APPLICATION OF A DAMAGE MODEL FOR HIGH RATE DEFORMATION OF COMPOSITE STRUCTURES

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SUMMARY: After implementing the model into ABAQUS/Explicit software, a plate with a hole is analyzed. First, mesh objectivity of the model in the context of finite element analysis is addressed, and it is shown, by analyzing four different mesh refinements, that the model is objective with respect to the mesh size. Then the model is employed to predict the failure load of a plate with a hole of various diameters. Two different techniques are investigated for failure load prediction namely: mesh and stress limiter. Mesh limiter is based on using a specific mesh size. The stress limiter is based on limiting the minimum stress after damage. Both techniques rely on a known failure load of a particular geometry to evaluate the limits. The failure loads predicted using both techniques conform reasonably well to those predicted by Whitney and Nuismer's model and experiment.

KEYWORDS: continuum damage, mesh sensitivity, high-rate deformation, graphite epoxy, rate dependence, isotropic damage tensor.

INTRODUCTION

Today, the most powerful tool for analyzing complex structures available to structural analysts is the finite element method. Hence, in order to get the most use of any material model, it must be incorporated into a finite element code. Although in engineering applications structures with complicated geometry may be used, they consist of some fundamental structural units. It is, therefore, rational to test models dealing with damage and failure analysis using simple structural units, which contain a component that introduces a non-homogenous state of stress. These components can be referred to as "stress raisers," for they increase the stress in their neighborhood and cause stress gradients. Amid these stress raisers are holes, notches and slits. With the help of the theory of elasticity, the stress gradient can be characterized; however, it is difficult to predict the failure load in the presence of stress raisers without considering the redistribution of stress. Therein lies the challenge of damage models as how to account for this material behavior.

One very important structural unit is a plate with a hole, where the hole is the stress raiser. From a practical point of view, their importance becomes especially clear when it comes to the use of composite materials. Since structures made of these materials cannot be joined by welding, the most

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widely used alternative joining method is the use of bolts which require drilling holes in the structure. Therefore, a plate with a hole was selected as a suitable application of the developed model. Several approaches have been followed by researchers to investigate the failure of notched composite laminates (in many references notch is used to refer to all stress raisers such as holes, slits, etc.). A comprehensive review of the previous works is given in [1].

One of the early approaches, employing linear elastic fracture mechanics concepts, was proposed by Waddoups *et al.* [2]. They developed a two-parameter model consisting of a stress intensity factor and a characteristic length (which was referred to as the length of the intense energy region). From the results of a test on a laminate with a particular hole size, the strength of a laminate containing any hole size could be predicted. Whitney and Nuismer [3,4] proposed a two-parameter model which used the unnotched strength and a “characteristic distance” based on the normal stress distribution ahead of a hole. They suggested two failure criteria: 1) point stress criterion (PSC), 2) average stress criterion (ASC). Failure is assumed to occur when the normal stress at some characteristic distance or the average stress over some characteristic distance ahead of the hole becomes equal to the unnotched strength of the material. Following this study, several other models emerged focusing basically on the characteristic distance [5,6].

Another set of models use the basic properties of composite laminates such as the unnotched strength and an entity which represents a characteristic length, whether it is termed as the length of the intense energy region or critical damage zone [7,8]. The underlying idea behind the introduction of a characteristic length in all these models seems to be avoiding the use of the high stress concentration at the notch edge (or in the case of a crack: infinite stress at the crack tip) as a measure for failure prediction. This concept has also been used in local approach solutions to fracture mechanics problems: a crack is assumed to grow when some critical value of a physical quantity is reached at some critical distance ahead of the crack tip [9].

Although reasonable agreement with experimental results has been obtained by these models, they lack versatility to deal with other types of stress raisers or a unit with a combination of the same stress raisers. However, being specialized for certain problems, some of these models, like that of Whitney and Nuismer give simpler and quicker solutions.

Another class of models referred to as progressive damage models, use conventional failure criteria for different damage modes in composite laminates on an element basis [10,11]. Good agreement with experimental results have been reported. Unlike the previously mentioned models, they are more versatile. However, since the progressive damage models do not consider the degradation of elastic parameters until the failure point (sudden failure), they cannot model accurately the behavior of the material when the behavior is governed by considerable nonlinearity caused by damage.

Except for a few models such as that of Chow and Sze [12], the application of continuum damage models for analysis of notched structures has not been adequately explored despite the fact that the numerous continuum damage models have been developed. The aim of this study is to demonstrate the applicability of a continuum damage model for analysis of composite laminate structures.

RATE DEPENDENT MODEL

Static and dynamic experimental results, presented in Ref. [13], show that the quasi-isotropic laminates of graphite-epoxy material are rate sensitive even at moderate rates of strain. This type of material system sustains higher strain and stress to failure as the strain rate is increased. Other types of composite materials, metals, rocks, and concrete are also reported to show rate dependent behavior [14,15]. Generally, higher strength and strain to failure are obtained for higher rate of loading.

For these materials, one hypothesis to account for such behavior is based on damage evolution in the material and its dependency on the rate of loading. As the rate of loading is increased, according to this hypothesis, there is less time for damage to develop. And, therefore, the amount of accumulated damage at a particular strain level decreases as the strain rate increases. This, in turn, causes the material to withstand higher load and strain to failure for higher rates. The hypothesis is supported by experimental results [16,17].

To account for rate dependency in the developed damage model, the damage evolution equations presented in Ref. [18] are modified so that the damage rate is a function of both deformation and rate of deformation as expressed in the following equations:

$$\frac{d\delta}{dt} = c_1 sign(I_{1e}) * \left(\frac{\dot{J}_1}{\dot{J}_\circ}\right)^q \left(\frac{J_1 - J_{1t}}{J_{1t}}\right)^m \qquad \text{if } J_1 > J_{1t} \tag{1}$$

$$d\delta = 0 \qquad \text{if } J_1 \le J_{1t} \text{ or } \dot{J}_1 < 0$$

$$\frac{d\beta}{dt} = c_2 \left(\frac{\dot{J}_2}{\dot{J}_\circ}\right)^r \left(\frac{J_2 - J_{2t}}{J_{2t}}\right)^n \qquad \text{if } J_2 > J_{2t} \tag{2}$$

$$d\beta = 0 \qquad \text{if } J_2 \le J_{2t} \text{ or } \dot{J}_2 < 0$$

where δ and β are the independent members of the fourth-rank isotropic damage tensor, J_1 and J_2 are the "damage forces" which are measures of deformation (strain invariants), $\dot{J}_1$ and $\dot{J}_2$ are the deformation rates, J_{1t} and J_{2t} are the thresholds below which no damage occurs, and the four parameters q, m, r, and n are the material parameters. $\dot{J}_\circ$ is used in the denominator to normalize the deformation rate. Having the dimension of T^{-1}, the constants c_1 and c_2 establish the dimensional equivalence of two sides of equations. In this study, the constants c_1, c_2, and $\dot{J}_\circ$ are considered to be equal to one.

The way in which the evolution equations are postulated here, does not necessarily yield the increase of strength or strain to failure with the increase of deformation rate. In fact, depending on the values of powers q and r, the opposite may hold. It can be shown that reaching the same strain (or J) with different rates can result in different damage accumulation (β) depending on the value of r:

if $r < 1$: smaller damage with higher rates (or higher strength with higher rates),
$r > 1$: higher damage with higher rates (or lower strength with higher rates),
$r = 1$: no effect of strain rate on damage parameters.

The further r is from unity, the more rate sensitivity is represented. Rate dependent materials, to the author's knowledge, generally fall in the first category, requiring $r < 1$.

To demonstrate the rate-dependency behavior, simulations based on the one-dimensional form of the rate-dependent model were performed using the MATLAB software. Figure 1 shows the results of the simulations: stress-strain curves for three different rates, setting q and r at 0.95. The stress-strain curves in this graph are based on the use of material parameters determined for quasi-isotropic graphite-epoxy laminates by experiment. The strain rate of 200 s^{-1} represents the approximate average strain rate of the dynamic experiments

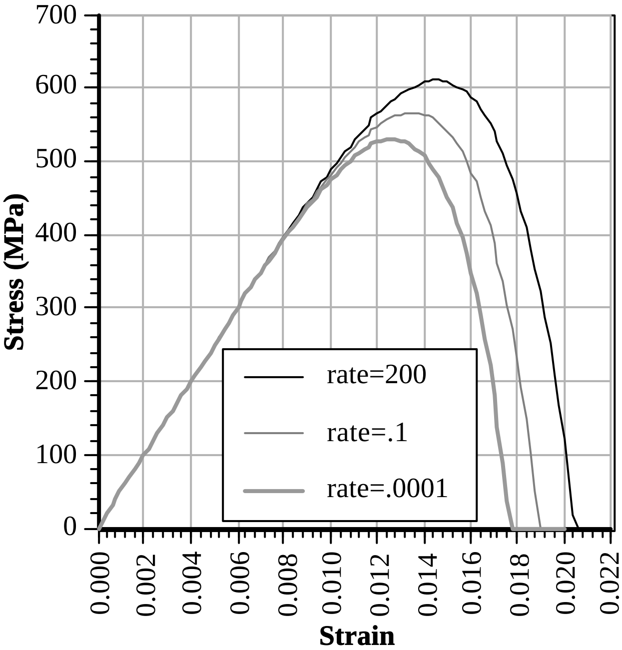

Fig. 1 Simulation of rate dependent behavior of the quasi-isotropic graphite-epoxy laminate.

in [13] and the strain rate of 0.0001 s^{-1} represents the static tests.

EXPERIMENT

The phenomenological model uses new material parameters which have to be determined by experiments. In addition, experiments were performed to validate the model. The experiments involve material and structural tests at a low strain-rate with a hydraulic MTS machine and at a high strain-rate using the newly developed Hopkinson bar technique for tension testing. A description of the Hopkinson bar, test procedure, and material test results can be found in Ref. [13]. The composite system investigated is a quasi-isotropic graphite-epoxy laminate. The results of material tests are the stress-strain relationship; which are used to validate and determine the material parameters introduced in the model. The results of structural tests are failure loads, for a plate with a hole subjected to tensile load, which are used to evaluate the applicability of the model for structural analysis. The results of structural tests are compared with predictions of the model in a later section. The term material testing is used for cases when maximum possible homogeneity of deformation in the specimen is desired to obtain the fundamental material properties. Structural testing here refers to deliberately induced inhomogeneity of deformation in the specimen by means of holes, notches etc.

STRUCTURAL ANALYSIS

As mentioned before, in order for the model to be tested and useful for structural analysis, it must be incorporated into a finite element code. For this purpose, the model was programmed in FORTRAN language and supplied as a subroutine to the ABAQUS/EXPLICIT finite element analysis software. Before introducing complexities of a structure, the model was verified using a single element. This way, potential problems and errors (due to programming and interfacing) can be traced in an easier manner, ensuring that for later structural analysis a correct model is used.

Finite Element Model

In this section, a description of the model geometry, boundary conditions, loading and other details are laid out.

Geometry

Dimensions of the structure modeled for the mesh sensitivity investigation are shown in Figure 2a. It should be noted that these dimensions correspond to those of the specimens employed in the experimental study, and hence, the limitations on the specimen’s dimensions due to Hopkinson bar testing are also imposed for numerical analysis. For failure load analysis, the same geometry as shown in Figure 2a is used, except for the hole diameter. Different hole diameters were considered for this study: 3.00, 3.58, 3.94, and 4.94 mm.

Boundary Conditions and Loading

Due to symmetry only a quarter of the structure is modeled as shown in Figure 2b. The nodes on the left end are subjected to time-varying displacement in the negative x-direction, simulating a displacement control experiment. The profile of the displacement versus time, simulating experimental deformation rate to which the specimens were subjected, is extracted from the results of Hopkinson bar testing.

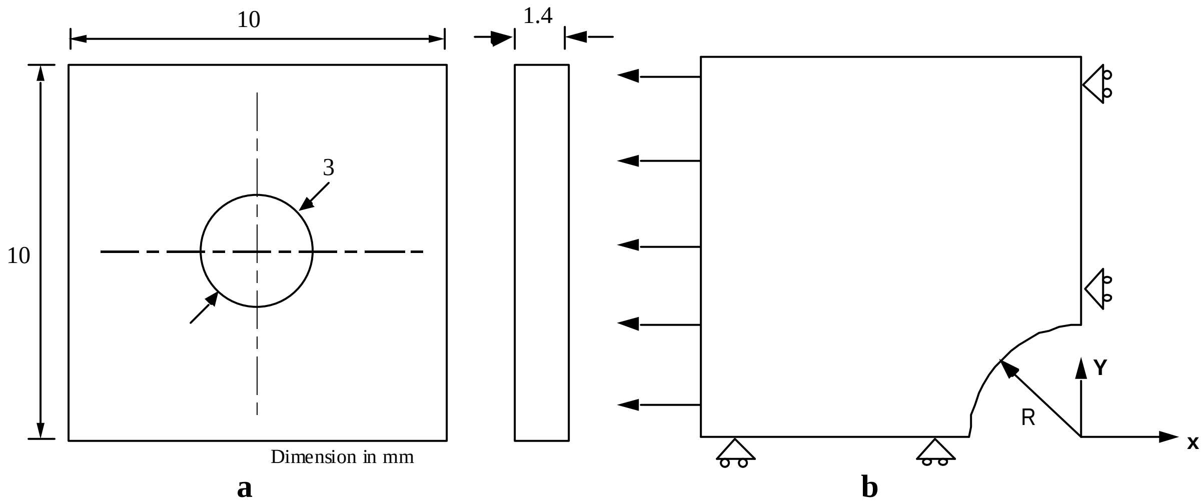

Fig 2 a) The geometry used in constructing the F.E. meshes. b) Boundary conditions for a quarter of the specimen

Element Type and Mesh

Due to the small thickness of the plate, plane stress deformation was assumed, and consequently four-node plane stress elements were used to create the appropriate meshes. Four meshes, having the same geometry but different mesh refinement, were created to investigate the mesh objectivity of the present damage model. These meshes have a total number of 110, 300, 500, and 810 elements each. Four other meshes, having the same mesh refinement but different hole diameter, were constructed in order to predict the failure load. These four meshes include a 3.00, 3.58, 3.94, and 4.94 mm diameter hole in a 10 by 10 mm plate each.

Mesh Objectivity

One of the problems associated with continuum damage models when used in conjunction with finite element analysis is the mesh sensitivity, that is dependency of the solution on the mesh size without convergence to a reasonable solution. This difficulty can be due to strain softening which is the decrease of stress with increasing strain under deformation control [19]. As the strain softening occurs, the damage tends to localize in a narrow region (usually the size of the element used) as the mesh is refined, causing the dissipated energy to approach zero. Several solutions have been proposed to overcome problems of mesh sensitivity associated with damage localization. The most notable models can be classified in three groups: a) non-local damage theories, b) higher gradient theories, c) rate-dependent theories. Both higher gradient and non-local models are based on the fact that the interaction between microstructural damage is not local. It should be noted that the rate dependency approach cannot be used for the sole reason of overcoming the mesh sensitivity if it is not the intrinsic behavior of the material itself. A brief review of these approaches and the pertinent references are given in Ref. [20].

The developed model here falls in the category of rate-dependent theories, however, it was found that the damage localization, and hence mesh sensitivity, still occurs to some extent. The mesh objectivity in the present model is achieved through the use of a technique (here referred to as stress-limiter technique) in an attempt to simulate the stress redistribution, due to damage, at the vicinity of a notch. This technique is explained in detail in the next section.

Figure 3 depicts the failure load versus number of elements. The failure load is marked by the peak of the load-displacement or load history curves. It is seen that the failure load converges as the number of elements increases. The failure load obtained by an 810-element mesh is only 0.7% and

0.5% less than that of 500-element mesh for dynamic and static analysis respectively. Figure 4 shows the extent of damage (represented by damage parameter β) for different meshes at time 58.6 μs. It is seen that the extent of damage is independent of the mesh size. In other words, damage localization has not occurred.

Failure Load Prediction

In order for the implemented model to correctly predict the failure load, a damage process which causes redistribution of the stress gradient in the vicinity of hole is simulated. For the problem at hand, when the load is increased so that the tensile stress at the edge of the hole reaches the threshold

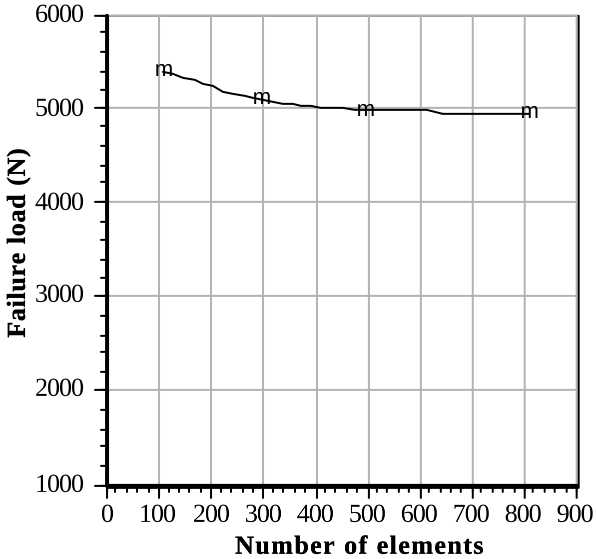

Fig. 3 Results of mesh objectivity study on five meshes for dynamic simulation

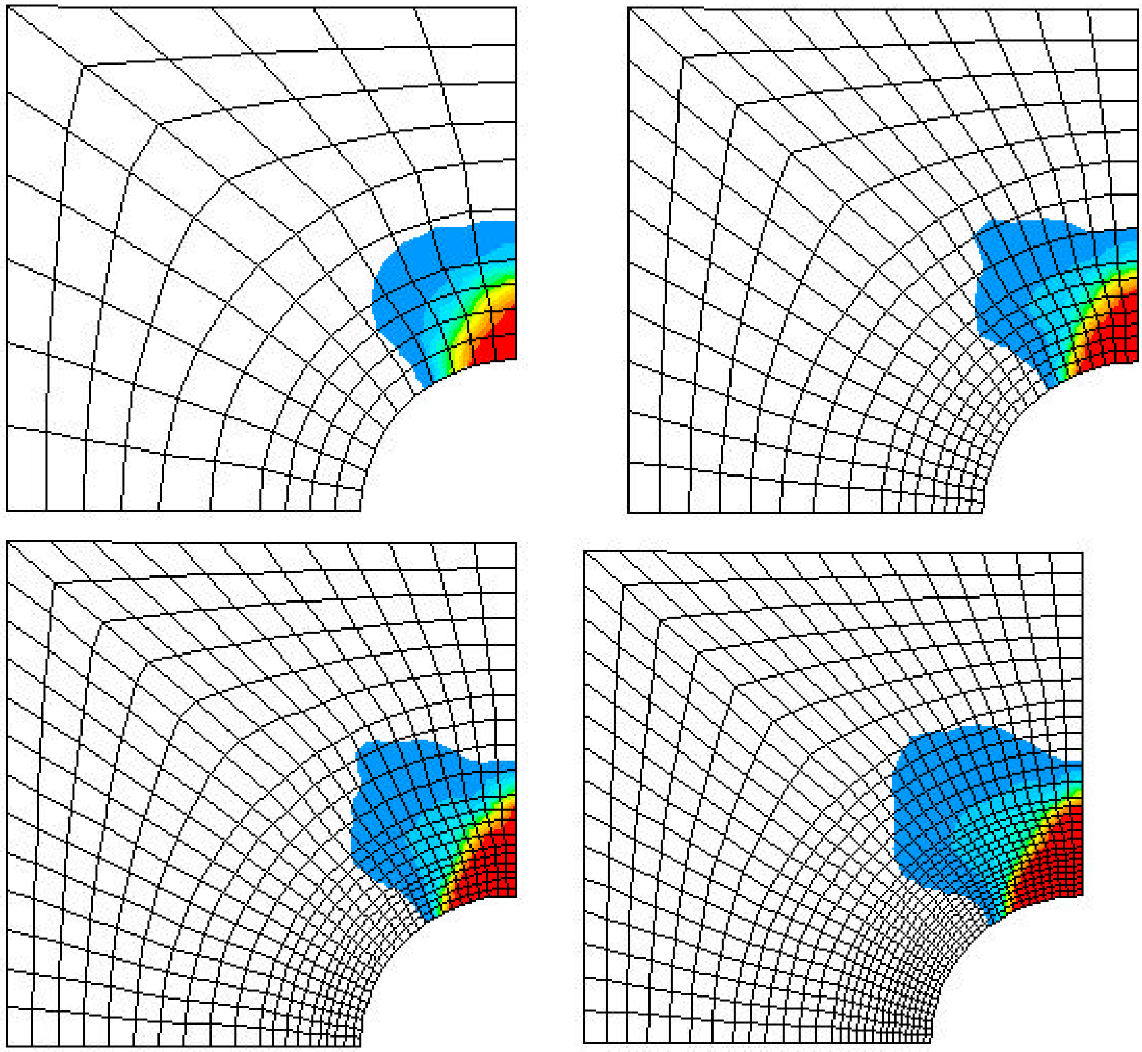

Fig. 4. The extent of damage propagation for four different mesh refinements.

stress, a damage zone starts forming. As the load is increased, the damage zone propagates further into the laminate and the stress at the edge of the hole reaches the strength of the unnotched laminate (material), σ_o, followed by stress relaxation at this region. Therefore, at this time, the stress level at the edge of the hole is lower than σ_o and the stress level at some point ahead of the damage zone is equal to σ_o. In composite materials, the stress relaxation occurs mainly due to different damage mechanisms such as fiber debonding and matrix cracking. Like plastic flow in metals, these damage mechanisms cause some degree of notch insensitivity.

For the present damage model, to simulate the explained damage process, in the context of finite element analysis, a technique here referred to as "stress limiter technique" is used. In the stress limiter method, the damage parameters are not permitted to increase to values causing the stress to reach zero in regions of high stress concentration. Imposing limits on the damage parameter has the following consequence: when the stress in an element in the high-stress region exceeds the unnotched strength of the material, due to damage, stress decreases but the element does not lose its total stiffness, in contrast to a perfectly brittle material which does lose its total stiffness. The limit values of the damage parameters are determined based on the experimental failure load for 3-mm diameter hole (adjusting the failure load to its experimental counterpart by setting the limits for damage parameters). The evaluated damage parameters can then be used to predict the failure load for other geometries. The limit values of damage parameters were determined to be: β=0.32 and δ=0.18. This method has also been employed by Murakami and Liu [21] who used a moderately low value for critical damage parameter to suppress the damage localization in studying creep fracture.

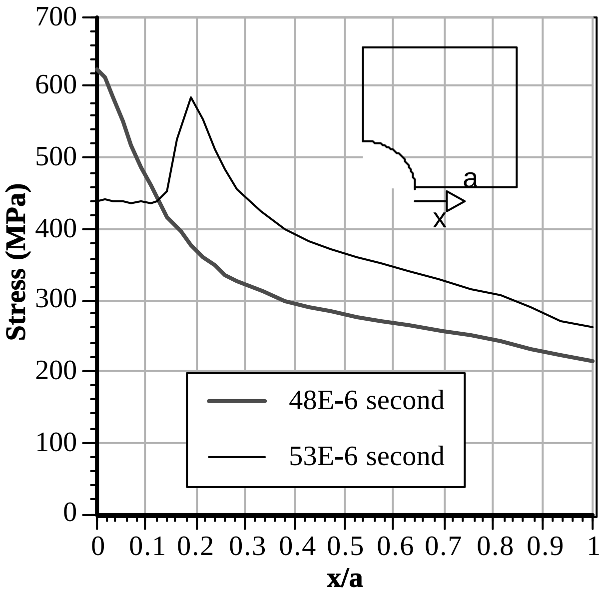

Fig. 5 Stress distribution ahead of the hole before and after reaching the material strength

Figure 5 shows the stress distribution ahead of the hole toward the edge of the laminate, the high-stress path, before and after the time at which stress reaches to the unnotched tensile strength σ_o, for dynamic tests. Figure 6 shows the pattern of damage zone corresponding to the failure load. As it is seen, the intense damage zone extends through the whole laminate width. The stress-strain curve which yields the damage pattern shown in this figure, corresponding to limit values mentioned above, is depicted in Figure 7. If no limit is imposed on the damage parameters and therefore, the stress is allowed to drop to zero, a perfectly brittle behavior is then simulated for which damage is confined in a narrow region ahead of the hole through the laminate width and perpendicular to the tensile load. An alternative technique, although not as attractive as the stress limiter technique, consists of imposing a limit on the element size. The technique can be referred to as mesh limiter by analogy with the stress limiter technique. The technique is based on the "characteristic length" concept used in many models as was observed in Whitney and Nuismer's model. By limiting the element size, which one expects to be close to the characteristic length or distance, stress is averaged over a larger area. A high stress concentration at the edge of a stress raiser is thereby avoided. The element size is usually fixed, using a known failure load for one specific geometry, and then the same element size is used to predict the failure load for other geometries. The technique has been used in local approaches to fracture problems [9] where the local mesh size is fixed through "critical length" considerations or statistics of defects in the volume element.

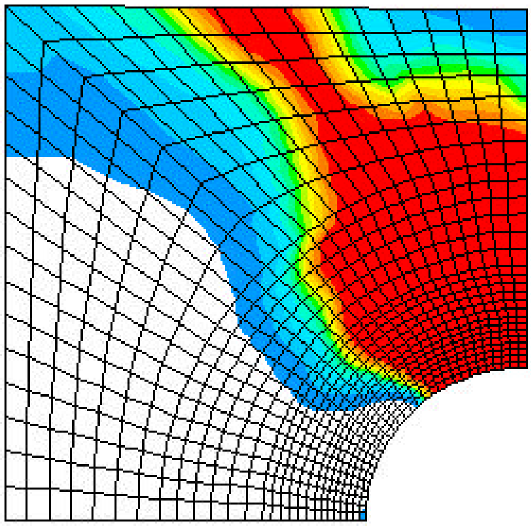

Fig. 6 The patterns of damage zone, corresponding to failure load when

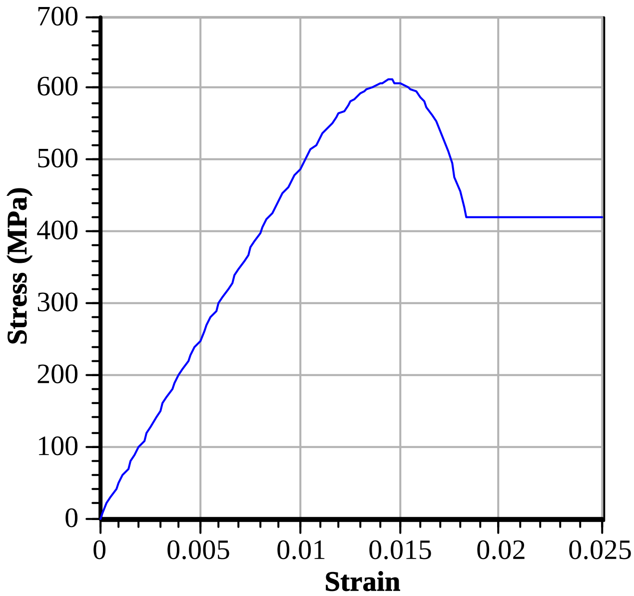

Fig. 7 Stress-strain curve in damaged elements when the stress-limiter technique is used.

To demonstrate the application of this technique for the problem in hand, two meshes were created for 3 and 5mm diameter holes.

The critical elements are of the same size for both meshes, having dimensions of 0.7 by 0.6 mm which are approximately the same as the characteristic distance used in the application of Whitney and Nuismer's model.

To have a comparison with predictions of other models, the model developed by Whitney and Nuismer is selected. The point stress criterion of this model assumes that failure occurs when the normal stress at some characteristic distance ahead of the hole becomes equal to the unnotched strength of the material. The characteristic distance, d_0 , was obtained by corresponding the failure load with that of experimental results for one specific hole size (3 mm diameter hole). It was then used to calculate the failure load for other hole sizes. d_0 was found to be 0.7 mm. Whitney and Nuismer's model is also used to predict failure load for dynamic simulations. The characteristic distance was assumed to be the same as that used for static simulations; however, the material strength σ_0 is taken as that obtained in dynamic experiments (dynamic material strength).

Predicted failure loads, for various hole diameters, using the two techniques (stress limiter and mesh limiter) are compared with those of Whitney and Nuismer's model and those predicted using the stress concentration factor criterion for dynamic and static analysis in Figure 8. It is observed from these figures that a good agreement between predictions of the model using the stress limiter technique and those of Whitney and Nuismer's model is obtained. The SCF and notch insensitive curves represent the lower and upper bound of the failure load, respectively. The SCF curve is obtained by applying the full stress concentration factor in calculating failure loads. The notch insensitive curve is obtained simply by multiplying the material strength by the net area of the section containing the hole diameter. The more notch sensitive a material is, typical of brittle materials, the more its characteristic curve is shifted toward the SCF curve. As it is observed from the results presented here, the graphite-epoxy composite laminates are relatively resistant to notch or stress raisers.

In addition, experimental results of structural testing are included in these figures. The main aim of the experiments was to set the maximum limit of the damage parameters and to determine the characteristic distance for the material system used in this study. These experiments were performed using specimens with 3 mm diameter holes for both static and dynamic loading. Moreover, an extra data point was obtained in static tests for a 5.1 mm diameter hole to check the predictions of different methods with experiment. This data point is in close agreement with finite element predictions.

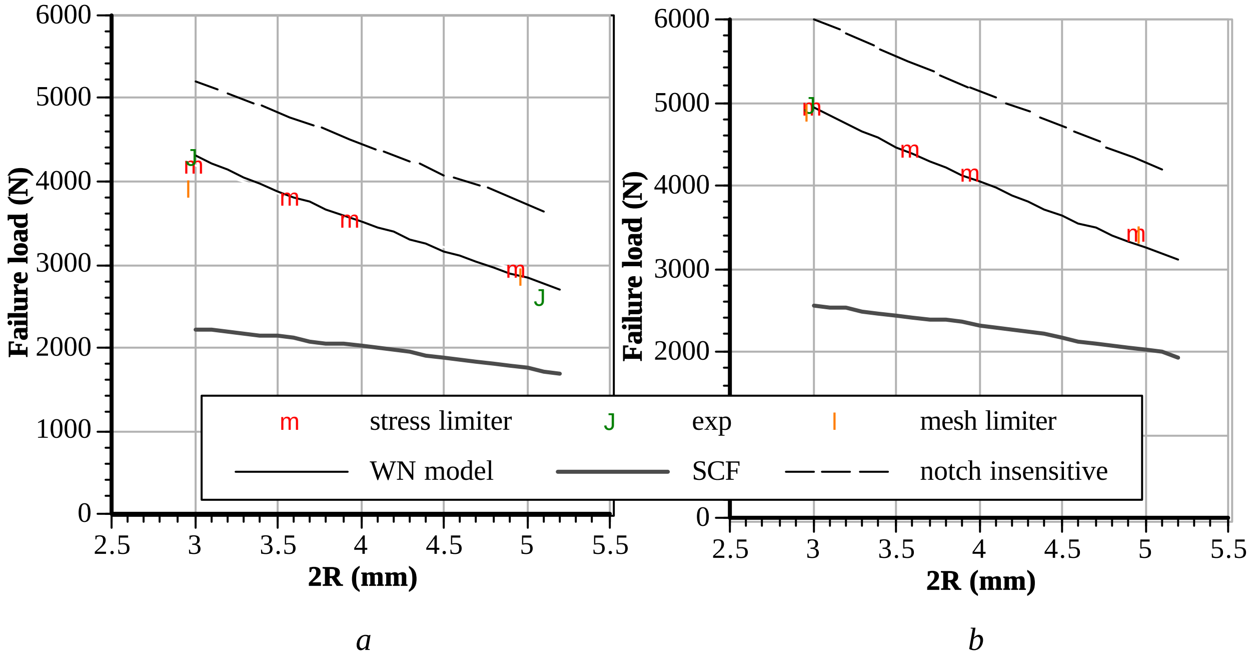

Fig. 8 Failure loads of various hole sizes as predicted by different models and measured for : a) static analysis and b) dynamic analysis

CONCLUSIONS

The applicability of a rate dependent continuum damage model for deformation and failure analysis of composite laminate structures under both static and dynamic loading is demonstrated. It is shown that in order for the model to correctly predict the failure load, a damage process which causes redistribution of the stress gradient in the vicinity of hole has to be simulated. This is achieved by using a technique referred to as stress limiter technique which inhibits stress from falling below a specific level in the damage zone. It is also shown that as a result of employing this technique, in the context of finite element analysis, the model is objective with respect to the mesh size. The predicted failure loads using this technique agree reasonably well with those predicted by Whitney and Nuismer's model and experiment.

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