SUBSTRUCTURE METHOD FOR AN ASSEMBLY OF STRUCTURES INVARIANT IN ONE DIRECTION

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SUMMARY: This study deals with the modeling by finite elements of an elastomeric composite structure. The main difficulties lie in the heterogeneity of the piece and in the geometrical non-linearity due to the centrifugal forces. We use the combination of two different methods of level sub-structuring. The first one is a multi-level sub-structuring method [1] and the second is a more classical sub-structuring [2]. After computation, the complete solution in displacement, the complete stress field and the global dumping of the structure (by an energetic method) are obtained.

KEYWORDS: finite elements method, sub-structure method, dumping, large number of degrees of freedom, heterogeneity, geometric non linearity, elastomer, composite material.

INTRODUCTION

The Elastomeric Flex Beam (E.F.B.) is developed in aeronautics industry by the Eurocopter Company. This part is made of one hundred sticks of composite materials (unidirectional carbon or unidirectional glass) put into different elastomeric matrices. It has to ensure the liaison between the rotor and the blade. It is substituted to the three articulations of the classical rotor, in order to decrease the aerodynamic stress. Due to the differences between the characteristics of each part, many mechanical problems appear like cracking, fatigue and stress concentration, which involve an accurate knowledge of mechanical results like the stress field, the displacements solution or the global dumping of the structure.
Numerically, the insufficient number of sticks and the heterogeneity of the different materials make it difficult to run a homogenization computation. The complete 3D problem can be solved, using different sub-structuring methods taking the geometrical non-linearity due to the centrifugal forces, into account.

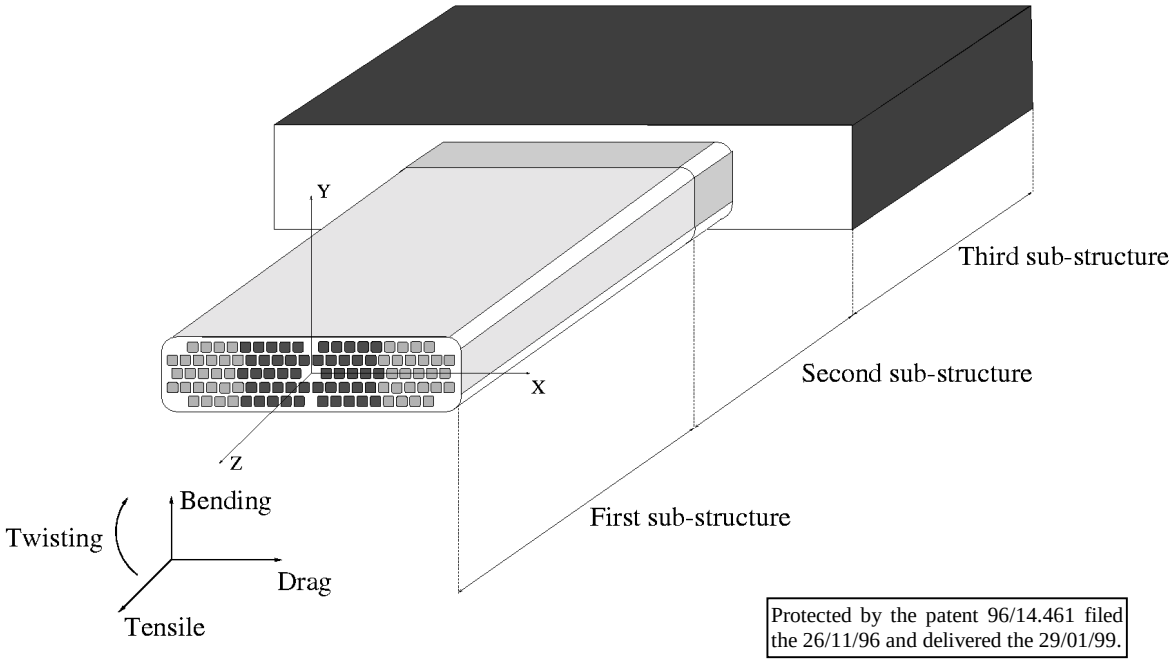

Fig. 1: Elastomeric Flex Beam.

1. Substructuring methods

Actually, the sub-structuring methods are more and more applied, because of their advantages:

-They allow the division of the construction and the verification of large structures.

-Once the complete model is defined, each sub-structure can be modified independently of the other one.

-Sub-structure methods are often the solution for the large problems (several millions of degrees of freedom).

1.1. Classical Method [1] [2]

Considering a structure which are subdivided in one level of sub-structures (Fig. 2).

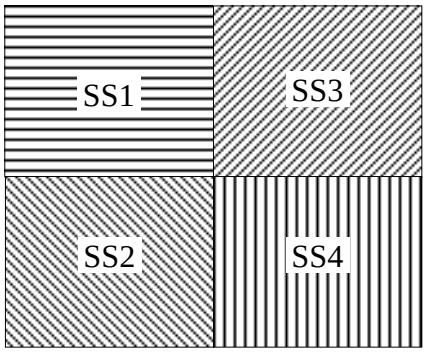

Fig. 2: Example of level one substructuring.

For one substructure, internal (q_i^s) and liaison (q_j^s) degrees of freedom are distinguished. This allow the writing of the stiffness matrix:

$$\begin{bmatrix} K_{ii}^s & K_{ij}^s \\ K_{ji}^s & K_{jj}^s \end{bmatrix} \begin{Bmatrix} q_i^s \\ q_j^s \end{Bmatrix} = \begin{Bmatrix} F_i^s \\ F_j^s \end{Bmatrix} \tag{1}$$

The condensed stiffness matrix is defined by:

$$\overline{K}_{jj}^s = K_{jj}^s - K_{ji}^s K_{ii}^{s^{-1}} K_{ij}^s \tag{2}$$

and the condensed second member is:

$$\overline{F}_j^s = F_j^s - K_{ji}^s K_{ii}^{s^{-1}} F_i^s \tag{3}$$

and so:

$$\boxed{\overline{K}_{jj}^s q_j^s = \overline{F}_j^s} \tag{4}$$

(4) can be written for each substructure:

$$\begin{cases} \mathrm{q}_\mathrm{j}^\mathrm{s} : \text{dof of boundary of the substructures} \\ \overline{\mathrm{q}}_\mathrm{j} : \text{dof of boundary of the complete structure} \end{cases} \tag{5}$$

Then, if β^s is the matrix of localization for the substructure s:

$$q_j^s = \beta^s \overline{q}_j \tag{6}$$

The global condensed stiffness matrix is given by:

$$\overline{K}_{jj} = \sum_s \beta^{s^T} \overline{K}_{jj}^s \beta^s \tag{7}$$

and the global condensed second member by:

$$\overline{F}_j = \sum_s \beta^{s^T} \overline{F}_j^S \tag{8}$$

to get the global system:

$$\boxed{\overline{K}_{jj} \overline{q}_j - \overline{F}_j = 0} \tag{9}$$

Finally, the complete solution is obtained by:

$$\begin{cases} q_j^s = \beta^s \overline{q}_j \\ q_i^s = \Phi_{ij}^s q_j^s + q_{i0}^s \end{cases} \tag{10}$$

with:

$$\begin{cases} \Phi_{ij}^s = -K_{ii}^{-1} K_{ij} \\ q_{i0} = K_{ii}^{-1} F_i \end{cases} \tag{11}$$

The method can be subdivided into three phases and represented in Fig. 3.

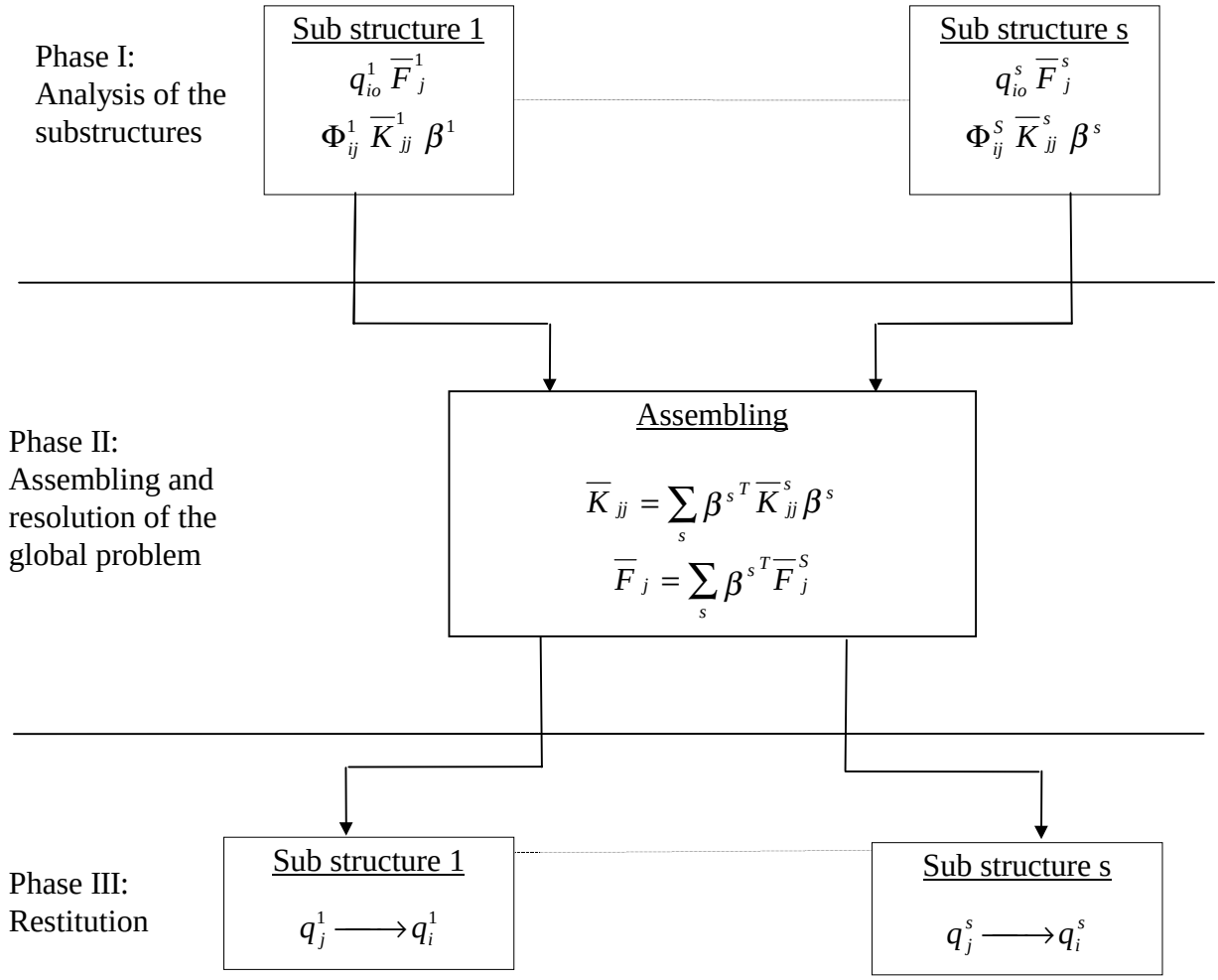

Fig. 3: Classical sub-structure method.

1.2. Multi-levels substructuring method [3]

This method is applied to all the structures invariant in one direction. In general terms the idea is to assemble two identical layers, and to eliminate the middle nodes. The first level layer has just one element in the invariant direction. After n steps of this method, the stiffness matrix condensed on the extremity of 2^n starting sub-structures (Fig. 4) is obtained.

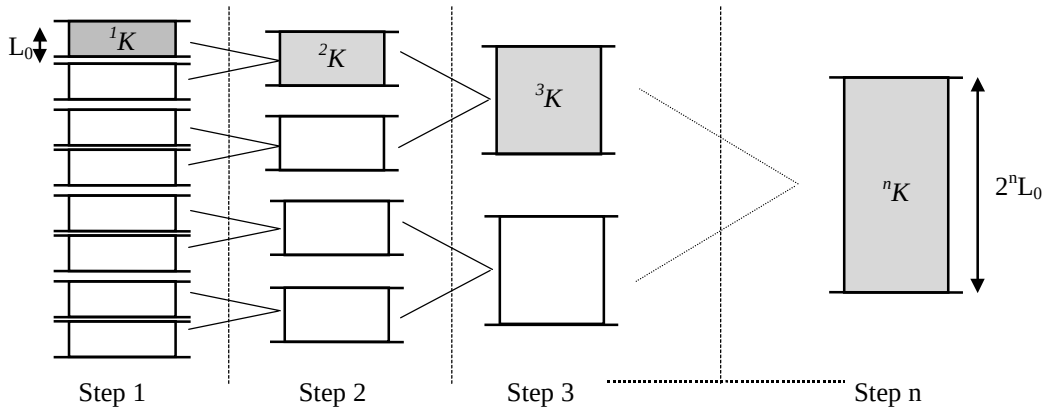

Fig. 4: multi-level substructuring method

It is established by recurrence that at the step i, the stiffness matrix $\lfloor {}^{i}K \rfloor$ condensed on the extremities, could be decomposed as shown in (12):

$$ {}^{i}K = \begin{bmatrix} {}^{i}S^{g} & {}^{i}S^{gd} \\ {}^{i}S^{gd} & {}^{i}S^{g} \end{bmatrix} + \begin{bmatrix} {}^{i}Q^{g} & {}^{i}Q^{gd} \\ -{}^{i}Q^{gd} & -{}^{i}Q^{g} \end{bmatrix} \tag{12} $$

with

$$ {}^{i}S^{g} = {}^{i-1}S^{g} - \frac{1}{2}\left({}^{i-1}S^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}S^{gd} - {}^{i-1}Q^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}Q^{gd} \right) \tag{13} $$

$$ {}^{i}Q^{g} = {}^{i-1}Q^{g} - \frac{1}{2}\left({}^{i-1}Q^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}S^{gd} - {}^{i-1}S^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}Q^{gd} \right) \tag{14} $$

$$ {}^{i}S^{gd} = -\frac{1}{2}\left({}^{i-1}S^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}S^{gd} - {}^{i-1}Q^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}Q^{gd} \right) \tag{15} $$

$$ {}^{i}Q^{gd} = -\frac{1}{2}\left({}^{i-1}Q^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}S^{gd} - {}^{i-1}S^{gd}\, {}^{i-1}S^{g^{-1}}\, {}^{i-1}Q^{gd} \right) \qquad 16) $$

We can consider that after n steps in the sense of [4] a <u>"superelement"</u> ($2^{n}L_0$ in length) is obtained. It contains only nodes on its extremities with a stiffness matrix ${}^{n}K$.
The complete solution can be obtained recursively if the n matrices ${}^{i}K$ for $i \in \{1,...,n\}$ are stored by

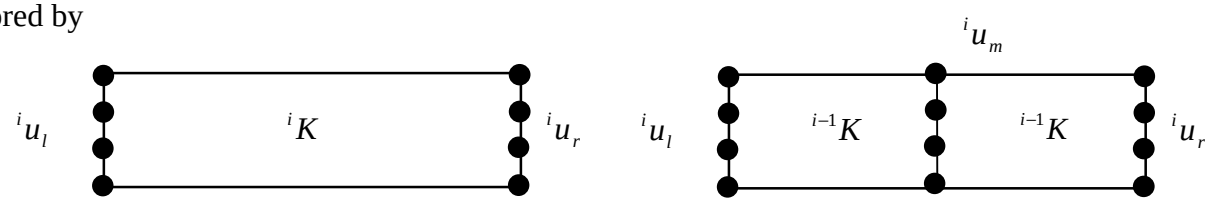

Fig. 5: Deduction of the middle dof.

$$ \boxed{ {}^{i}U_m = \frac{1}{2}\, {}^{i-1}S^{g} \left(\left(-{}^{i-1}S^{gd} + {}^{i-1}Q^{gd} \right) {}^{i}U_d - \left({}^{i-1}S^{gd} + {}^{i-1}Q^{gd} \right) {}^{i}U_g \right) } \tag{17} $$

To take the geometric non-linearity into account, and according with [4] the stiffness matrix can be decomposed in:

$$ {}^{i}K = {}^{i}K_0 + {}^{i}K_\sigma + {}^{i}K_L \tag{18} $$

with $\begin{cases} {}^{i}K_0 \text{ is the usual small displacements stiffness matrix,} \\ {}^{i}K_\sigma \text{ is a symmetric matrix dependent on the stress level,} \\ {}^{i}K_L \text{ is the large deformation matrix due to the large displacement.} \end{cases}$

If the displacements are small enough, ${}^{i}K_L$ can be neglected. Moreover we can demonstrate that ${}^{i}K_\sigma$ can be written like ${}^{i}K_0$, i.e.:

$$ {}^{i}K_{\sigma} = \begin{bmatrix} {}^{i}S_{\sigma}^{g} & {}^{i}S_{\sigma}^{gd} \\ {}^{i}S_{\sigma}^{gd} & {}^{i}S_{\sigma}^{g} \end{bmatrix} + \begin{bmatrix} {}^{i}Q_{\sigma}^{g} & {}^{i}Q_{\sigma}^{gd} \\ -{}^{i}Q_{\sigma}^{gd} & -{}^{i}Q_{\sigma}^{g} \end{bmatrix} \tag{19} $$

So, globally this treatment does not modify the method previously exposed

1.3. Combination of classical and multi-level substructuring methods

The E.F.B. can be divided in three parts (Fig. 1): they are composed of composite sticks put into three kinds of matrices, the first is an elastomeric matrix, the second one is a more rigid elastomeric matrix and the last one is an assembly of elastomer and composite materials.

All these parts are invariant in one direction (z-axis). They will be substituted by their equivalent "superelements" determinated by the multi-level sub-structuring method. From this phase the three stiffness matrices ${}^{n_1}K^1$, ${}^{n_2}K^2$, ${}^{n_3}K^3$ are deduced, where n_1, n_2, n_3 are the number of steps of the multi-level sub-structuring. Consistent with the notation of (12), we have:

$$ {}^{n_s}K^s = \begin{bmatrix} {}^{n_s}S^{g\,s} & {}^{n_s}S^{gd\,s} \\ {}^{n_s}S^{gd\,s} & {}^{n_s}S^{g\,s} \end{bmatrix} + \begin{bmatrix} {}^{n_s}Q^{g\,s} & {}^{n_s}Q^{gd\,s} \\ -{}^{n_s}Q^{gd\,s} & -{}^{n_s}Q^{g\,s} \end{bmatrix} s \in \{1;2;3\} \tag{20} $$

So considering the complete structure as a new assembling of three "superelements", the three phases of the classical sub-structuring method can be used, ${}^{n_s}q_{i0}^{s}$, ${}^{n_s}\overline{F}_{j}^{s}$, ${}^{n_s}\Phi_{ij}^{s}$, ${}^{n_s}\overline{K}_{jj}^{s}$, ${}^{n_s}\beta^{s}$, $s \in \{1;2;3\}$ are determined on each sub-structures during the first phase. The solution in displacement is obtained on the three "superelements". Then if ${}^{n}K^{s}$ with $s \in \{1;2;3\}$ and $n \in \{1 \ldots n_s\}$ are stored, the complete solution is determined on all the E.F.B.

Clearly, this method consists in adding one phase before and one after the three ones of a classical sub-structuring method, see Fig. 6.

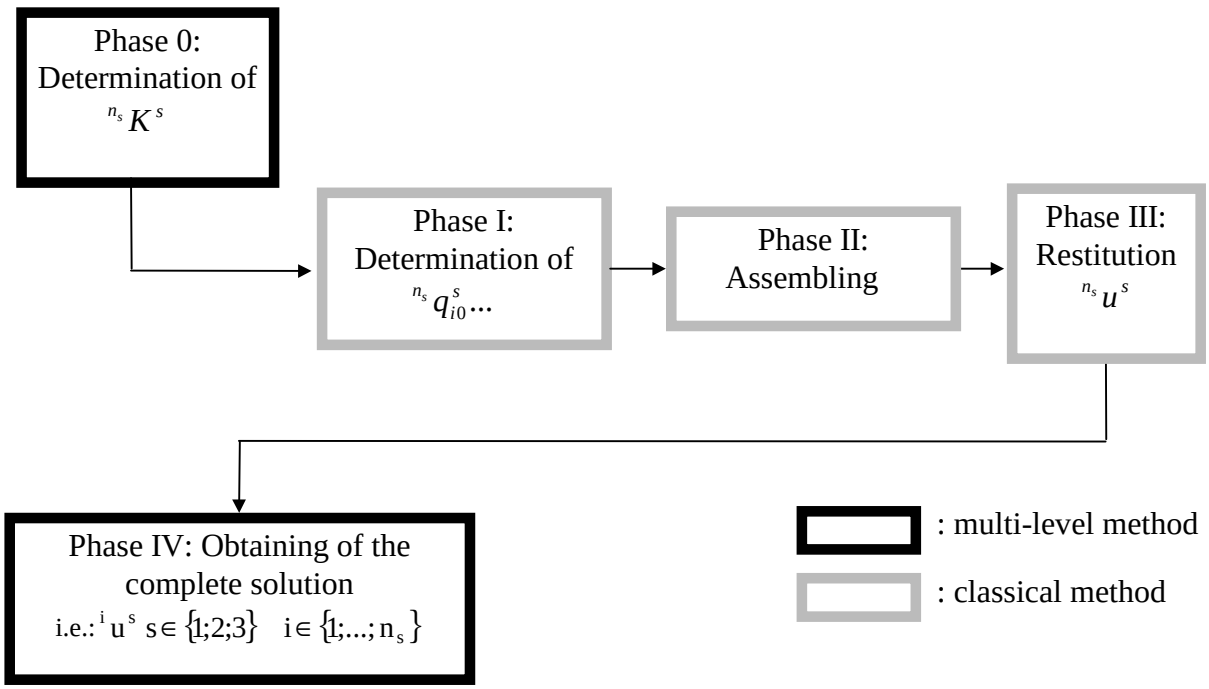

Fig. 6: Combination of the two sub-structuring methods.

2. Results and conclusions

Different loading are applied all under centrifugal forces on the E.F.B, as illustrated in Fig. 1:
- tensile,
- drag,
- twisting
- and bending.

As said previously, an accurate mesh must be used. Its description is given Fig. 7; using symmetric and antisymmetric boundary conditions just one quarter of the structures is considerated. So a layer for the current and the intermediate parts presents 10406 dof and for the fixation 14754 dof. Respectively 64, 16 and 256 layers are adopted. The meshes are realized with iso-parametric 3D elements. All the numerical implementations are made on SIC (system Interactif de Conception) [5]. If the complete 3D problem is solved this leads to a size of approximately two millions of degree of freedom for the numerical problem. So to overcome this difficulty, the combination of the two sub-structuring methods seems to be a good solution.

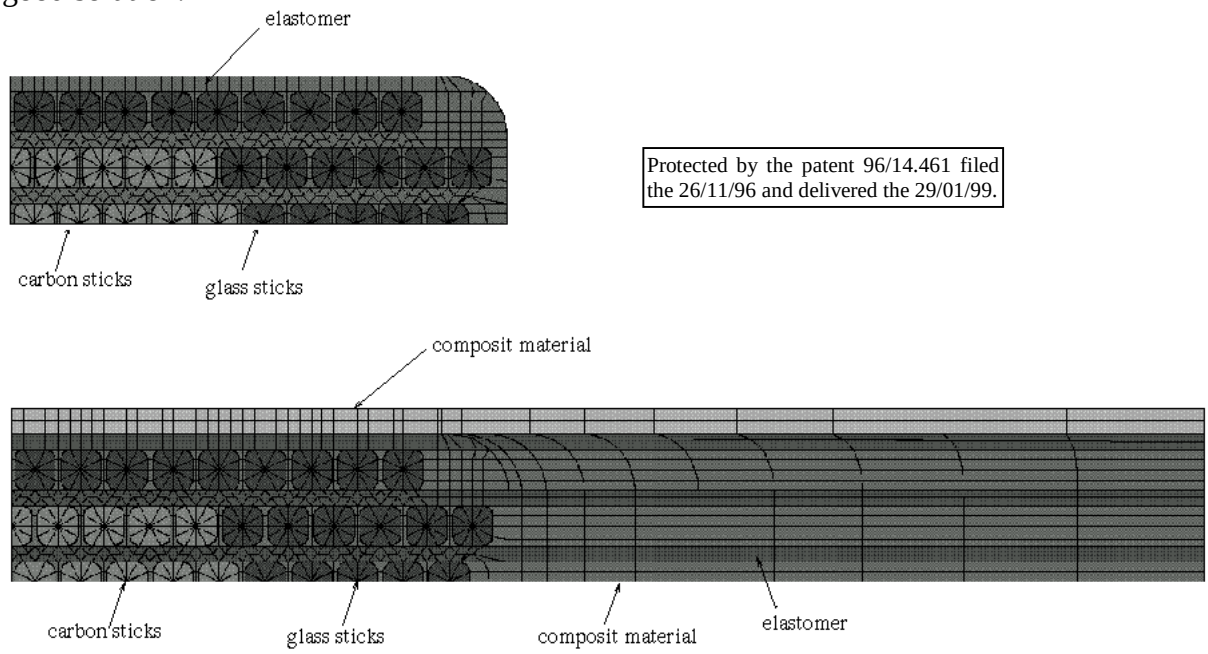

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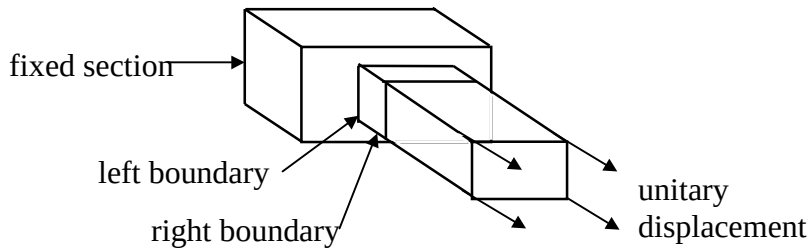
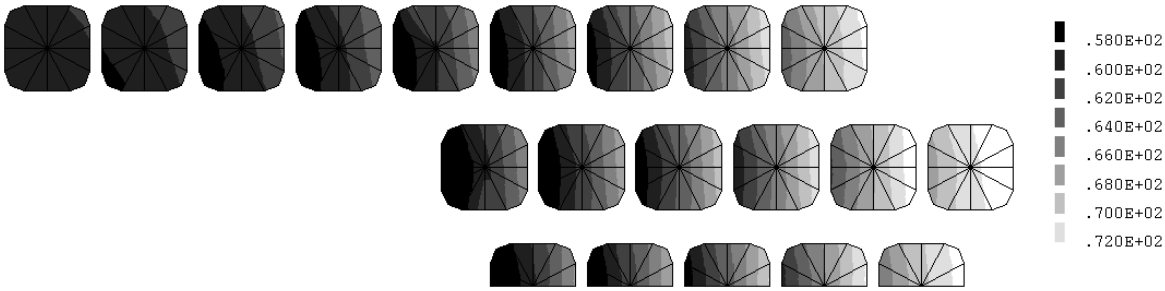
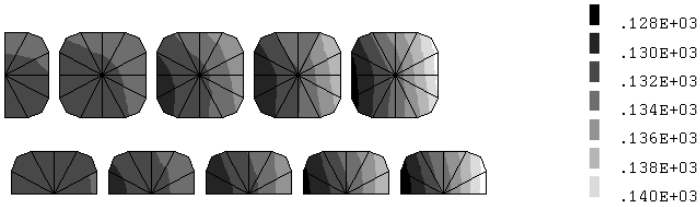
Fig. 7: Mesh description of the current part and the fixation section.

As illustration, different mechanical results are given on the boundaries of the different parts i.e.: the more sensible zones. We impose a unitary displacement on the right extremity of the third substructure, and the left extremity of the first one is fixed.





σ_{zz} in the glass sticks (MPa).



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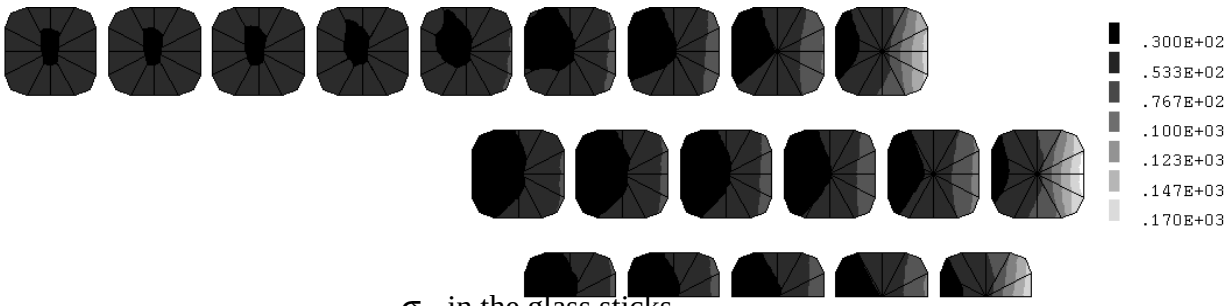
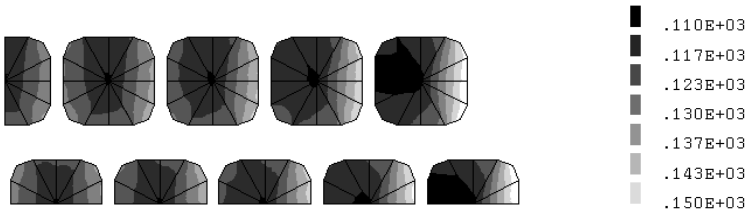
σ_{zz} in the carbon sticks (MPa).

Fig. 8: Stress field on the right boundary under tensile.



σ_{zz} in the glass sticks (MPa).



σ_{zz} in the carbon sticks (MPa).

Fig. 9: Stress field on the left boundary under tensile.

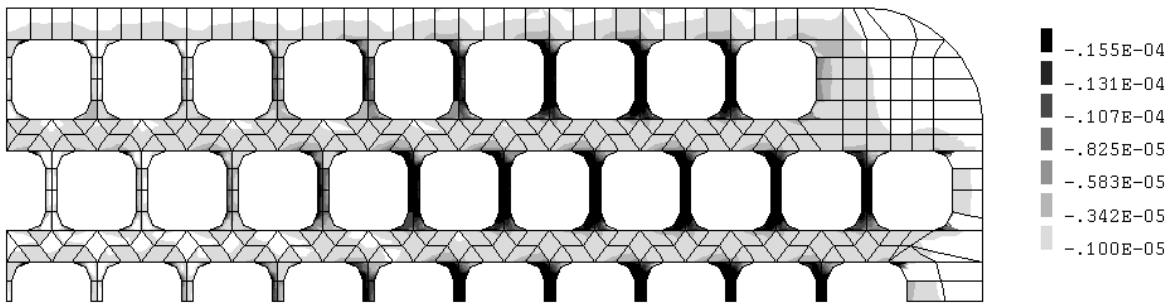


Fig. 10: Second deformations invariant on the right boundary in the elastomer.

On Fig. 8 and Fig. 9, we note that under a tensile loading, the right extremity sticks are the more constrained ones. It is in accordance with experimental results [3].

Furthermore, we can see important concentration of the second deformations invariant $\left(I_2 = \frac{1}{2}\left(Tr(1+2\varepsilon)\right)^2 - Tr(1+2\varepsilon)^2 \right)$ (Fig. 10) in this zone. This involves important deformations in the elastomer.

These phenomena prove the heterogeneous comportment of our structure, and hence the necessity of a complete simulation.

To sum-up, we developed a method enable to simulate some large problems taking geometrical non-linearity into account. The complete structure must be an assembly of structures invariant in one direction (a beam, a plate...). This method uses a combination of different substructuring methods, and leads to the classical results of finite elements calculation like the stress field or the solution in displacement.

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