ELASTOPLASTIC ANALYSIS
OF INTERFACIAL STRESS IN MODEL
COMPOSITES

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SUMMARY: The nonlinear relation between stress and strain is analysed for the fibre and the matrix in model composite. The shear stress distribution in the interface is studied in an elastoplastic state. The linear equation is deduced between the load and the interfacial shear stress.

KEYWORDS: model composite, elastoplastic interface, shear stress distribution

INTRODUCTION

Studies of the interfacial mechanical behavior for model composites are very active over the years. The characterisation of the fibre-matrix interface has made great progress in theory and experiment\cite{1,2}. The study on interfacial properties is progressively developed from elasticity to elastoplasticity\cite{3}. For the metal matrix composite, the plastic property of the interphase is further paid attention to\cite{4}. The property of the interphase is different from the matrix. Both can even appear great difference sometimes. In this paper the model to study is the single fibre pull-out specimen, by which the interfacial elastoplastic problem is discussed.

BASIC EQUATION

The single-fibre pull-out specimen is a typical model composite (Fig.1). By employing the shear-lag theory, the equilibrium equations for fibre and matrix are

\[ \frac{d\sigma_i}{dz} + \frac{2\tau_i}{a} = 0, \quad r\tau_i = a\tau_i \tag{1} \]

In the interface the strain-displacement relations can be simply represented by

\[ \varepsilon_i = \frac{dw_i}{dz} = \frac{\partial w}{\partial z} \bigg|_{r \rightarrow a}, \quad \gamma_i = \frac{\partial w}{\partial r} \bigg|_{r \rightarrow a} \tag{2} \]

Where w is for the axial displacement. The radial displacement can be neglected.

The material of matrix and interphase is generally of elastoplastic deformation that obeys the power-hardening law. There is a turning point \((\tau_T, \gamma_T)\) from elastic state to plastic state. So the stress-strain relations in matrix and interphase are as follows

\[ \frac{\tau}{\tau_T} = \frac{\gamma}{\gamma_T} \quad (\tau < \tau_T) \tag{3} \]

\[ \frac{\tau}{\tau_T} = \left(\frac{\gamma}{\gamma_T}\right)^n \quad (\tau > \tau_T) \tag{4} \]
When the model composite is loaded, shear deformation of the matrix near the applied load is larger. While applied load $P$ reaches $P^o$, initial plastic deformation occurs in the interface at upper end. The shear stress is expressed as $\tau_0 (z \rightarrow 0)$, $\tau_0 = \tau_r$. When $P$ reaches $P^*$, there is a turning point of elastoplastic deformation at $z \rightarrow T$. The interface becomes into elastic area $(T < z < L)$ and plastic area $(0 < z < T)$. The applied stress of the fibre is $\sigma^o$ while $P = P^o$, or $\tau^*$ while $P = P^*$, i.e. $P^o = \pi a^2 \sigma^o$, $P^* = \pi a^2 \sigma^*$. The physical relations are

$$
\sigma_f = E_f \frac{d w}{d z}, \quad \tau_{ie} = G_i \frac{r}{a} \frac{\partial w}{\partial r}, \quad \tau_{ip} = \frac{\tau_r}{\gamma_f} \frac{\partial w}{\partial r} \bigg|_{r=a}
$$

(5)

Where $\sigma_f$, $\tau_{ie}$ and $\tau_{ip}$ are the fibre stress, elastic and plastic shear stress respectively. $G_i$ is the shear modulus of the interphase.

For the elastic area, $z > T$, suppose $w(r, z)$ to be

$$
w = A \ln \frac{R}{r} \exp(- \frac{kz}{a}) + C
$$

(6)

Substituting eqn.(6) into eqn.(5), we have

$$
\sigma_f = -E_f \frac{kA}{a} \ln \frac{R}{r} \exp(- \frac{kz}{a})
$$

(7)

$$
\tau_{ie} = -G_i \frac{A}{a} \exp(- \frac{kz}{a})
$$

(8)

If $L/a \gg 1$, then at the fibre tip, $z \rightarrow L$, $\sigma_f \rightarrow 0$. Substituting eqns. (7) and (8) into eqn.(1), we have

$$
k^2 \ln \frac{R}{a} = \frac{2G_i}{E_f}
$$

(9)

At $z = T$, the interfacial shear stress is expressed as $\tau_r$. Then from eqn. (8) we have

$$
\frac{A}{a} = -\frac{\tau_r}{G_i} \exp(\frac{kT}{a})
$$

(10)

The interfacial shear stress of the elastic area can be expressed as

$$
\tau_{ie} = \tau_r \exp(\frac{kT}{a} - \frac{kz}{a})
$$

(11)

For the plastic area ($z < T$), suppose the displacement function to be

$$
w = B \ln \frac{R}{r} \left( \frac{L - z}{L - T} \right)^t + D
$$

(12)

Substituting eqn. (12) into eqn.(5), we have

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Fig. 1 Schematic representation of the single-fibre pull-out specimen
\[
\sigma_f = -E_f \frac{Bs}{L-T} \ln \frac{R_i}{a} \left( \frac{L-z}{L-T} \right)^{s-1} \tag{13}
\]
\[
\tau_{ip} = \tau_T \left( -\frac{B}{a\gamma_T} \right)^m \left( \frac{L-z}{L-T} \right)^{ms} \tag{14}
\]

At \( z = T \), \( \tau_{ip} = \tau_T \), from eqn.(14) we have
\[
\frac{B}{a} = \gamma_T = -\frac{\tau_T}{G_i} \tag{15}
\]

Then \( \tau_{ip} \) can be expressed as
\[
\tau_{ip} = \tau_T \left( \frac{L-z}{L-T} \right)^{ms} \tag{16}
\]

Substituting eqns.(13) and (16) into eqn.(1) and simplifying, we have
\[
\frac{(L-T)^2 - (L-z)^2}{a} \left( \frac{L-z}{L-T} \right)^{2+ms-s} = \frac{s(s-1)E_f}{2G_i} \ln \frac{R_i}{a} \tag{17}
\]

Then it must be that
\[
s = \frac{2}{1-m}, \quad \left( \frac{L-T}{a} \right)^2 = \frac{s(s-1)E_f}{2G_i} \ln \frac{R_i}{a} \tag{18}
\]

At \( z = T \), the fibre stress \( (\sigma_f) \) determined from eqn.(7) is equal to that from eqn.(13). And using eqns. (10) and (15), we have
\[
\ln \frac{R_i}{a} = \frac{k}{s} \left( \frac{L-T}{a} \right) \ln \frac{R}{a} \tag{19}
\]

At \( z = 0 \), \( \sigma^o \) and \( \sigma^* \) can be gotten by eqns.(7) and (13). And using above-named equations, we have
\[
\frac{P^*}{P^o} = \frac{\sigma^*}{\sigma^o} = \frac{kL}{(s-1)a} \left( \frac{L}{L-T} \right)^{ms} \tag{20}
\]

At the beginning of plastic deformation, \( T \to 0 \), \( P^* \to P^o \), so it must be that
\[
\frac{kL}{(s-1)a} = 1 \quad \text{or} \quad k = (s-1) \frac{a}{L} = \left( \frac{1+m}{1-m} \right) \frac{a}{L} \tag{21}
\]

Therefore, eqn.(16) can be written for
\[
\tau_{ip} = \tau_T \frac{P^*}{P^o} (1 - \frac{z}{L})^{ms} \tag{21}
\]

This shows that the interfacial shear stress in the plastic area is proportional to the load exerted on the fibre.

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