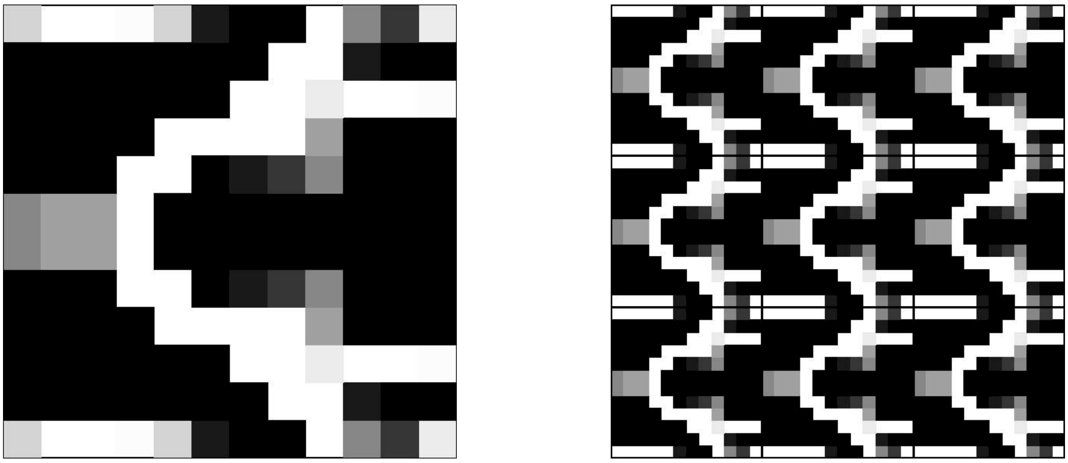

Fig. 2: Microstructures and periodic microstructures for maximum damping in example 2

These facts show the tradeoff between stiffness and damping in the composites and show that the microstructure design gives as large damping as possible in the specified direction while maintaining the stiffness at the desired level.

CONCLUSIONS

Optimal design of microstructures of viscoelastic composites is presented using the inverse homogenization approach. The numerical examples show that the improvements in damping characteristics of the viscoelastic composites can be achieved by the inverse homogenization approach based on the given materials instead of designing new materials. Damping capacity of a viscoelastic composite, which is represented by the value of loss tangent at specified frequencies, is optimized by designing the topology of the microstructure in the composite while maintaining its stiffness within the specified bounds.
The designed microstructures show the tradeoff between stiffness and damping in the viscoelastic composites. More critical problem resides in manufacturing of the microstructures. Advanced manufacturing technology should be developed so that the design and manufacturing of microstructures in viscoelastic composites are guaranteed in the viewpoint of costs and manufacturing abilities. Newly developed technologies such as micromachining and fabrication of molecular composites by micro-network may be the possible candidates. For example, works on fabricating molecular composites at the molecular level is in progress now [8].

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