

## Prediction of Yarn Structure of 3D-Braided Composites

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In order to design braiding sequences for 3D-braided components, a method to predict the yarn structure is necessary, since the mechanical properties of the component are dependent on the yarn structure. Satisfactory methods for predicting yarn structure based on braiding sequence are as yet unavailable, although a recent effort [1] based on a finite element approach has yielded some positive results. In this paper we present a relatively efficient method based on an optimization approach. In our approach, the basic idea is to exploit the fact that the yarn structure of the braid must be the one that minimizes the total yarn length in the braid. Thus, the yarn structure can be predicted by minimizing the yarn length, subject to the condition that the yarns cannot intersect each other.

### The yarn model

To model the yarns, we chose second order non-uniform rational B-splines of the form

$$p(u) = \left( \sum_{i=0}^n h_i \cdot p_i \cdot N_{i,k}(u) \right) / \left( \sum_{i=0}^n h_i \cdot N_{i,k}(u) \right), \text{ with } k=3 \text{ for second order} \quad (1)$$

Here,  $p(x, y, z)$  is position vector to the point,  $u$  is the parameter,  $n$  is the number of control vertices,  $p_i$  is the position  $(x_i, y_i, z_i)$  of the  $i^{\text{th}}$  control vertex and  $h_i$  its corresponding weight.  $N_{i,k}(u)$  is defined as follows:

$$\begin{aligned} N_{i,k}(u) &= 1 \text{ if } t_i \leq t_{i+1} \\ N_{i,k}(u) &= (u - t_i) * N_{i,k-1}(u) / (t_{i+k-1} - t_i) + (t_{i+k} - u) * N_{i+1,k-1}(u) / (t_{i+k} - t_{i+1}) \text{ otherwise} \\ \text{where } t_i &= 0 \quad \text{if } i \leq k \\ t_i &= 1 - k + 1 \quad \text{if } k \leq i \leq n \\ t_i &= n + k + 2 \quad \text{if } i \geq n \end{aligned}$$

### The non linear programming problem (NLP)

The optimization problem for obtaining the yarn structure can then be formulated as an NLP:

- The design variables are the coordinates of the control vertices and the weights. Note that the end points are fixed in the  $z$  direction (braid axis) to prevent the yarn length from going to zero. Moreover, since we want to know the yarn structure of the preform at steady state, the yarn structure is assumed to be periodic. Therefore, the  $(x, y)$  coordinates of control vertices at the bottom of the sample are not defined as variables but are set equal to the  $(x, y)$  coordinates of the corresponding control vertices at the top of the braid.
- The cost function is the total yarn length in the braid. This can be easily computed given the position of the control vertices by using Eq. (1):

$$\text{total\_length} = \sum_{\text{yarn}} \left( \sum_{k=0}^{k=m} \left\| p(k \cdot u_{\max} / m) - p((k+1) \cdot u_{\max} / m) \right\| \right) \quad (2)$$

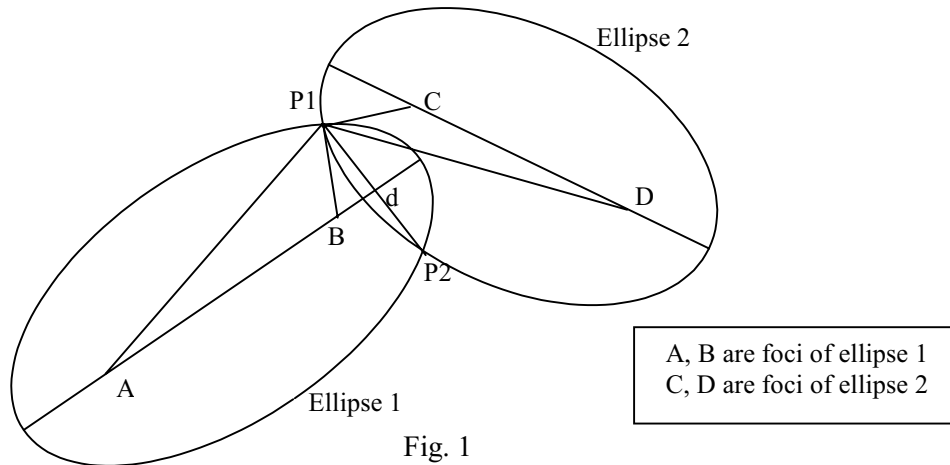
where  $u_{\max}$  is the maximum value of the parameter  $u$  for each yarn and  $m$  is the number of yarn segments used to estimate the length. Increasing  $m$  increases the length precision.

- Non-intersection constraints: Since the yarns have to be non-intersecting at the solution, we check for intersections of the yarn cross sections at a number of planes perpendicular to the braid axis. These cross sections are ellipses and the constraint is evaluated as follows: First, we decide on the number of planes for this check and determine their  $z$  positions. Then, for each given  $z$ , we calculate the parameter  $u$  for each yarn from Eq. (1).

From this value of  $u$ , we find the position of the center of the ellipse and the derivative of the B-spline at this point, whence we obtain the major axis and the angle of the ellipse in the  $(x, y)$  plane. The minor axis of the ellipse is the radius of the yarn. We then compute the intersection points (if any) and the distance  $d$  (Fig. 1), which is taken to be the constraint violation value. In order compute the intersection points, we use the property that P1 and P2 (Figure 1), should both satisfy

$$|AP_i| + |BP_i| = c_1 \text{ and } |CP_i| + |DP_i| = c_2 \text{ for } i=1..2, c_1 \text{ and } c_2 \text{ being known constants.}$$

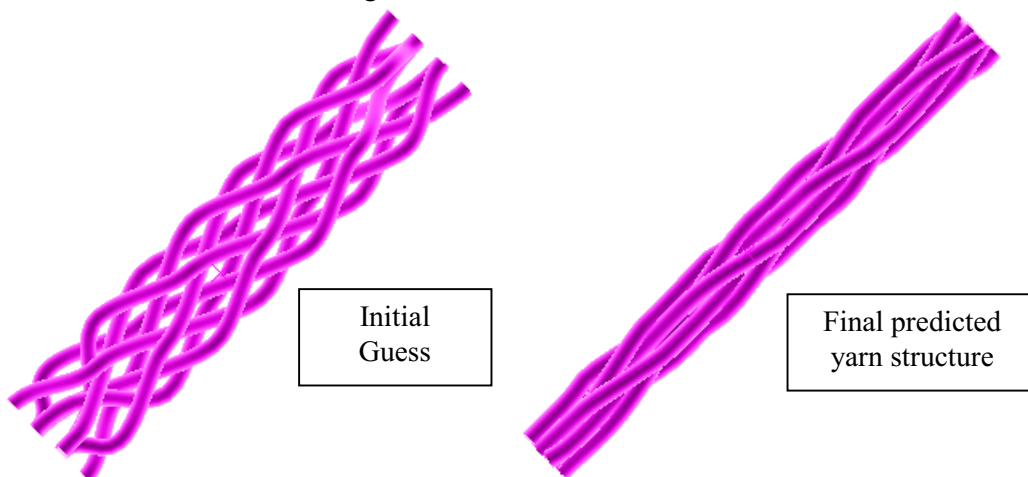
These two equations can be converted to one linear and one quadratic equation, which can be solved directly to obtain the locations of P1 and P2, from which we can compute  $d$ .



The numerical optimization is done using an exterior penalty function approach with a directed grid search for unconstrained minimization. To generate the initial guess, we take the spatial positions of the yarn carriers at each step, compute a piecewise linear approximation and scale it with a factor  $k \geq 1$  to avoid intersections. Finally, we increase this scaling factor until the B-splines fitting these piecewise linear approximations are all non-intersecting.

### Example

We consider four pitches of a 4-step braiding cycle with 8 yarns. Using 11 control vertices per yarn results in 268 design variables and 100 intermediate planes were used for evaluating the non-intersection constraint. The result, which was obtained in 10 minutes on a 400MHz Pentium II machine, is shown in Fig.2 below.



### References

1. Wang, Y.Q., and Sun, X.K., "Digital Element Simulation of Textile Processes", submitted to the 13<sup>th</sup> International Conference on Composite Materials, Beijing, June 2001