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Strain Invariant Failure Criteria for Fiber Reinforced Polymeric Composite Materials

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Physics-based failure criteria for the polymeric phase of fiber reinforced composite materials are introduced in this paper. The initiation of damage is determined through the use of critical dilatational and distortional strain invariants of the lamina. Critical dilatational behavior is characterized by the first invariant of strain (volumetric strain). Critical distortional behavior is characterized by the equivalent strain. Both lamina invariants are effective with respect to the general state of deformation of the loaded lamina. The homogenous lamina solution is modified through the use of strain amplification factors and superimposed thermal strains extracted from finite element micromechanical models. All analysis described in this paper involves the finite element method. As a result, all micromechanical modification of the homogenous finite element lamina solution is conducted in post-processing. Since the second invariant of strain is a function of both the first invariant of strain and the equivalent strain it can be used to determine the extent of matrix damage propagation once damage has been initiated. The second invariant of strain describes the post-damage initiation behavior of the lamina, therefore it is not micromechanically modified. In this paper, the validity of the strain invariant failure criteria is demonstrated primarily with test data from the literature.

The Strain Invariant Failure Criteria

Experimentally derived plane stress failure envelopes for both thermosetting and thermoplastic polymers have shown that, in the first quadrant, there is a significant deviation from the distortional dependent failure ellipse evident for most ductile metals [1]. The author has shown that both the first invariant of strain and the equivalent strain are needed to numerically reproduce the experimentally derived failure envelopes from the literature [2]. Therefore, the critical deformation of polymers cannot be described by only one mechanism (distortion). There are two failure mechanisms, one dilatational and one distortional. The critical material properties associated with these two failure mechanisms are the first invariant of strain (volumetric strain) and the equivalent strain.

Both the first invariant of strain and the equivalent strain are functions of the three principal strains. The first invariant of strain is the sum of the three principal strains,

$$J_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3. \qquad (1)$$

The equivalent strain is also a function of the three principal strains,

$$\varepsilon_{\text{equivalent}} \text{ (or } \varepsilon_e) = [0.5((\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_1-\varepsilon_3)^2 + (\varepsilon_2-\varepsilon_3)^2)^{0.5}. \qquad (2)$$

The second invariant of strain is a function of the sum of the products of the three principal strains,

$$J_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3. \qquad (3)$$

Using equations (1) and (2) one can show that the second invariant of strain is also a function of J_1 and ε_e,

$$J_2 = (J_1^2 - \varepsilon_e^2)/3. \tag{4}$$

Equation (4) allows J_2 to be used as a damage functional to describe the extent of damage propagation within the matrix phase of composite materials once damage has been initiated (as determined by the critical values of J_1 and ε_e).

Verification of the Strain Invariant Failure Criteria

The validity of the strain invariant failure criteria is demonstrated with test data from the literature (3), (4) and (5). Supplemental data from the Boeing Co. is used as needed. The unidirectional material systems utilized include; T300/5208, IM6/3501-6 and IM7/PETI-5. All test data utilized in this section involved the following tests; transverse tension, transverse compression, angle-ply tension, off-axis tension, in-plane compression and static indentation. The critical effective invariant properties are extracted from some of the test results and the invariant failure criteria is validated with the remaining test results. A simple example involving the determination of the $\varepsilon_e^{critical}$ (or ε_e^{cr}) for T300/5208 is given below. In Table 1, both the global axial strains and the ε_e^{cr}'s at failure are shown for lamina angles of 10° and 30°. As can be seen, the global axial strains are significantly different whereas the ε_e^{cr}'s nearly the same.

Table 1 Extraction of the Critical Equivalent Strain for T300/5208

Off-Axis Angle	Global Axial Strain	Equivalent Strain
10	0.0057	0.0342
30	0.0070	0.0334

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