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THE THERMAL CONDUCTIVITY OF MISALIGNED SHORT-FIBER-REINFORCED POLYMER COMPOSITES

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ABSTRACT: Short-fiber-reinforced polymer (SFRP) composites are often made with extrusion compounding and injection moulding techniques. In the final composite products a fiber length distribution (FLD) and a fiber orientation distribution (FOD) usually exist. In this paper, the thermal conductivity of SFRP composites is studied in detail taking into account the effects of the FLD, the FOD and the fiber volume fraction. It is shown that thermal conductivity can be significantly enhanced by the incorporation of short fibers into a polymer resin. The thermal conductivity of SFRP composites increases almost linearly with fiber volume fraction; it also increases with mean fiber length (or mean aspect ratio) but decreases with fiber orientation angle relative to the measured direction. The latter two effects depend highly on the thermal conductivity of the short fibers. Comparison is also made with existing experimental results and is found to be satisfactory.

KEYWORDS: Thermal Conductivity, Short Fiber Reinforced Polymer Composites

1. INTRODUCTION

Short fiber reinforced polymer (SFRP) composites have many applications as a class of structural materials because of their ease of fabrication, relatively low cost and mechanical properties which are superior to those of relevant polymer resins. In the final composites, there are a fiber length distribution (FLD) and a fiber orientation distribution (FOD). Studies on the mechanical properties of SFRP composites have shown that the FLD and the FOD play very important roles in determining the mechanical properties. SFRP composites are also attractive materials for electronic packaging applications where the combination of reinforcement with high thermal conductivity embedded in a resin matrix with low thermal conductivity is desirable to maintain a low temperature environment for thermally sensitive electronic packaging components. A number of models have been developed to predict the thermal conductivity of short fiber reinforced composites [1-9]. Nonetheless, these models were focused on aligned short fiber composite [1-4], completely random short fiber composites [5,6] or short fiber composites with constant fiber length [7-9]. However, the effects of the FLD and the FOD on thermal conductivity of short fiber composites have not been studied in any detail.

In the present work, an expression for the thermal conductivity of SFRP composites is derived using the laminate analogy approach (LAA). The thermal conductivity of SFRP composites is studied taking into account the effects of the fiber volume fraction, the FLD and the FOD. Moreover, comparison of the theoretical results with experimental data is made and is found to be satisfactory.

2. THEORY

The laminate analogy approach (LAA) will be used here to evaluate the thermal conductivity of SFRP composites. In the LAA, a SFRP composite can be simulated as a sequence of a stack of various laminae with different fiber orientation and different fiber length. Fiber orientation can be defined by a pair of angles (θ,ϕ) as shown in Fig. 1. The fiber length and orientation distribution density functions, $f(L)$ and $g(\theta)$, are [10-13]:

$$f(L) = a\, b\, L^{b-1}\exp(-a\, L^{b}) \qquad \text{for } L > 0 \tag{1}$$

$$g(\theta) = (\sin\theta)^{2p-1}(\cos\theta)^{2q-1}/\int_{\theta_{\min}}^{\theta_{\max}}(\sin\theta)^{2p-1}(\cos\theta)^{2q-1}\, d\theta \tag{2}$$

where a and b are size and shape parameters, respectively, for determining the size and shape of fiber length distribution curves. p and q are the shape parameters which can be used to determine the shape of the fiber orientation distribution curve and $0 \le \theta_{\min} \le \theta \le \theta_{\max} \le \pi/2$. The mean fiber length is given by:

$$L_{mean} = \int_{L_{\min}}^{L_{\max}} Lf(L)dL = a^{-1/b}\Gamma(1/b+1) \tag{3}$$

where $\Gamma(x)$ is the gamma function. And the mean fiber orientation angle is:

$$\theta_{mean} = \int_{\theta_{\min}}^{\theta_{\max}} \theta g(\theta)d\theta \tag{4}$$

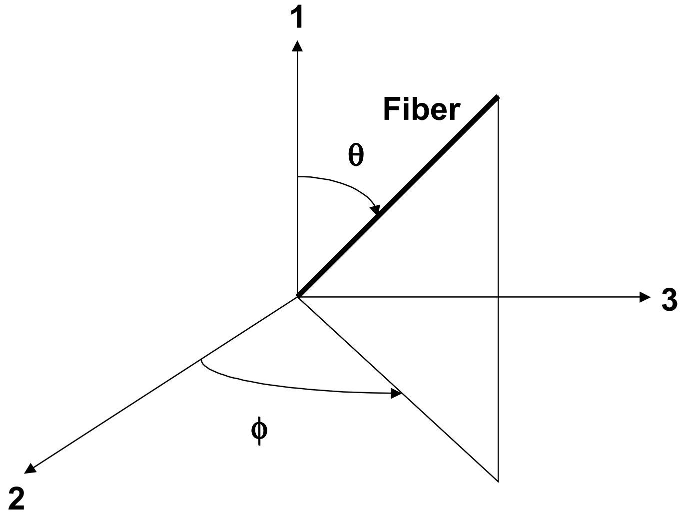

Fig. 1 The definition of spatial fiber orientation angles θ and ϕ.

Similar to the prediction of the elastic modulus of SFRP composites [11], a 3-dimensional (3D) misaligned SFRP composite is simulated as a laminated plate in order to evaluate the thermal conductivity of the SFRP composite in the "1" direction using the laminate analogy approach (LAA), which depends only on the orientation distribution of the angle the fibers make with the "1" direction. The successive development of the laminated plate model of 3D misaligned short fiber reinforced polymer is shown in Fig. 2. A SFRP composite with a 3D spatial fiber orientation distribution function $g(\theta,\phi)$ [$= g(\theta)\cdot g(\phi)/\sin\theta$] having fiber ends in the 3 visible planes (see Fig. 2a) is first replaced by an SFRP with the same $g(\theta)$ but $\phi = 0$ having no fiber ends in the 1-2 plane or no fibers in the out-of-planar direction (represented by the 3-axis) (see Fig. 2b). Then according to the fiber length distributions, this composite is regarded as a combination of laminates, each comprising of fibers having the same fiber length (see Fig. 2c, "L(L_i), i=1,2,...n" denotes the ith laminate containing fibers of the same length L_i). Each laminate with the same fiber length is then treated as a stacked sequence of laminae, each lamina consists of fibers having the same fiber length and the same fiber orientation (see Fig.

2d, "L(L_i, θ_j), j=1,2,...m" denotes the *j*th lamina containing fibers having the same length L_i and the same angle θ_j).

In the LAA, we need first to evaluate the thermal conductivity of corresponding aligned short fiber composites assuming all the fibers lie in the 1-2 plane. Although a few models have been proposed for the thermal conductivity of aligned short fiber composites, they are all quite complicated and are not favored to be used here. On the other hand, Bigg [14] and Choy et al [8,9] have shown that the Halpin-Tsai equation can be used to describe the thermal conduction of aligned short fiber composites. For a unidirectional lamina, the thermal conductivities parallel (K_1) and perpendicular (K_2) to the fiber direction are given by [15,16]

$$K_1 = \frac{1+2\alpha\mu_1 V_f}{1-\mu_1 V_f} K_m \tag{5}$$

$$K_2 = \frac{1+2\mu_2 V_f}{1-\mu_2 V_f} K_m \tag{6}$$

where $\alpha = L/d_f$ in which d_f is the fiber diameter. V_f is the fiber volume fraction. K_m is the thermal conductivity of the matrix. And μ_1 and μ_2 are given by

$$\mu_1 = \frac{K_{f1}/K_m - 1}{K_{f1}/K_m + 2\alpha} \tag{7}$$

$$\mu_2 = \frac{K_{f2}/K_m - 1}{K_{f2}/K_m + 2} \tag{8}$$

where K_{f1} and K_{f2} are the thermal conductivity of the fiber in the direction parallel and transverse to the fiber axis direction, respectively. For a unidirectional lamina, if fibers of the lamina are aligned parallel to the 1-axis ($\theta = 0$), the linear relationship between the heat flux and temperature gradient in the direction parallel and perpendicular to the fiber direction, namely in the 1-2 coordinate, is given by

$$q_i = -K_i \nabla T_i \qquad i = 1,2 \tag{9}$$

If the fibers of the lamina are oriented at an angle θ ($\theta \neq 0$) relative to the 1-axis (defined here as the 1' axis) and a new 1'-2' coordinate system is obtained, the linear relationship between heat flux and temperature gradient in the new coordinates is:

$$q_i^{'} = -K_i^{'} \nabla T_i^{'} \qquad i = 1,2 \tag{10}$$

Let us introduce a transformation tensor X_{ij} defined by:

$$Y_i = X_{ij} Y_j^{'} \tag{11}$$

where Y_i and $Y_j^{'}$ are the components of a vector *Y* in the 1-2 and 1'-2' coordinates, respectively. Then, we can have:

$$X_{ij}^{-1} q_i^{'} = -K_i X_{ij} \nabla T_j^{'} \tag{12}$$

So,

$$K_i^{'} = X_{ij}^{-1} K_i X_{ij} \tag{13}$$

where the coordinate transformation tensor X_{ij} is given by

$$X_{ij} = \begin{Bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{Bmatrix} \tag{14}$$

X_{ij}^{-1} can be obtained as

$$X_{ij}^{-1} = \begin{Bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{Bmatrix} \tag{15}$$

So we finally obtain:

$$K_1^{'} = K_1 \cos^2\theta + K_2 \sin^2\theta \tag{16}$$

This expression is also given in References [8,9,13]. The total heat flux along the 1' axis for a multi-laminate is then:

$$Q_1^{'} = \sum_{k=1}^{M} q_1^{'} h_k = -\sum_{k=1}^{M} K_1^{'} \nabla T_1^{'} h_k \tag{17}$$

where M represents the number of plies in the laminate, k is the serial index of the ply in the laminate, and h_k is the thickness fraction of the k^{th} ply. Since the temperature gradient is continuous across the thickness, so eqn (17) reduces to

$$Q_1^{'} = -K_c \nabla T_1^{'} \tag{18}$$

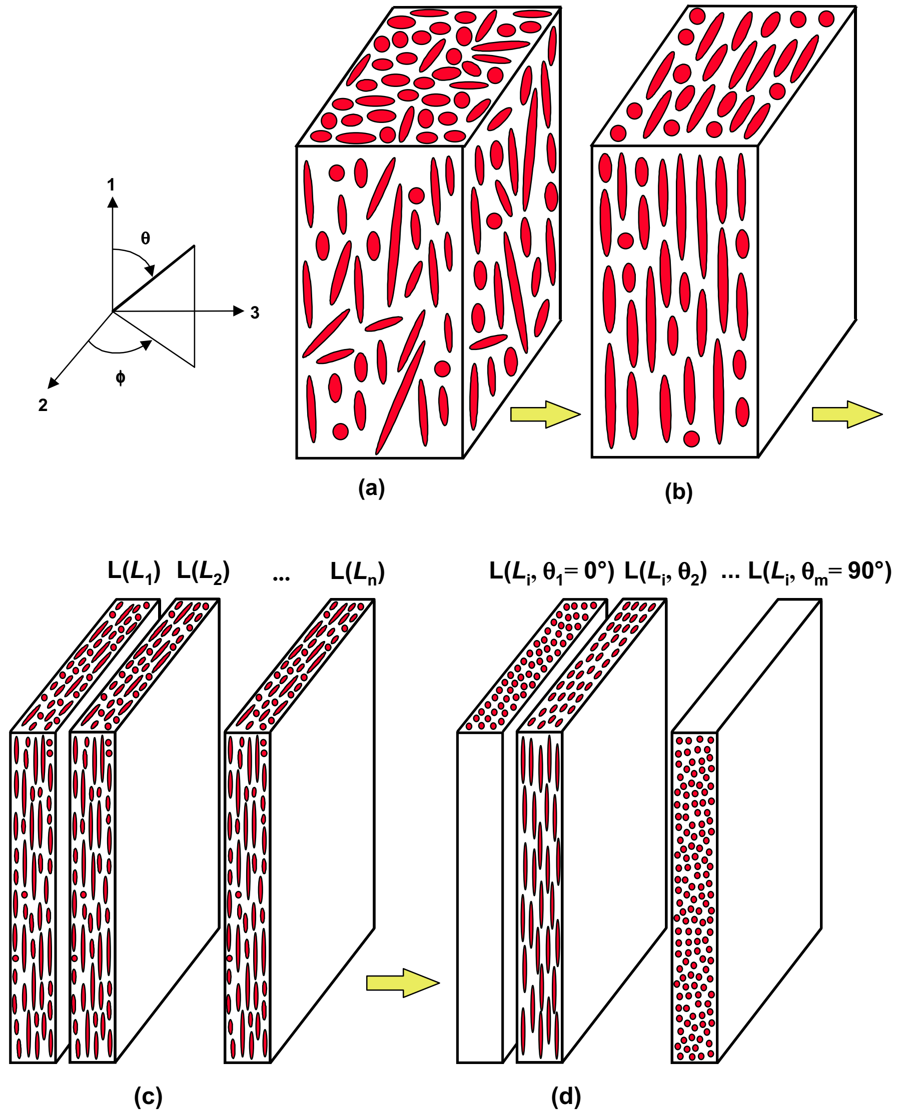

Fig. 2 Simulated progress of the laminated plate model of a 3D misaligned short fiber reinforced polymeric composite: (a) the real 3D SFRP, (b) the supposed SFRP, (c) the supposed SFRP is considered as combination of laminates, each laminate has the same fiber

length and (d) each laminate is treated as a stacked sequence of laminae, each lamina has the same fiber length and the same fiber orientation.

where the thermal conductivity of the composite laminate K_c is given by

$$K_c = \sum_{k=1}^{M} K_1^{'} h_k = \int_{L=L_{\min}}^{L_{\max}} \int_{\theta=\theta_{\min}}^{\theta_{\max}} K_1^{'} f(L) g(\theta) \mathrm{d}L \mathrm{d}\theta \tag{19}$$

The thermal conductivity of a SFRP composite with a FLD and a FOD can then be evaluated using eqn (19).

Moreover, equation (19) can also be used to evaluate the thermal conductivity of a SFRP composite in any given direction (Θ,Φ) but the angle θ must be replaced by the angle δ in equation (16), where the angle δ is that between the fiber axial direction (θ,ϕ) and the direction (Θ,Φ) given by [10]

$$\cos\delta = \cos\Theta\cos\theta + \sin\Theta\cos(\phi - \Phi) \tag{20}$$

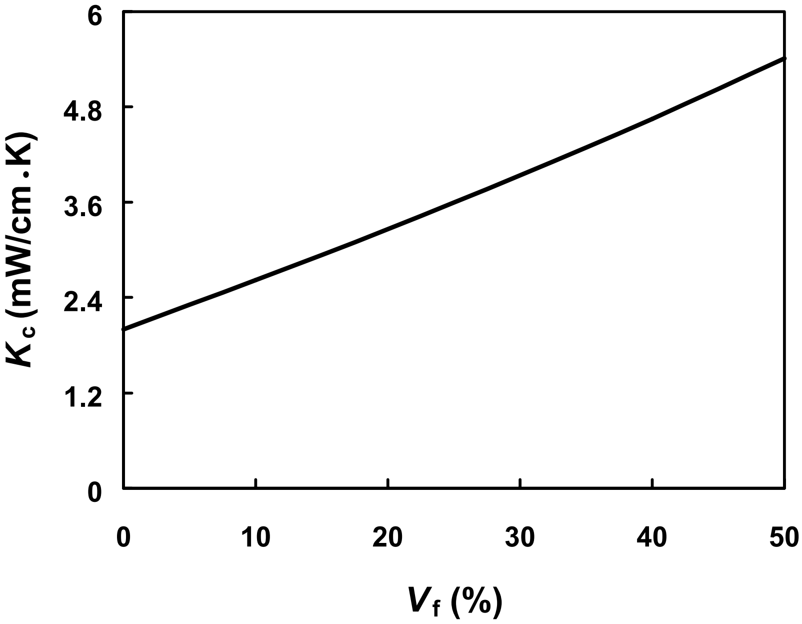

Fig. 3 The effect of fiber volume fraction on the thermal conductivity of SFRP composites.

3. RESULTS AND DISCUSSION

The effect of fiber volume fraction on the thermal conductivity of SFRP composites is shown in Fig. 3, where $d_f = 10$ µm, $K_{f1} = K_{f2} = 10.4$ mW/cm·K [for short glass fiber (SGF), see Ref 8], $K_m = 2$ mW/cm·K [for poly (phenylene sulfide) (PPS), see Ref 8], $L_{mean} = 424$ µm ($a = 2.6$ and $b = 1.2$) and $\theta_{mean} = 35.95^{o}$ ($p = 0.6$ and $q = 1$). It is observed that the thermal conductivity of the composites increases almost linearly with the increase of fiber volume fraction. This is obvious because the thermal conductivity of fibers is much higher than that of the resin matrix. So, the incorporation of short fibers can significantly enhance the thermal conductivity.

Fig. 4 shows the effect of mean fiber length (or mean aspect ratio) on the thermal conductivity of SFRP composites, where the parameters are the same as in Fig. 3 except V_f is fixed at 0.3, and the thermal conductivities of carbon fiber are $K_{f1} = 94$ mW/cm·K and $K_{f2} = 6.7$ mW/cm·K [8] and L_{mean}/d_f varies by changing a (b is fixed at 1.2). It can be seen that the thermal conductivity of short glass fiber reinforced polymer composites increases slightly with the increase of mean fiber aspect ratio when $L_{\mathrm{mean}}/d_{\mathrm{f}}$ is small (< ~ 10) and becomes insensitive to fiber aspect ratio when $L_{\mathrm{mean}}/d_{\mathrm{f}}$ is large (> ~20). However, the thermal conductivity of short carbon fiber reinforced polymer composites increases significantly with the increase of mean fiber aspect ratio especially when $L_{\mathrm{mean}}/d_{\mathrm{f}} < \sim 50$. This is because the thermal conductivity of carbon fiber is much higher than that of glass fiber in the fiber axis direction.

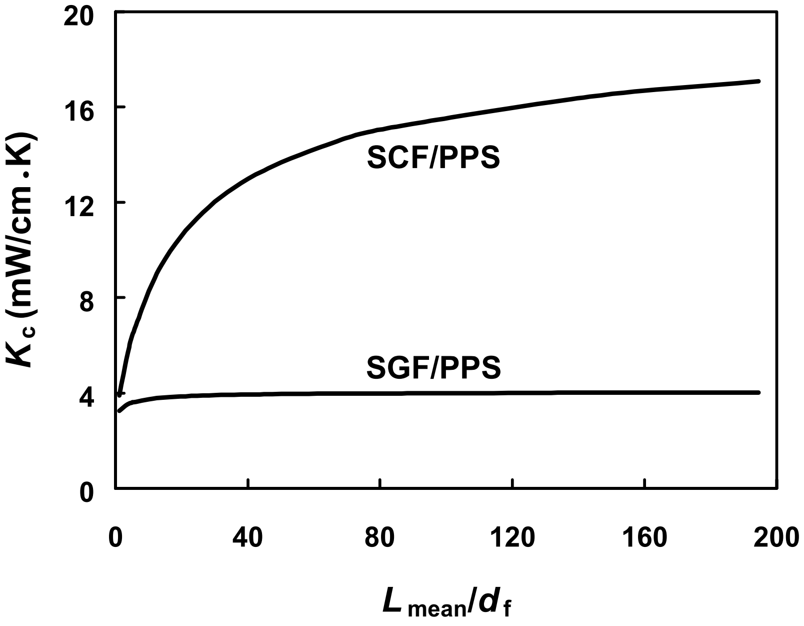

Fig. 4 The effect of mean fiber aspect ratio on the thermal conductivity of SFRP composites.

Fig. 5 displays the effect of mean fiber orientation angle on the thermal conductivity of SFRP composites, where the parameters are the same as in Fig. 4 except $L_{\mathrm{mean}} = 424$ μm and θ_{mean} varies by changing p (q is fixed at 1). It is observed that the thermal conductivity of short glass fiber reinforced polymers decreases slowly with the increase of mean fiber orientation angle. However, the thermal conductivity of short carbon fiber reinforced polymers increases significantly with the decrease of mean fiber orientation angle θ_{mean}. This is again caused by the fact that the thermal conductivity of carbon fiber is much higher than that of glass fiber in the fiber axis direction.

Comparison between the predicted results and the experimental data is shown in Table 1 for the thermal conductivity of short glass fiber and short carbon fiber reinforced poly (phenylene sulfide) (PPS) composites [8] and poly (ether ether ketone) (PEEK) composites [9]. The thermal conductivity of the surface layer and the skin layer was measured separately. The data for the parameters are [8,9]: $K_m = 2.43$ mW/cm·K for PEEK and $K_m = 2$ mW/cm·K for PPS, $K_1 = K_2 = 10.4$ mW/cm·K for glass fiber, $K_1 = 94$ mW/cm·K for the carbon fiber used in PPS and $K_1 = 80$ mW/cm·K for the carbon fiber used in PEEK and $K_2 = 6.7$ mW/cm·K. An alternative expression for the fiber orientation distribution density function is given by: $g(\theta) = -\lambda \cdot \exp(-\lambda\theta)/[1-\exp(-\lambda\pi/2)]$ [8,9] and the values of λ are given in Table 1. Clearly, the theoretical results are in good agreement with the experimental values.

4. CONCLUSIONS

The effects of fiber volume fraction, fiber length and orientation distributions on the thermal conductivity of short fiber reinforced polymer composites have been studied. The composite thermal conductivity increases almost linearly with the increase of fiber volume fraction. When the fiber thermal conductivity is high, the composite thermal conductivity increases significantly with increase of mean fiber aspect ratio (or mean fiber length) and decrease of mean fiber orientation angle. When the fiber thermal conductivity is low, the composite thermal conductivity increases slowly with increase of mean fiber aspect ratio (or mean fiber length) and decrease of mean fiber orientation angle. Also, theoretical predictions agree well with experimental results.

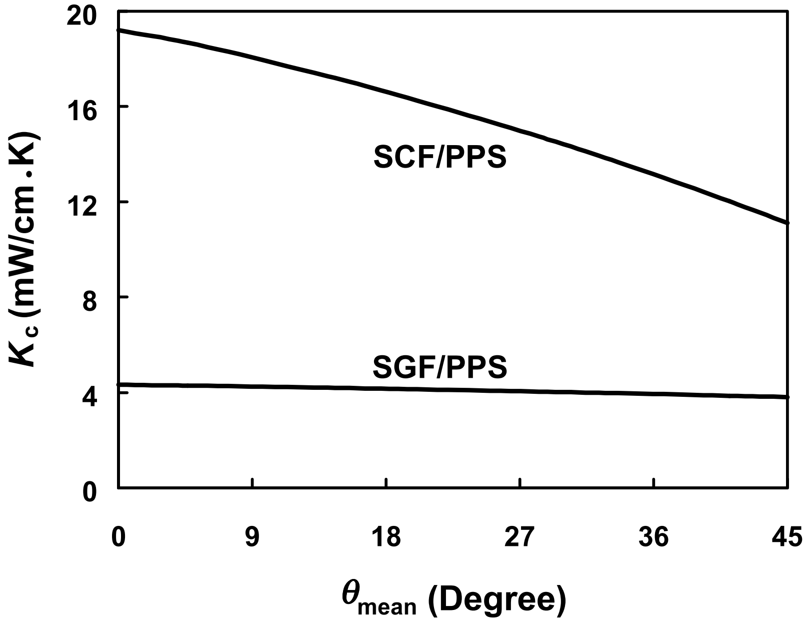

Fig. 5 The effect of mean fiber orientation angle on the thermal conductivity of SFRP composites.

Table 1. Comparison between the predictions and the experimental results [8,9] for the thermal Conductivity K_c (mW/cm·K) of four composites

Composites	V_f	Average Aspect Ratio	λ		K_c			
			Surface Layer	Middle Layer	Surface Layer		Middle Layer	
					Exp.	The.	Exp.	The.
PEEK30cf*	0.214	17	3.3	2.4	11.6	10.4	10.8	9.9
PPS30cf	0.243	21	4.1	1.9	15.2	12.6	12.4	11.1
PPS40cf*	0.335	16	4.7	2.1	17.2	16.0	15.6	14.0
PPS40gf*	0.264	17	5.3	2.9	4.08	3.93	3.99	3.85

*30cf, 40cf and 40gf denote 30 wt % carbon fiber, 40 wt % carbon fiber and 40 wt % glass fiber, respectively.

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