EXPERIMENTAL AND NUMERICAL DETERMINATION OF STRESS INTENSITY FACTOR IN COMPOSITE MATERIALS

D. Cerniglia[1], V. Nigrelli[2], and A. Pasta[1]

[1] *Dipartimento di Meccanica e Aeronautica,*
Viale delle Scienze, 90128 Palermo, Italy
[2] *Dipartimento di Meccanica, Strutture, Ambiente e Territorio,*
Via G. Di Biasio, 43, 03043 Cassino (FR), Italy

SUMMARY : A procedure to obtain stress intensity factors (SIF) employing electrical strain gages, as used for homogeneous isotropic materials, is extended to composite materials. A single edge notched (SEN) specimen of orthotropic carbon-epoxy laminate is used.
A suitable computer program gives SIF value by means of measured strain. Least squares method is applied to minimize the errors, using a number of strain gages greater than the bare necessities. The obtained SIF value is compared with that calculated by a program based on the dual boundary element method (DBEM) and the J-integral. Experimental and numerical SIF values are very close.
DBEM has allowed to point out that the effects of the orthotropy on SIF are very little.

KEYWORDS : fibre-reinforced composites, fracture, boundary element method, strain gages, stress intensity factor.

INTRODUCTION

Linear elastic fracture mechanics describes the behaviour of cracked bodies and relates the stress field near to the crack tip to remote stress, to size, shape and orientation of the crack and to material properties. In all structural components made with homogeneous and isotropic material, subjected to opening mode, the stress distribution is an invariant and its magnitude can be determined by a single parameter K_I, called stress intensity factor (SIF).
The fracture mechanics concepts employed for homogeneous isotropic materials can be applied to composite materials under the following conditions [1]:

1. crack orientation with respect to the material principal axes must be known;
2. SIF expression for anisotropic case must satisfy also the isotropic one;
3. crack orientation must coincide with one principal direction of elastic symmetry.

Irwin suggested [2] to determine SIF in isotropic materials by strain gages. The method has been applied by Shukla et alii [3] to orthotropic materials, using only one strain gage properly located and oriented. Authors [3] do not rely very much on their results because of a fairly large region around the crack tip where microcracking occurs which might influence the strain

field close to the crack tip. Besides, close to the crack tip, the three dimensional effects become prominent so affecting the results.
In this work a procedure to obtain stress intensity factor employing electrical strain gages, as used for homogeneous isotropic materials, is extended to composite materials. In order to obtain a more reliable measurement, the strain gages are located in a region not very close to the process zone. Since in such a region it is necessary to increase the terms of the two series that supply the stress function, three terms for each series are employed and therefore it is necessary to use six strain gages at least. A number of strain gages greater than the bare necessities and the least-squares method [4] are employed in a suitable computer program to minimize the errors. SIF value obtained in such a way is compared with that obtained by an appropriate software based on dual boundary element method (DBEM) and the J-integral [5].

ANALYTICAL FORMULATION

For an orthotropic material plane body, under plane stress conditions ($\sigma_z = \tau_{yz} = \tau_{xz} = 0$), the stress-strain relations are [3]:

$$\begin{aligned} \varepsilon_{xx} &= a_{11}\cdot\sigma_x + a_{12}\cdot\sigma_y \\ \varepsilon_{yy} &= a_{12}\cdot\sigma_x + a_{22}\cdot\sigma_y \\ \gamma_{xy} &= a_{66}\cdot\tau_{xy} \end{aligned} \tag{1}$$

where: $a_{11}=1/E_L$, $a_{12}= -\nu_{LT}/E_L=-\nu_{TL}/E_T$ and $a_{66}=1/G_{LT}$; L and T, principal material axes, are shown in Fig. 1; E_L, E_T, G_{LT} and ν_{LT} are longitudinal modulus, transverse modulus, shear modulus and Poisson ratio respectively.

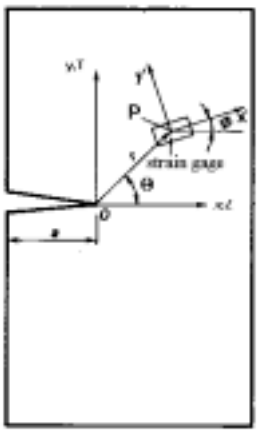

Fig. 1 : Coordinate system

The equilibrium equations are not influenced by anisotropy, so Airy stress function, F, can be considered to define the stress :

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2} \qquad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{2}$$

The compatibility equation through Eqns 1 and 2 gives:

$$a_{11}\frac{\partial^4 F}{\partial y^4} + (a_{66} + 2a_{12})\frac{\partial^4 F}{\partial y^2 \partial x^2} + a_{22}\frac{\partial^4 F}{\partial x^4} = 0 \tag{3}$$

Stress function, F, that determines a balanced stress state, is determined so as to satisfy the compatibility Eqn 3, that is a constant coefficient biharmonic equation. Integration coefficients are obtained by imposing boundary conditions.
For an orthotropic material plane body with crack subject to opening mode, under plane stress condition, Airy function was suggested by Irwin [6]:
where :

$$F = \frac{1}{2}\left(\mathrm{Re}\,\bar{\bar{Z}}_1 + \mathrm{Re}\,\bar{\bar{Z}}_2\right) - \frac{\beta}{2\alpha}\left(\mathrm{Re}\,\bar{\bar{Z}}_1 + \mathrm{Re}\,\bar{\bar{Z}}_2\right) - \frac{\beta}{2\alpha}\left(\mathrm{Re}\,\bar{\bar{Y}}_1 - \mathrm{Re}\,\bar{\bar{Y}}_2\right) \tag{4}$$

$$\alpha = \left[\frac{1}{2}\left(\frac{a_{66}+2a_{12}}{2a_{11}} - \sqrt{\frac{a_{22}}{a_{11}}}\right)\right]^{\frac{1}{2}} \qquad \beta = \left[\frac{1}{2}\left(\frac{a_{66}+2a_{12}}{2a_{11}} + \sqrt{\frac{a_{22}}{a_{11}}}\right)\right]^{\frac{1}{2}}$$

$$\overline{Z}_1 = \overline{Z}(z_1) \qquad \overline{Z}_2 = \overline{Z}(z_2) \qquad \overline{Y}_1 = \overline{Y}(z_1) \qquad \overline{Y}_2 = \overline{Y}(z_2)$$

$$z_1 = x + iy_1 = x + i(\beta+\alpha)y = r_1 e^{i\vartheta_1}$$

$$z_2 = x + iy_2 = x + i(\beta-\alpha)y = r_2 e^{i\vartheta_2}$$

$$\overline{Z} = \frac{d\overline{\overline{Z}}}{dz} \qquad \mathrm{Z} = \frac{\mathrm{d}\overline{\mathrm{Z}}}{\mathrm{dz}} \qquad \mathrm{Z}' = \frac{\mathrm{dZ}}{\mathrm{dz}}$$

$$\overline{Y} = \frac{d\overline{\overline{Y}}}{dz} \qquad \mathrm{Y} = \frac{\mathrm{d}\overline{\mathrm{Y}}}{\mathrm{dz}} \qquad \mathrm{Y}' = \frac{\mathrm{dY}}{\mathrm{dz}}$$

The stress field equation is obtained deriving Eqn 4 according to Eqns 2 and using the Cauchy-Riemann equations:

$$\frac{\partial \operatorname{Re} Z}{\partial x} = \frac{\partial \operatorname{Im} Z}{\partial y} = \operatorname{Re} Z' \qquad \frac{\partial \operatorname{Re} Y}{\partial x} = \frac{\partial \operatorname{Im} Y}{\partial y} = \operatorname{Re} Y'$$

$$\frac{\partial \operatorname{Im} Z}{\partial x} = -\frac{\partial \operatorname{Re} Z}{\partial y} = \operatorname{Im} Z' \qquad \frac{\partial \operatorname{Im} Y}{\partial x} = -\frac{\partial \operatorname{Re} Y}{\partial y} = \operatorname{Im} \mathrm{Y}'$$

It is obtained:

$$\sigma_x = \frac{\beta^2-\alpha^2}{2\alpha}\left[(\alpha+\beta)\operatorname{Re} Z_1 + (\alpha-\beta)\operatorname{Re} Z_2\right] + \frac{\beta}{2\alpha}\left[(\alpha+\beta)^2 \operatorname{Re} Y_1 - (\beta-\alpha)^2 \operatorname{Re} Y_2\right]$$

$$\sigma_y = \frac{\alpha-\beta}{2\alpha}\operatorname{Re} Z_1 + \frac{\alpha+\beta}{2\alpha}\operatorname{Re} Z_2 - \frac{\beta}{2\alpha}\operatorname{Re} Y_1 + \frac{\beta}{2\alpha}\operatorname{Re} Y_2$$

$$\tau_{xy} = \frac{\alpha^2-\beta^2}{2\alpha}(\operatorname{Im} Z_1 - \operatorname{Im} Z_2) - \frac{\beta}{2\alpha}\left[(\beta+\alpha)\operatorname{Im} Y_1 - (\beta-\alpha)\operatorname{Im} Y_2\right]$$

Replacing these relations in Eqns 1 the strain expressions are obtained:

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\alpha-\beta}{2\alpha}\left[a_{12} - a_{11}(\alpha+\beta)^2\right]\operatorname{Re} Z_1 + \frac{\alpha+\beta}{2\alpha}\left[a_{12} - a_{11}(\beta-\alpha)^2\right]\operatorname{Re} Z_2 + \\
&\quad \frac{\beta}{2\alpha}\left[a_{11}(\alpha+\beta)^2 - a_{12}\right]\operatorname{Re} Y_1 + \frac{\beta}{2\alpha}\left[a_{12} - a_{11}(\beta-\alpha)^2\right]\operatorname{Re} Y_2 \\
\varepsilon_{yy} &= \frac{\alpha-\beta}{2\alpha}\left[a_{22} - a_{12}(\alpha+\beta)^2\right]\operatorname{Re} Z_1 + \frac{\alpha+\beta}{2\alpha}\left[a_{22} - a_{12}(\beta-\alpha)^2\right]\operatorname{Re} Z_2 + \\
&\quad \frac{\beta}{2\alpha}\left[a_{12}(\alpha+\beta)^2 - a_{22}\right]\operatorname{Re} Y_1 + \frac{\beta}{2\alpha}\left[a_{22} - a_{12}(\beta-\alpha)^2\right]\operatorname{Re} Y_2 \\
\gamma_{xy} &= \frac{a_{66}}{2\alpha}\left(\alpha^2-\beta^2\right)(\operatorname{Im} Z_1 - \operatorname{Im} Z_2) - \frac{a_{66}\beta}{2\alpha}\left[(\beta+\alpha)\operatorname{Im} Y_1 - (\beta-\alpha)\operatorname{Im} Y_2\right]
\end{aligned} \tag{5}$$

To obtain the strain relative to the coordinate system x' y' that has origin in a point P (r, θ) and is at an angle ϕ with the xy system (Fig. 1), the complex strain transformation equation:

$$\left(\varepsilon_{y'y'} - \varepsilon_{x'x'}\right) + i\gamma_{x'y'} = \left(\varepsilon_{yy} - \varepsilon_{xx} + i\gamma_{xy}\right)e^{2i\phi} \tag{6}$$

and the first strain invariant:

$$\varepsilon_{x'x'} + \varepsilon_{y'y'} = \varepsilon_{xx} + \varepsilon_{yy} \tag{7}$$

are used.
The strain in x' direction is obtained by substituting Eqn 5 into Eqns 6 and 7. It is found:

$$\begin{aligned}
\varepsilon_{x'x'} = & \frac{\alpha-\beta}{2\alpha}\left[\cos^2\phi\left(-a_{11}(\alpha+\beta)^2 + a_{12}\right) + sin^2\phi\left(-a_{12}(\alpha+\beta)^2 + a_{22}\right)\right]\mathrm{Re}\,Z_1 + \\
& \frac{\alpha+\beta}{2\alpha}\left[\cos^2\phi\left(-a_{11}(\alpha-\beta)^2 + a_{12}\right) + sin^2\phi\left(-a_{12}(\alpha-\beta)^2 + a_{22}\right)\right]\mathrm{Re}\,Z_2 + \\
& a_{66} sin\phi\cos\phi\left[\frac{\alpha^2-\beta^2}{2\alpha}\left(\mathrm{Im}\,Z_1 - \mathrm{Im}\,Z_2\right)\right] + \\
& \frac{\beta}{2\alpha}\left[\cos^2\phi\left(a_{11}(\alpha+\beta)^2 - a_{12}\right) + sin^2\phi\left(a_{12}(\alpha+\beta)^2 - a_{22}\right)\right]\mathrm{Re}\,Y_1 + \\
& \frac{\beta}{2\alpha}\left[\cos^2\phi\left(-a_{11}(\beta-\alpha)^2 + a_{12}\right) + sin^2\phi\left(-a_{12}(\beta-\alpha)^2 + a_{22}\right)\right]\mathrm{Re}\,Y_2 - \\
& a_{66}\frac{\beta}{2\alpha} sin\phi\cos\phi\left[(\alpha+\beta)\mathrm{Im}\,Y_1 - (\beta-\alpha)\mathrm{Im}\,Y_2\right]
\end{aligned} \tag{8}$$

The stress functions Z_1, Z_2, Y_1 and Y_2, for a body with single edge notch, can be represented by the following truncated series:

$$\begin{aligned}
Z_1(z_1) &= \sum_{n=0}^{N} A_n z_1^{n-1/2} \qquad & Z_2(z_2) &= \sum_{n=0}^{N} A_n z_2^{n-1/2} \\
Y_1(z_1) &= \sum_{m=0}^{M} B_m z_1^{m} \qquad & Y_2(z_2) &= \sum_{m=0}^{M} B_m z_2^{m}
\end{aligned} \tag{9}$$

It has been shown [7] that stress intensity factor is related to the coefficient A_0 from relation $K_I = A_0\sqrt{2\pi}$.
The strain can be obtained exactly only if an infinite number of terms in series (9) is considered. In such a way an infinite number of unknown coefficients A_n and B_m would be obtained. It is necessary therefore to cut the two series. The unknown coefficients A_n and B_m can be determined solving the system constituted from the Eqn 8 written for every point where the strain value is measured by strain gage.
Such a system can be expressed in a matrix form:

$$\{c\} = [D]\,\{A\ B\} \tag{10}$$

As above mentioned, in this work three terms for every series are considered and therefore six strain gages are necessary.
To minimize the errors, as it is carried out in [8] for isotropic material, a greater number of strain gages is used and therefore a system is obtained in which the number of equations exceeds the number of unknown coefficients. The system is solved applying the least-squares method and the solution is given by the set of coefficients {A B} that minimizes the vector:

$$\{r\} = \{c\} - [D]\,\{A\ B\}$$

The solution for the unknown coefficients, that is unique for a given set of n data points, is given by:

$$\{A\ B\} = ([D]^T [D])^{-1} [D]^T \{c\}$$

A software routine has been set up, following the above described procedure; it enables to determine K_I by introducing the strain values, the polar coordinates and the elastic characteristics of the laminate.

EXPERIMENTAL PROCEDURE AND RESULTS

The experimental tests have been performed on a specimen (Fig. 2) that was obtained from symmetrical balanced laminated cross-ply $[90_2/0]_{2s}$. The laminate elastic constants are shown in Table 1.

Table 1 : Laminate elastic constants

E_L=44650 MPa
E_T=93500 MPa
G_{LT}=4450 MPa
ν_{LT}=0.0241

The experimental strain values have been found using (Fig. 2) one chain with ten strain gages oriented at ϕ=81° to the x axis. Two strain gages have been placed away enough from the crack tip, on opposite faces and in longitudinal direction, in order to point out undesired bending effects. A half bridge circuit has been realised using a compensator strain gage located, with equal direction to that of the chain, on a dummy unloaded specimen that has the same geometry of the previous one. An HBM UPM 100 type equipment has been used to acquire strain gage signals.

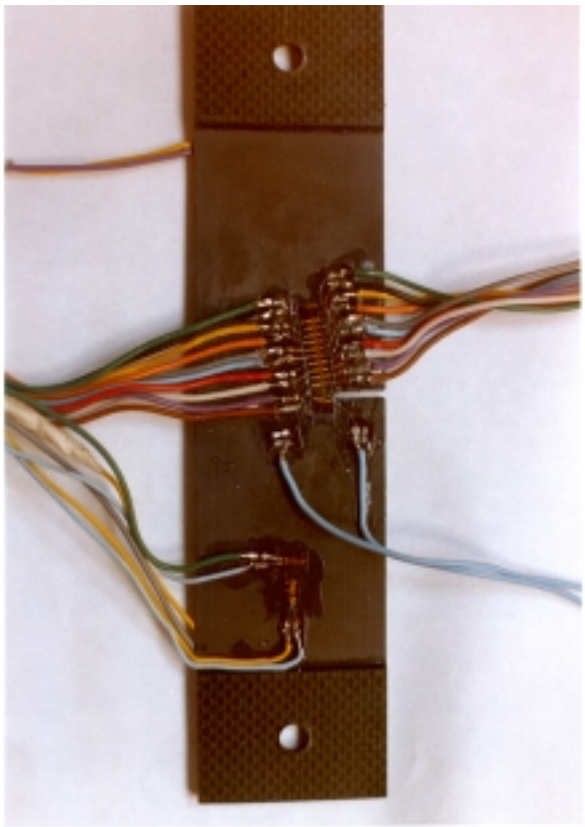

Fig. 2 : Specimen with strain gages

The measured strain values have been recorded for several values of the load. Radial positions of the strain gages and the relative strain values to a load of 1030 N are given in Table 2. By providing these data to the program a K_I = 4.72 MPa $\sqrt{m}$ is obtained.

Table 2 : Experimental strain values at ϕ=81° and different radial positions to a load of 1030 N

Strain gage	r (mm)	$\varepsilon_{x'x'}$ (μm/m)
1	3.25	358
2	5.25	294
3	7.25	272
4	9.25	256
5	11.25	239
6	13.25	234
7	15.25	231
8	17.25	225
9	19.25	222
10	21.25	220

NUMERICAL ANALYSIS AND RESULTS

SIF value has also been estimated using a program that enables to resolve problems of fracture mechanics both for anisotropic and for orthotropic materials; it is based on the DBEM and on the J-integral [5]. The DBEM incorporates two independent equations: the displacement and the traction boundary integral equations. The modelling strategy adopted in the dual boundary element method can be summarized as follows:

- the traction boundary integral equation is applied for collocation on one of the crack boundaries;
- the displacement boundary integral equation is applied for collocation on the opposite crack boundary and remaining boundaries;
- the crack boundaries are discretized with quadratic discontinuous elements;
- continuous quadratic boundary elements are used along the remaining boundaries, except at the intersection between crack and edge, where discontinuous elements are required.

The J-integral represents the energy for a unit surface of the crack available to the fracture and, in plane stress state, is related to K_I from relation $J=K_I^2/E$.
In order to determine J the program defines a line of circular contour around the crack tip and a set of points on such a line in which it determines the stress state and therefore J.
Among the input data there are the elastic characteristics of the material. For the case under investigation, orthotropic material and crack aligned with one of the principal axes, it is sufficient to supply E_L, E_T, G_{LT} and ν_{LT}. The output data are displacements, stress and stress intensity factor. By this procedure it has been obtained K_I =5 MPa $\sqrt{m}$.
The stress values at internal points corresponding to the center points of the strain gages have been calculated also. From these stress values, through Eqns 1, 6 and 7, the strains $\varepsilon_{x'x'}$ shown in Table 3 are obtained. These values are very close to experimental ones (Table 2).

ORTHOTROPY INFLUENCE

DBEM program has been employed to evaluate the orthotropy influence on SIF value. K_I values have been obtained varying the ratio $\chi = E_L/E_T$ between 0.1 and 4.5. It has been posed $E_L = G_{LT}(\chi+2\nu_{LT}+1)$.

The obtained values are shown in Table 4; χ=0.477 is relating to the examined case. DBEM value obtained in the isotropic case (χ =1) is very close to 5.22 MPa$\sqrt{m}$ that is the theoretic value obtained by $K_I = Y\sigma\sqrt{a}$ where Y depends on the ratio between crack length and specimen width.
Results show that the orthotropy influence is very little. This is in agreement with the results obtained by Kageyama [9].

Table 3 : Numerical strain values at internal points corresponding to the center point of the strain gages (load 1030 N; ϕ=81°)

r (mm)	$\varepsilon_{x'x'}$ (μm/m)
3.25	365
5.25	305
7.25	277
9.25	262
11.25	249
13.25	237
15.25	229
17.25	223
19.25	218
21.25	216

Table 4 : Stress intensity factor for various χ

χ	K_I (MPa$\sqrt{m}$)
0.1	5.220
0.3	5.243
0.477	5.090
0.7	5.254
1	5.253
1.5	5.240
2.5	5.230
3.5	5.210
4.5	5.192

CONCLUSIONS

The stress intensity factor has been determined by means of strain gages on laminate in carbon-epoxy, extending a procedure that was used for isotropic materials. Through a program that applies the dual boundary element method and the J-integral, the K_I value has been numerically determined also. The obtained values of K_I are close enough.
The strain values experimentally found and those numerically obtained are in a good agreement.
It has been pointed out that orthotropy influence on SIF value is very little.

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