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THE STRESS TRANSFER THEORY FOR SHORT FIBER REINFORCED METAL MATRIX COMPOSITES

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SUMMARY: The stress transfer mechanism of short fiber reinforced metal matrix composites was researched theoretically and numerically. The traditional shear-lag theory[1] of stress transfer was initially modified. Some factors that could not be well considered in the existed theories were involved in the revised theory. These factors included the normal stress transfer at fiber end and the bond of interface. Some new formulas were gained. Then, in order to discuss the reasonability of the revised formula, the results of these formulas were compared with the stress distributions obtained by the numerical simulation of Finite Element Method (FEM). It is shown that the revising for the traditional model is reasonable and necessary.

KEYWORDS: Metal Matrix Composites, Short Fiber, Stress Transfer, Shear-lag Model

INTRODUCTION

The shear-lag theory was first developed by Cox[1] in 1952 and was used to describe the stress transfer between the fiber and matrix in short fiber composites. The shear stress caused by the unmatched elastic deformation between fiber and matrix at interface acted as the means of stress transfer from the matrix to the fiber. With some assumptions and simplifications, the distribution formulae of fiber axial stress σ_f and interfacial shear stress τ_i were gained by Cox[1]. Rosen[2] simplified the results of Cox further. However, the normal stress at fiber end and the interfacial bond could not be well considered in their analysis. Fukuda-Chou[3] took the fiber's end stress into the stress transfer and developed the advanced shear-lag theory. However, this theory was too complicated to be used easily. Thus, the traditional shear-lag theory was revised by considering the normal stress at fiber end and the interfacial bond in this work. In order to discuss the reasonability of the revised formulae, the stress distributions obtained from the revised theory were compared with those calculated by FEM.

AN EQUIVALENT SHEAR-LAG MODEL

In order to take the effects of normal stress at fiber end and interfacial bonding on the stress transfer in short fiber reinforced metal matrix composites (SFRMMCS) into account easily, an equivalent shear-lag model was suggested. In the equivalent model, the distribution of the fiber axial stress and interfacial shear stress was deduced again by considering the equilibrium of a small element of fiber as shown in Fig. 1. In this model, an interfacial layer was inserted. The bonds between the interfacial with the fiber and the matrix were assumed to be perfect. The state of interfacial bond was represented by the elastic modulus of the interfacial layer material. They were different with the Cox's model[1]. However, when the elastic modulus of interfacial layer was adopted to be equal to that of matrix material, this model was simplified to be similar to the Cox's model.

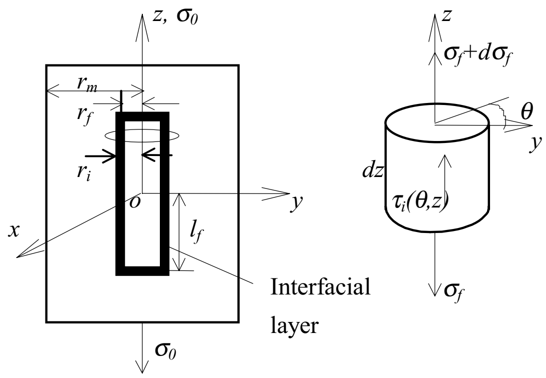


Fig.1 Equivalent shear-lag model with an interfacial layer

With an applied stress of σ_0, the distribution formulae of the fiber axial stress σ_f and the interfacial shear stress τ_i are deduced here. The force equilibrium of an infinitesimal length, dz, requires

$$\int_0^{2\pi} \tau_i d\theta + \pi r_f \, {d\sigma_f}/{dz} = 0 \tag{1}$$

where σ_f is the fiber stress in the axial direction, τ_i is the shear stress on the cylindrical fiber-matrix interface, and r_f is the fiber radius. Now, the key-step of derivation is how to find the formula of τ which is a function of r_f. The equilibrium of matrix between r_f and r requires

$$2\pi r \tau(r) = Const. = 2\pi r_f \tau(r_f) \tag{2}$$

The shear strain of matrix at r is

$$dw/dr = \tau(r)/G_m = \tau(r_f) r_f / G_m r \tag{3}$$

Integrating the Eq. 3 from r_f to r_m, we get

$$\Delta w = v - u = \frac{\tau(r_f) r_f}{G_m} \ln\left(r_m / r_f \right) \tag{4}$$

where v is the axial displacement of matrix at r_m, u is the axial displacement of matrix at r_f. If the interface is assumed to be perfect, u can be taken as the axial displacement of fiber. Differentiating Eq. 4 yields

$$\frac{d\tau(r_f)}{dz} = \frac{G_m}{r_f \ln\left(r_m / r_f \right)} \left(dv/dz - du/dz \right) = \frac{G_m}{r_f \ln\left(r_m / r_f \right)} \left(\sigma_0 / E_m - \sigma_f / E_f \right) \tag{5}$$

The effect of fiber has become very weak at r_m and can be neglected. In Eq. 5, σ_0 is the applied load in the model. Differentiating Eq. 1 yields

$$d^2\sigma_f / dz^2 = -\frac{2}{r_f} \, d\tau(r_f)/dz \tag{6}$$

Substituting Eq. 5 into Eq. 6, we obtain

$$d^2\sigma_f / dz^2 - \eta^2 \sigma_f + \xi^2 = 0 \tag{7}$$

where $\xi^2 = \dfrac{2G_m\sigma_0}{r_f^2 \ln\left(r_m / r_f \right) E_m}$ and $\eta^2 = \dfrac{2G_m}{r_f^2 \ln\left(r_m / r_f \right) E_f}$.

Solving Eq. 7 yields

$$\sigma_f = ASinh\eta z + BCosh\eta z + \frac{\xi^2}{\eta^2} \tag{8}$$

where A and B are constants and are obtained from the boundary condition at fiber's end. If the boundary condition is $\sigma_f|_{Z=-l_f} = \sigma_f|_{Z=l_f} = 0$, then

$$\begin{cases} \sigma_f = E_f \frac{\sigma_0}{E_m}\left(1 - \frac{Cosh\eta z}{Cosh\eta l_f}\right) \\ \tau_i = E_f \frac{\sigma_0}{E_m} \frac{Sinh\eta z}{Cosh\eta l_f}\sqrt{\frac{G_m}{E_f 2\ln\left(\frac{r_m}{r_f}\right)}} \end{cases} \tag{9}$$

where $\tau_i = \tau(r_f)$. it is found that the results of Eq. 9 are equivalent to those obtained by Cox.

STRESS TRANSFER OF SFRMMCS

Effects of the Normal Stress at Fiber End

In SFRMMCS, especially when the interfacial bonding was perfect, the normal stress at fiber end would have a significant effect on stress transfer between fiber and matrix, and could not be neglected. Fukuda-Chou[3] had considered this factor in their theory. However the formulae gained were too complex to be used. In order to take this factor into account, the boundary condition in Eq. 8 should be taken as $\sigma_f|_{Z=-l_f} = \sigma_f|_{Z=l_f} = \sigma_{f0}$. The σ_{f0} is the normal stress at fiber's end. Thus, Eq. 9 becomes to be

$$\begin{cases} \sigma_f = E_f \frac{\sigma_0}{E_m}\left(1 - \frac{Cosh\eta z}{Cosh\eta l_f}\right) + \sigma_{f0}\frac{Cosh\eta z}{Cosh\eta l_f} \\ \tau_i = \left(E_f \frac{\sigma_0}{E_m} - \sigma_{f0}\right)\frac{Sinh\eta z}{Cosh\eta l_f}\sqrt{\frac{G_m}{E_f 2\ln\left(\frac{r_m}{r_f}\right)}} \end{cases} \tag{10}$$

where ξ and η are the same as those in Eq. 9. the results of Eq. 10 are equivalent to those of Cox when $\sigma_{f0}=0$.

Effects of the Interface

From the model as shown in Fig. 1, the effects of interface on stress transfer were discussed. The interface between fiber and matrix was taken to be a layer of interfacial material. The state of interfacial bond was characterized by the elastic modulus of interfacial layer. Here, the Eq. 6 is still satisfied. Because the interfacial layer is very thin, the shear stress of interfacial layer can be assumed to be uniform at different $r(r_i \geq r \geq r_f)$. Thus, we get

$$\tau(r_i) = \tau(r_f) = \frac{v_i - u_i}{r_i - r_f}G_i \tag{11}$$

where G_i is the shear modulus of interfacial layer, v_i is the axial displacement of interfacial layer near the matrix(at r_i), and u_i is the axial displacement of interfacial layer near the fiber(at r_f). The axial displacements of interfacial layer and matrix are assumed to be same at r_i, and the axial displacements of interfacial layer and fiber are also assumed to be same at r_f. Considering the equilibrium of the matrix at r_i and $r(r \geq r_i)$ yields

$$2\pi r_i \tau(r_i) = Const. = 2\pi r \tau(r) \tag{12}$$

The shear strain of matrix at $r(r \geq r_i)$ is

$$\frac{dw}{dr} = \frac{\tau(r)}{G_m} = \frac{\tau(r_i)r_i}{G_m r} \tag{13}$$

Integrating the Eq. 13 from r_i to r_m, we get

$$\Delta w = v - v_i = \tau(r_i)r_i \Big/ G_m \ln\left(r_m / r_i\right) \tag{14}$$

Differentiating Eq. 11 and substituting Eq. 14 into, the following formula is obtained

$$d\tau(r_f) \Big/ dz = \frac{G_i G_m}{G_m(r_i - r_f) + G_i r_i \ln\left(r_m / r_i\right)} \left(\frac{\sigma_0}{E_m} - \frac{\sigma_f}{E_f} \right) \tag{15}$$

Substituting it into Eq. 6 yields

$$d^2\sigma_f \Big/ dz^2 - \eta_1^2 \sigma_f + \xi_1^2 = 0 \tag{16}$$

where $\xi_1^2 = \dfrac{2G_i G_m \sigma_0}{\left[r_f G_m (r_i - r_f) + G_i r_i r_f \ln\left(r_m / r_i\right) \right] E_m}$;

and $\eta_1^2 = \dfrac{2G_i G_m}{\left[r_f G_m (r_i - r_f) + G_i r_i r_f \ln\left(r_m / r_i\right) \right] E_f}$.

Solving Eq. 16 and taking the boundary condition as $\sigma_f \big|_{z=-l_f} = \sigma_f \big|_{z=l_f} = 0$ yields

$$\begin{cases} \sigma_f = \xi_1^2 \Big/ \eta_1^2 \left(1 - Cosh\eta_1 z \Big/ Cosh\eta_1 l_f \right) = E_f \dfrac{\sigma_0}{E_m} \left(1 - Cosh\eta_1 z \Big/ Cosh\eta_1 l_f \right) \\ \tau_i = \dfrac{1}{2} E_f \dfrac{\sigma_0}{E_m} r_f \eta_1 \, Sinh\eta_1 z \Big/ Cosh\eta_1 l_f \end{cases} \tag{17}$$

If the boundary condition at fiber end is taken as $\sigma_f \big|_{z=-l_f} = \sigma_f \big|_{z=l_f} = \sigma_{f01}$, we get

$$\begin{cases} \sigma_f = E_f \dfrac{\sigma_0}{E_m} \left(1 - Cosh\eta_1 z \Big/ Cosh\eta_1 l_f \right) + \sigma_{f01} \, Cosh\eta_1 z \Big/ Cosh\eta_1 l_f \\ \tau_i = \dfrac{1}{2} (E_f \dfrac{\sigma_0}{E_m} - \sigma_{f01}) r_f \eta_1 \, Sinh\eta_1 z \Big/ Cosh\eta_1 l_f \end{cases} \tag{18}$$

where σ_{f01} is the normal stress at fiber end and is related to the modulus of interfacial layer.

DISCUSSION

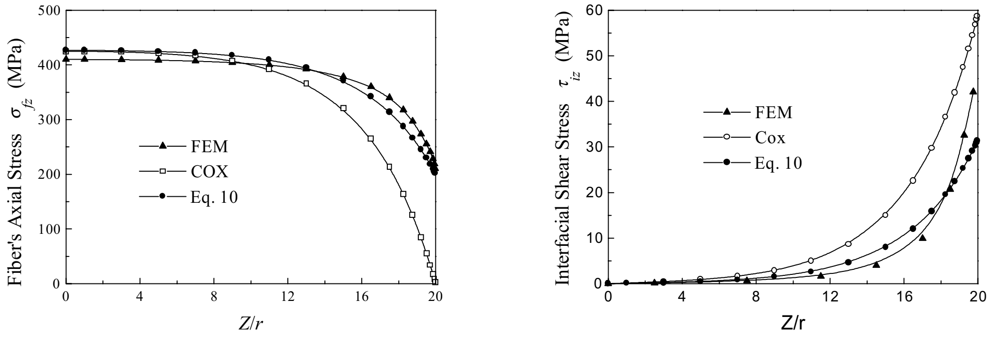
Finite Element Method

In order to discuss the reasonability of the revised theory, the stress distributions of fiber axial stress and interfacial shear stress were calculated by FEM in the same condition as that used in theoretical analysis. The model was the same as that shown in Fig. 1, but mesh of FEM was not shown here. The code was ALGOR Finite Element Analysis System and the element is 8-nodes axi-symmetrical element. The number of total elements was 608 and the number of total nodes was 660. The size of model was the same as that used in theoretical analysis: r_m=10r_f, r_i-r_f=0.05r_f, l_f=20r_f, l_m=l_f+10r_f.The details were given in Reference[4]

Effects of the Normal Stress at Fiber's End

When E_f=300GPa, E_i=E_m=70GPa, v_f=0.2, v_i=v_m=0.33, and the applied stress σ_0=100MPa, the distributions of fiber axial stress σ_{fz} and interfacial shear stress τ_{iz} obtained by theoretical formulae and numerical simulation are shown in Fig. 2. It can be concluded that the stress distribution curves gained from Eq. 10 (in which the σ_{f0} is equal to 2σ_0[5]) are closer to the curves calculated by FEM than the results of Cox. This meant that, when the interfacial bonding is perfect, the modified shear-lag theory, in which the normal stress at fiber's end is considered, is more reasonable than Cox's shear-lag theory, especially near the fiber's end.

From Fig. 2, it can also be concluded that the normal stress has no effect on the stresses in the middle of the fiber.



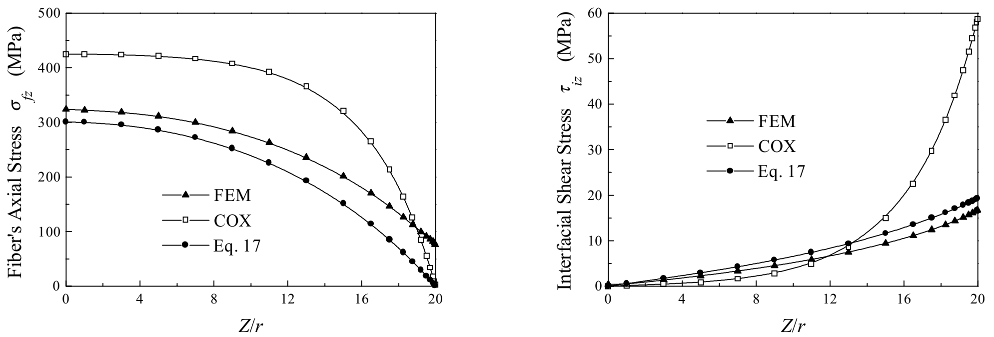
a)σ_{fz} b) τ_{iz}

Fig. 2 The distributions of fiber axial stress σ_{fz} and interfacial shear stress τ_{iz}

($E_i = E_m$, σ_0=100MPa)

Effects of Interfacial Layer

When E_i=0.2GPa and other parameters were the same as those taken in former section, the distributions fiber axial stress σ_{fz} and interfacial shear stress τ_{iz} were calculated by theoretical formulae and FEM. The results are given in Fig. 3. It can be concluded that from Fig. 3: (1) Since the interface is assumed to be perfect and the weak interface cannot be taken into account, the results of Cox have large differences with the stress distributions obtained by FEM, especially in the middle of the fiber. (2) With the weak interfacial bond considered, the results of the modified theory agree well with that calculated by FEM. Thus, the modifying to the traditional shear-lag theory is necessary and reasonable.



a)σ_{fz} b) τ_{iz}

Fig. 3 The distribution of fiber axial stress σ_{fz} and interfacial shear stress τ_{iz}

(σ_0=100MPa, E_i=0.2GPa)

In Fig. 3, the fiber's normal stress at fiber's end is not included in theoretical analysis. Thus, there is some difference between the results of Eq. 17 and those of FEM, especially at fiber's end. As mentioned in former section of this paper, the fiber's normal stress at fiber end σ_{f01} was related to the modulus of interfacial layer E_i. Thus, the relation between σ_{f01} and E_i was studied by FEM here. The result is shown in Fig. 4. It is concluded that the fiber's normal stress at fiber end σ_{f01} increases with the increase in the modulus of interfacial layer E_i. However, the rate of the increment of σ_{f01} is not same for all of the moduli of interfacial layer $E_i(E_i \leq E_m)$. When E_i is small, with the elevation of E_i the fiber's normal stress at fiber end σ_{f01}

increases rapidly. When E_i is near the modulus of matrix E_m, with the elevation of E_i the σ_{f01} increases very slowly and mostly does not change.

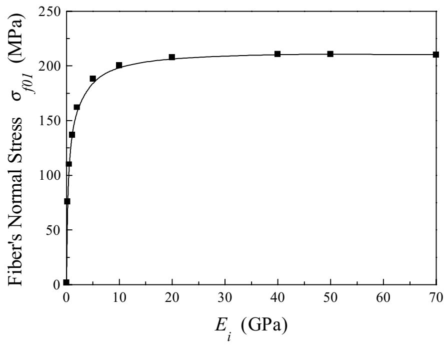
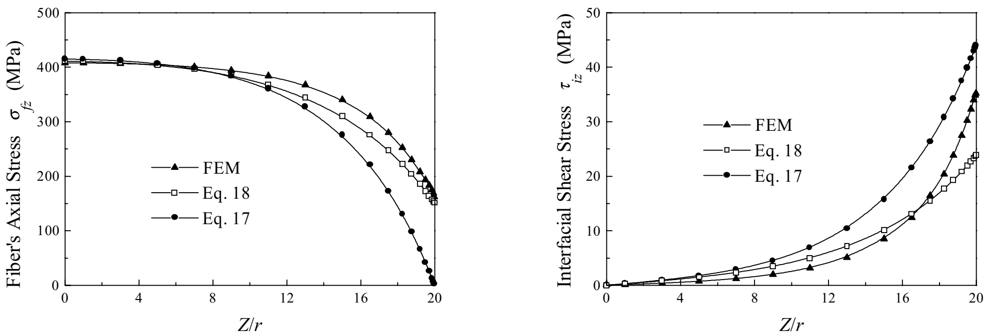


Fig. 4 The relative curve of the fiber's normal stress at fiber end σ_{f01} and the E_i (σ_0=100MPa)

According to Fig. 4, when E_i=2GPa, the σ_{f01} may be taken to be 160MPa. With this value of σ_{f01}, the distribution curves of the fiber axial stress σ_{fz} and the interfacial shear stress τ_{iz} gained from Eq. 18 and Eq. 17 are given in Fig. 5. It is indicated that the results of Eq. 18 agree very well with those gained by FEM.



a)σ_{fz} b) τ_{iz}

Fig. 5 The distributions of fiber axial stress σ_{fz} and interfacial shear stress τ_{iz} (σ_0=100MPa, E_i=2GPa)

CONCLUSION

Through comparing with the results gained from FEM, it is found that the modified shear-lag theory is more reasonable than the traditional shear-lag theory. The revisions for the fiber's normal stress at fiber end and the interfacial bond are necessary to the analysis of the stress transfer in short fiber reinforced metal matrix composites (SFRMMCs). The normal stress at fiber end σ_{f01} increases with the increment of the modulus of interfacial layer E_i. However, the rate of the increase of σ_{f01} is not the same for all of the interfacial moduli E_i($E_i \leq E_m$). The normal stress at fiber end σ_{f01} has no effect on the stresses in the middle of fiber.

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