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# GEOMETRIC ANALYSIS OF 3D WOVEN STRUCTURES AND DETERMINATION OF FIBER VOLUME FRACTION

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**SUMMARY:** Analyses of 3D woven structure discovered that the fibrous architecture of the woven structure varies significantly with binding patterns, which are classified as angle- or orthogonal-interlock, with through-the-thickness or layer-to-layer binding. To describe fibrous architecture of 3D woven structures and, consequently, to determine the contributions of different yarn systems inside the 3D structure to the fiber volume fraction, a unit cell model is developed. Parameters are defined to identify yarns geometry along their lengths within the cell. By assuming yarn cross section of racetrack shape, yarn configurations as an assemblage of linear and arc sections for binders and warps, and straight form for stuffers and wefts, the orientation angle and the length of constituent yarns within the cell can thus be calculated. This leads to the determination of fiber volume fraction of the woven composite. The calculated fiber volume fraction is compared with test results from 11 selected samples of 3D woven composites with fiberglass woven preform and polyester resin matrix. A good agreement is demonstrated between theoretic predictions and experiment results.

**KEYWORDS:** Textile composite, 3D woven structure, Fiber volume fraction

## INTRODUCTION

3D woven structures are used as reinforcements to provide an integrated fibrous architecture and to improve the inter-laminar fracture toughness of composites. To determine a suitable 3D fibrous architecture as the reinforcement for composites, it is imperative to understand the contributions of constituent yarn systems forming the fibrous architecture. Byun[1] made an initial effort to analyze 3D woven structures based on Peirce's model[2]. In the analysis, the yarn cross section was assumed to be of lenticular shape, as originally proposed by Grosberg[3] for 2D woven fabrics, and a kind of the tightest structure was implied that all the yarns were in contact with their neighboring ones. Another geometric model of 3D woven structures was presented by Ko[4] to establish a relationship between fiber volume fraction and processing parameters. Some design parameters for 3D reinforcement patterns, such as the number of yarns in each yarn system within a unit cell, fabric tightness factor and linear density ratio of yarns, have been considered. The model is suitable for woven structures with equal number of yarns in each yarn system.

A refined geometric model was proposed by Pochiraju[5] to describe fiber distributions inside 3D woven structures. The number of yarn layers integrated in the thickness direction and the tow size of yarns were considered as the structural parameters. A "unit cell decomposition" method was used to describe individually the centerline of yarns in the 3D architecture and an

"assembly model" was applied to establish the geometrical relationships among yarn systems within the unit cell. To simplify the analyses, the centerline of binders was assumed to follow a sinusoidal configuration, while wefts and stuffers were considered to be straight. The designability of 3D woven composites was emphasized in the study.

A number of investigations[6-14] indicated that the micro-structure of woven composites varies significantly with types of yarn, fabric setts, binding patterns and consolidation conditions. These studies demonstrated the necessity to develop a general purposed modeling method in dealing with varieties of 3D woven structures, by which governing factors on the performance of 3D woven composites could be discussed, in order to facilitate the structure design of 3D woven preforms.

### GEOMETRIC MODEL OF 3D WOVEN STRUCTURE

Yarn orientation angle and fiber volume fraction are two governing factors of the reinforcement in determining performances of composite materials. A good prediction of effects of the two factors, to a great extent, relies on a precise description of the geometric pattern of 3D fibrous architectures, by which the contributions of different yarn systems can be approached. For this purpose, a geometric model has been developed based on the following assumptions:

- The cross section of all yarns is of racetrack shape, uniform in packing density and geometry along their lengths.
- Stuffers and wefts are of straight configuration along their length, and
- Warps and binders are deflected along their length with sections of straight parts joined by arc ones.

A unit cell is composed of four yarn systems (binder, warp, stuffer and weft) termed as sub-unit cells. In each sub-unit cell the number of yarns can be determined according to the binding pattern. The sub-unit cells of non-straight yarns, such as binders and warps, can further be divided into micro cells of two types of yarn sections, linear and arc sections. For a structure of angle interlock with layer-to-layer binding, the formation of the unit cell with its constituent yarn systems is schematically illustrated in Fig.1. In the illustration  $n_b$ ,  $n_j$ ,  $n_s$  and  $n_w$  are the numbers of binder, warp, stuffer and weft, respectively, in the unit cell.

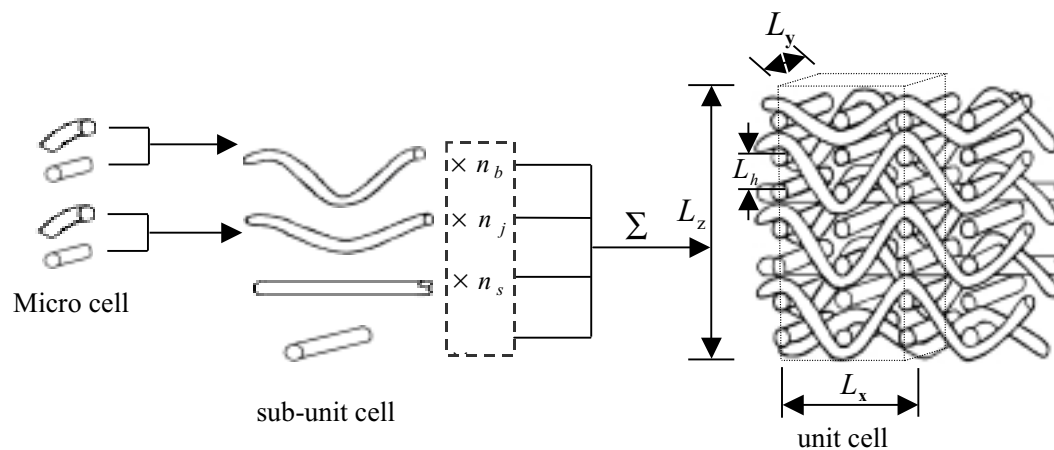


Fig.1 Geometric model of 3D woven structure

In Fig.1, the Cartesian coordinate axes of the unit cell coincide with the longitudinal, transversal and thickness directions of the 3D woven structure. The dimensions of the unit cell, illustrated by the dotted hexahedron, can be calculated as

$$\begin{aligned} L_x &= n_{fl} / P_w \\ L_y &= S_c / P_j \\ L_z &= L_h \cdot (n_f - 1) + 2B_b + B_w \end{aligned} \quad (1)$$

where,  $L_x$ ,  $L_y$  and  $L_z$  are the dimensions of the unit cell,  $L_h$  is the distance between two weft layers,  $P_j$  and  $P_w$  are warp density (ends/10cm) and weft density (picks/10cm),  $n_{fl}$  is the binding distance,  $n_f$  is the total weft layers,  $S_c$  is the distance of a binding couple moving to its neighbor one in the transversal direction within the unit cell,  $B_b$  and  $B_w$  are the short diameters of binder and weft, respectively.

With the racetrack yarn cross section, the area of the total fibers in the cross section  $S$  can be calculated by

$$\begin{cases} A = \lambda \cdot B \\ B = \sqrt{\frac{N \times 10^{-3}}{\rho \cdot \rho_f \cdot (\pi / 4 - 1 + \lambda)}} \\ S = (\pi / 4 + \lambda - 1) \cdot B^2 \end{cases} \quad (2)$$

where,  $A$  and  $B$  are the long and short yarn diameters,  $N$  is the yarn count in tex,  $\lambda$  is the aspect ratio of yarn cross section,  $\rho$  is the yarn packing density in percentage and  $\rho_f$  the fiber density in  $g/cm^3$ .

The geometry of the racetrack yarn cross section can be divided into two semicircles with a rectangle in between. The yarn packing density in the semicircle is around 0.75[4, 15] and that in the rectangle with square packing is 0.785[4]. Therefore, the yarn packing density,  $\rho$ , of the racetrack cross section would be in the range of 0.75~0.785. In the present calculation  $\rho = 0.77$  is applied.

According to the assumptions of the cell model, binders and warps are the assembly of straight and arc sections. An example illustrated in Fig.2 is the geometry of a binder in a 3D woven structure of an angle interlock with through-the-thickness binding.

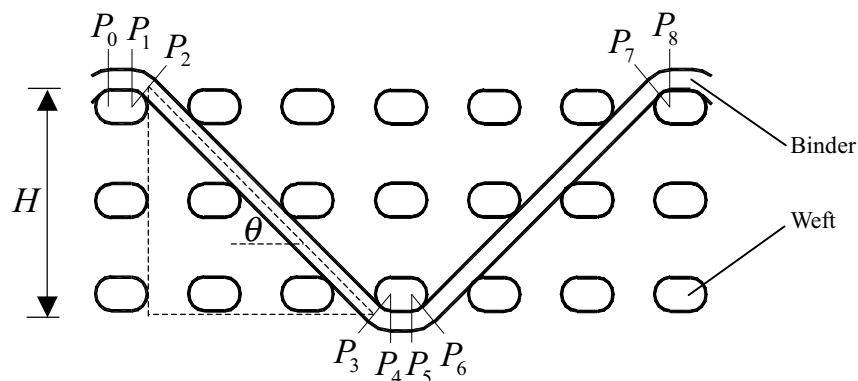


Fig.2 Geometry of a binder along its length in a unit cell

Within the unit cell, the shape of the binder can be divided into eight sections. Among them,  $P_0P_1$ ,  $P_2P_3$ ,  $P_4P_5$  and  $P_6P_7$  are the straight sections and  $P_1P_2$ ,  $P_3P_4$ ,  $P_5P_6$  and  $P_7P_8$  are the arc sections. The length of each section

$$\begin{aligned} P_0P_1 &= A_w - B_w \\ P_1P_2 &= \theta_b (B_w + B_b) / 2 \\ P_2P_3 &= H / \sin \theta_b \\ P_3P_4 &= P_5P_6 = P_7P_8 = P_1P_2, \quad P_4P_5 = P_0P_1, \quad P_6P_7 = P_2P_3 \end{aligned} \quad (3)$$

where,  $A_w$  and  $B_w$  are the long and short diameters of wefts,  $B_b$  is the short diameter of the binder and  $\theta_b$  the orientation angle of the binder.

Thus, the total length of the binder

$$L_b = \sum_{m=0}^7 P_m P_{m+1} \quad (4)$$

The total length of a binder in a unit cell varies with different binding patterns.

$$L_b = \begin{cases} 2(A_w - B_w + (\theta_b + \text{ctg } \theta_b)(B_w + B_b) + \frac{100(n_{fi} - 1)}{P_w \cdot \cos \theta_b}) & \text{for angle interlock} \\ 2(A_w - B_w + (B_w + B_b)(\theta_b + \frac{n_{fi} + \cos \theta_b - 1}{\sin \theta_b})) & \text{for orthogonal interlock} \end{cases} \quad (4a)$$

where,  $n_{fi}$  is the binding depth.

From the geometric pattern in Fig.3, the orientation angle of a binder can be obtained. For the reason of convenience, in the following parts of the paper, all the parameters of length are normalized by the long diameter of weft,  $A_w$ , and printed in small letters with respect to the absolute terms printed in capital.

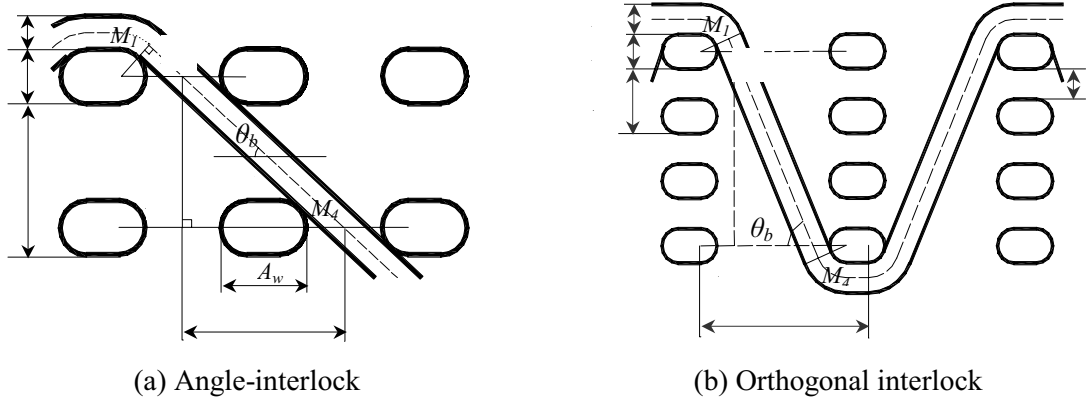


Fig.3 Geometry of binder and binding patterns

The orientation angle of a binder

$$\theta_b = \begin{cases} \arcsin\left(\frac{b_b + 1/\lambda_w}{1/p_w - 1 + 1/\lambda_w}\right) & \text{for angle interlock} \\ \arcsin\left(\frac{-\mu_1 + \sqrt{\mu_1^2 - 4\mu_0\mu_2}}{2\mu_0}\right) & \text{for orthogonal interlock} \end{cases} \quad (4b)$$

where,  $\lambda_w (= a_w/b_w)$  is the aspect ratio of weft yarns, and

$$\begin{aligned}\mu_0 &= (1/p_w + 1/\lambda_w - 1)^2 + (b_b + 1/\lambda_w)^2 (n_{ft} - 1)^2 \\ \mu_1 &= 2(b_b + 1/\lambda_w)(1 - 1/p_w - 1/\lambda_w) \\ \mu_2 &= n_{ft}(2 - n_{ft})(b_b + 1/\lambda_w)^2\end{aligned}$$

Similar geometric relationships are held for the calculation of warp yarns.

$$L_j = n_{ft}((A_w - B_w)(1 - \cos \theta_j) + (B_w + B_j)\theta_j + 100/P_w \times \cos \theta_j) \quad (5)$$

where,  $\theta_j$  is the orientation angle of warp yarns

$$\theta_j = \arcsin\left(\frac{b_b + 1/\lambda_w}{1/p_w - 1 + 1/\lambda_w}\right) \quad (5a)$$

For the straight form of stuffer and weft yarns, the lengths

$$L_s = 100/P_w \times n_{ft} \quad (6)$$

$$L_w = 100/P_w \times S_c \quad (7)$$

Thus, the fiber volume fraction in a unit cell

$$V_f = \frac{(n_b \cdot L_b \cdot S_b + n_j \cdot L_j \cdot S_j + n_s \cdot L_s \cdot S_s + n_w \cdot L_w \cdot S_w) \cdot \rho}{L_x \cdot L_y \cdot L_z} \times 100\% \quad (8)$$

where,  $L_b$ ,  $L_j$ ,  $L_s$  and  $L_w$  are the lengths of binder, warp, stuffer and weft in the unit cell with dimensions of  $L_x \times L_y \times L_z$ ,  $S_b$ ,  $S_j$ ,  $S_s$  and  $S_w$  are the areas of corresponding yarn cross section.

## EXPERIMENTS

To verify the cell model, samples of 3D woven composites were fabricated with selected 3D woven patterns. Fiberglass was used to form the reinforcement architecture and unsaturated polyester used as resin matrix. Densities of the sample were measured and, then, the fiber volume fraction of each sample

$$V_f = \frac{\rho_s - \rho_m}{\rho_f - \rho_m} \times 100\% \quad (9)$$

where,  $\rho_s$  is the tested density of the composite samples,  $\rho_f$  is the fiber density and  $\rho_m$  the resin density. For fiberglass,  $\rho_f = 2.54 \text{ g/cm}^3$  and for polyester resin  $\rho_m = 1.23 \text{ g/cm}^3$ .

In the experiments, eleven selected samples of 3D woven composites with typical binding patterns were included, such as angle interlock and orthogonal interlock, with through-the-thickness or layer-to-layer bindings. The comparison results are shown in Fig.4. Of each

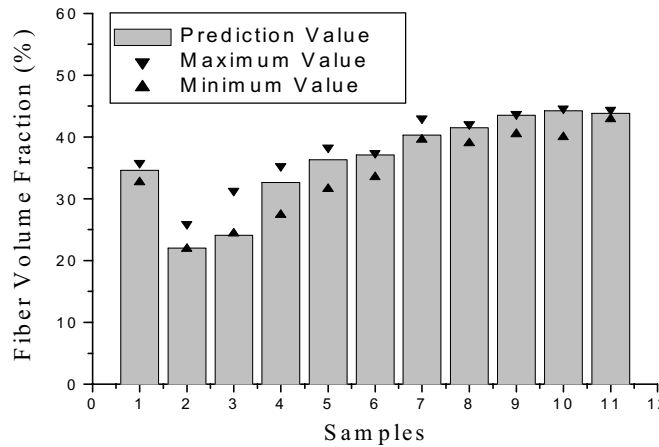


Fig.4 Fiber volume fraction – predictions vs. measurements

sample, the maximum and minimum values are from 6 individual tests. From the results, it can be seen that the model prediction values agree quite well with the sample measurements.

## CONCLUSIONS

A unit cell of a 3D woven structure is composed of four yarn systems, binder, warp, stuffer and weft. A model is developed by assuming that the geometry shape is of an assembly of linear and arc sections for binder and warp and straight form for stuffer and weft. The length of yarns within the unit cell can thus be calculated. With an additional assumption of racetrack yarn cross section to determine the area and packing density of yarns, the yarn orientation angle and fiber volume fraction can finally be determined.

Investigations[16] discovered that the fiber volume fraction varies with the binding pattern. With the same weft density, orthogonal interlock gives a higher fiber volume fraction than the angle interlock counterparts due to a more efficient utilization of spaces within the 3D fibrous architecture, especially under high weft sett. Also, layer-to-layer binding can offer a higher fiber volume fraction than through-the-thickness binding. Further application of the model can reveal the contributions of other parameters to the yarn orientation and packing, consequently, the fiber volume fraction.

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