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CURVED LAMINATE ANALYSIS

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SUMMARY: A general theory that describes the linear elastic response of a curved laminate subjected to in-plane loads and bending moments is developed in this paper. Similar to the classic 6x6 ABD matrix constitutive relation of a flat laminate, a new 6x6 matrix constitutive relation between stress resultants, moment resultants, mid-plane strains and deformed curvatures for a curved laminate is formulated. This new curved laminate theory will provide the fundamental basis for the analyses of curved laminate structures. The application of the curved laminate theory is demonstrated by the stress calculations of the curved laminate beam. The results show that the stresses predicted by the classic laminate theory and the curved laminate theory can be quite diverse as the ratio of height to radius, h/R, is getting larger.

KEYWORDS : Curved Laminate, Laminate Theory, Constitutive Relation

INTRODUCTION

The laminate theory originally presented by Pister and Dong [1], Reissner and Stavsky [2] and Dong et al. [3] has been widely applied to analyze the laminated composite structures. Thereafter, many books [4-7] were devoted to present the theory more completely and applied the theory to perform structure analyses of laminated beams, plates, columns and rods etc. with straight or flat geometries. However, it is frequent that the laminated composite structures are curved rather than straight or flat for many applications (e.g. airplane bracket and skin). Curved laminated beams and plates are instances where stresses and displacements must be determined on the basis of a theory that accounts for the non-flat geometries of the structures.

A general theory that describes the linear elastic response of a curved laminate subjected to in-plane loading and bending will be developed in the present analysis. Individual layers in the curved laminate are assumed to be homogeneous, orthotropic and in a state of plane stress. First, the strain-displacement equations between the in-plane strains and the mid-plane strains as well as the deformed curvatures are derived. Subsequently, the stresses in the individual layer can be evaluated from the plane stress constitutive relations. By integrating the stresses through the laminate thickness, the stress resultants (i.e., in-plane forces per unit length) and the moment resultants (i.e., in-plane moments per unit length) are obtained in terms of the mid-plane strains and the deformed curvatures. Finally, a new 6x6 ABCD matrix constitutive relation between stress resultants, moment resultants, mid-plane strains and deformed curvatures for a curved laminate is formulated. Similar to the classic ABD matrix constitutive formulation of a flat laminate, this new curved laminate constitutive relation will provide the fundamental basis to the analyses of curved laminate structures. The influence of the laminate lay-up sequences on the computation of the 6x6 ABCD matrix will be discussed.

The application of the curved laminate theory is demonstrated by the stress calculations of the curved laminate beam under the statically determined loading. Depending on the loading and the dimension of the cross section in relation to the location of the center of geometrical curvature, the stress predictions by the classic laminate theory and the curved laminate theory can be quite different. In situations where the h/R ratio is small, the classic laminate theory continues to give acceptable accuracy. However, as the h/R ratio is getting larger, the classic laminate theory can no longer predicts the stresses accurately and the curved laminate theory is essentially required for proper stress analyses.

CURVED LAMINATE THEORY

Consider a curved laminate of thickness h as depicted in Figure 1a. Here, the x-axis is passing everywhere through the centroid of the section and tangent to a circular arc of radius R, that is, ds=Rdθ, where θ is the angular variable associated with a change in location along the curved section. The z-axis lies along the local direction of the radius R with the y-axis such that a right-handed rectangular coordinate system is formed. As depicted in Figure 1b, the curved laminate has N layers numbered from bottom lamina to top lamina. Coordinates h_k are the vertical distance from the mid-plane to the interfaces and they have the sign conventions of the z coordinates.

Strain-Displacement Relationships

The curved laminate consists of perfectly bonded layers and its individual layer is assumed to be homogeneous, orthotropic and in a state of plane stress. Furthermore, the curved laminate deforms according to the Kirchhoff-Love hypothesis for stretching and bending of thin plates:

(1) A lineal element of the curved laminate extending through the laminate thickness is normal to the mid-plane (instantaneous xy plane). Upon application of load, the lineal element remains straight and normal to the deformed mid-plane.

(2) The lineal element does not change length.

Based upon the foregoing assumptions, the most general form for the displacements in the x and y directions is

$$u(x,\ y,\ z)=u_0(x,\ y)+z\alpha(x,\ y) \tag{1}$$

$$v(x,\ y,\ z)=v_0(x,\ y)+z\beta(x,\ y) \tag{2}$$

where u_0 and v_0 denote the mid-plane displacements in the x and y directions and α and β are notations which will be further defined later. From the assumption (2), requires that $\varepsilon_z=0$ and in turn means that the displacement in the z direction can be expressed as

$$w(x,\ y,\ z)=w_0(x,\ y)=\ w \tag{3}$$

where w_0 denotes the mid-plane displacements in the z direction.

By specializing the cylindrical coordinate strain-displacement relations to the present situation, the strain-displacement relations are

$$\varepsilon_x=\frac{1}{1+\kappa z}\left(\frac{\partial u}{\partial s}+Kw\right)=\frac{1}{1+\kappa z}\left(\frac{\partial u_0}{\partial s}+z\frac{\partial \alpha}{\partial s}+\kappa w\right) \tag{4}$$

$$\varepsilon_y=\frac{\partial v_0}{\partial y}+z\frac{\partial \beta}{\partial y} \tag{5}$$

$$\varepsilon_z=\frac{\partial w}{\partial z}=0 \tag{6}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{1}{1+\kappa z}\frac{\partial v}{\partial s} = \frac{\partial u_0}{\partial y} + z\frac{\partial \alpha}{\partial y} + \frac{1}{1+\kappa z}\left(\frac{\partial v_0}{\partial s} + z\frac{\partial \beta}{\partial s}\right) \tag{7}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} - \frac{\kappa}{1+\kappa z}u + \frac{1}{1+\kappa z}\frac{\partial w}{\partial s} = \frac{\partial u_0}{\partial z} + \alpha - \frac{\kappa}{1+\kappa z}(u_0 + z\alpha) + \frac{1}{1+\kappa z}\frac{\partial w}{\partial s} \tag{8}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial v_0}{\partial z} + \beta + \frac{\partial w}{\partial y} \tag{9}$$

where $\kappa=1/R$ is the geometrical curvature of the curved laminate. The assumption (1) requires that the shear deformations are zero (i.e., $\gamma_{xz}=0$ and $\gamma_{yz}=0$) and leads to

$$\alpha = \kappa u_0 - \frac{\partial w}{\partial s} \tag{10}$$

$$\beta = -\frac{\partial w}{\partial y} \tag{11}$$

Substituting Eqs. (10-11) into Eqs. (4-5) and (7), The in-plane strains can be expressed as

$$\varepsilon_x = \frac{\partial u_0}{\partial s} + \kappa w - \frac{z}{1+\kappa z}\left(\frac{\partial^2 w}{\partial s^2} + \kappa^2 w\right) = \varepsilon_x^0 + \frac{z}{1+\kappa z}\kappa_x^1 \tag{12}$$

$$\varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} = \varepsilon_y^0 + z\kappa_y^0 \tag{13}$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial s} - z\left(\frac{\partial^2 w}{\partial s \partial y} - \kappa\frac{\partial u_0}{\partial y}\right) - \frac{z}{1+\kappa z}\left(\frac{\partial^2 w}{\partial s \partial y} + \kappa\frac{\partial v_0}{\partial s}\right) = \gamma_{xy}^0 + z\kappa_{xy}^0 + \frac{z}{1+\kappa z}\kappa_{xy}^1 \tag{14}$$

where the mid-plane strains $\{\varepsilon^0\}$ and deformed curvatures $\{\kappa^0\}$ and $\{\kappa^1\}$ are defined as

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial s} + \kappa w \tag{15}$$

$$\kappa_x^1 = -\left(\frac{\partial^2 w}{\partial s^2} + \kappa^2 w\right) \tag{16}$$

$$\varepsilon_y^0 = \frac{\partial v_0}{\partial y} \tag{17}$$

$$\kappa_y^0 = -\frac{\partial^2 w}{\partial y^2} \tag{18}$$

$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial s} \tag{19}$$

$$\kappa_{xy}^{0} = -\left(\frac{\partial^2 w}{\partial s \partial y} - \kappa \frac{\partial u_0}{\partial y} \right) \tag{20}$$

$$\kappa_{xy}^{1} = -\left(\frac{\partial^2 w}{\partial s \partial y} + \kappa \frac{\partial v_0}{\partial s} \right) \tag{21}$$

Combining Eqs. (12-14) and Eqs. (15-21) in matrix form, we obtain

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} 0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} + \frac{z}{1+\kappa z} \begin{Bmatrix} \kappa_x^1 \\ 0 \\ \kappa_{xy}^1 \end{Bmatrix} \tag{22}$$

or more simply

$$\{\varepsilon\} = \{\varepsilon^0\} + z\{\kappa^0\} + \frac{z}{1+\kappa z}\{\kappa^1\} \tag{23}$$

Eq. (23) indicates that the in-plane strains {ε} at any z-location in the curved laminate are in terms of the mid-plane strains $\{\varepsilon^0\}$ and the deformed curvatures $\{\kappa^0\}$ and $\{\kappa^1\}$; it is a fundamental equation of curved laminate theory. The total strains are the sum of the mid-plane strains and the strains associated with deformed curvatures. It is noted that Eq. (23) is the result derived from Kirchhoff-Love assumptions on deformation and it is independent of material considerations. It means that the result of Eq. (23) is applicable to either isotropic materials or composite materials.

Stresses

The stresses at any z-location can be determined by substituting the strain equation of (22) into the plane stress constitutive equation, we have

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\overline{Q}]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{24}$$

where $[\overline{Q}]_k$ is the transformed reduced stiffness of the k^th^ lamina in the x,y coordinate system; and it can be related to the determined stiffness matrix $[Q]_k$ in the fiber direction coordinate system from standard coordinate transformation. Combining Eqs. (22) and (24) gives a general expression of stresses in the k^{th} lamina in terms of position z:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\overline{Q}]_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [\overline{Q}]_k z \begin{Bmatrix} 0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} + [\overline{Q}]_k \frac{z}{1+\kappa z} \begin{Bmatrix} \kappa_x^1 \\ 0 \\ \kappa_{xy}^1 \end{Bmatrix} \tag{25}$$

The first term in Eq. (25) corresponds to the stresses associated with the mid-plane strains, and the second and third terms correspond to the stresses associated with deformed curvatures. It is noted that $\{\varepsilon^0\}$, $\{\kappa^0\}$ and $\{\kappa^1\}$, which are associated with the mid-plane displacements and geometric curvature κ, are independent of z location.

Stress Resultants and Moment Resultants

The stress resultants {N} are defined as the through-thickness integrals of the planar stresses in the curved laminate. A similar interpretation can be given to the moment resultants {M}.

Thus, {N} and {M} in compact forms are, respectively, expressed as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz \tag{26}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdz \tag{27}$$

Substituting Eq. (25) into Eqs. (26-27) and the intervals are taken over the total laminate thickness by summing the intervals over each ply. Then, {N} and {M} become

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} dz + \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} 0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} zdz + \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} \kappa_x^1 \\ 0 \\ \kappa_{xy}^1 \end{Bmatrix} \frac{z}{1+\kappa z} dz \tag{28}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} zdz + \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} 0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} z^2 dz + \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \left[\overline{Q}\right]_k \begin{Bmatrix} \kappa_x^1 \\ 0 \\ \kappa_{xy}^1 \end{Bmatrix} \frac{z^2}{1+\kappa z} dz \tag{29}$$

Remembering $\{\varepsilon^0\}$, $\{\kappa^0\}$ and $\{\kappa^1\}$ are independent of z coordinate and the material properties are constant over each individual ply. Thus, the only variable inside the integrals is z and the integrals are easy to carry out. For example,

$$\int_{h_{k-1}}^{h_k} \frac{z}{1+\kappa z} dz = \frac{1}{\kappa}\left(h_k - h_{k-1} - \frac{1}{\kappa} \ln \frac{1+\kappa h_k}{1+\kappa h_{k-1}} \right) \tag{30}$$

$$\int_{h_{k-1}}^{h_k} \frac{z^2}{1+\kappa z} dz = -\frac{1}{\kappa^2}\left(h_k - h_{k-1} - \frac{1}{\kappa} \ln \frac{1+\kappa h_k}{1+\kappa h_{k-1}} \right) \tag{31}$$

Curved Laminate Constitutive Relations

By carrying out the integrals in Eqs. (28) and (29), the fundamental equation of curved lamination theory can be written in the following form:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B & C \\ & & \\ B & D & -C/\kappa \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa^0 \\ \kappa^1 \end{Bmatrix} \tag{32}$$

where A, B and D matrices are given in the classic lamination theory, and the C matrix is defined as

$$C_{ij} = \frac{1}{\kappa}\left(A_{ij} - \frac{1}{\kappa} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \ln \frac{1+\kappa h_k}{1+\kappa h_{k-1}} \right) \tag{33}$$

Eq. (32) can be written in expanded form as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & C_{11} & C_{12} & C_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & C_{21} & C_{22} & C_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & C_{61} & C_{62} & C_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & -C_{11}/\kappa & -C_{12}/\kappa & -C_{16}/\kappa \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & -C_{21}/\kappa & -C_{22}/\kappa & -C_{26}/\kappa \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & -C_{61}/\kappa & -C_{62}/\kappa & -C_{66}/\kappa \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ 0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \\ \kappa_x^1 \\ 0 \\ \kappa_{xy}^1 \end{Bmatrix} \tag{34}$$

The form of Eq. (34) is not suitable for the matrix operations. Recalling Eqs. (19-21), we obtain the following relationship

$$\gamma_{xy}^0 = \kappa_{xy}^0/\kappa - \kappa_{xy}^1/\kappa \tag{35}$$

Substituting Eq. (35) into Eq. (34), the fundamental equation of curved laminate theory can be expressed in the following 6x6 matrix form

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{12} & B_{16}+A_{16}/\kappa & C_{11} & C_{16}-A_{16}/\kappa \\ A_{21} & A_{22} & B_{22} & B_{26}+A_{26}/\kappa & C_{21} & C_{26}-A_{26}/\kappa \\ A_{61} & A_{62} & B_{62} & B_{66}+A_{66}/\kappa & C_{61} & C_{66}-A_{66}/\kappa \\ B_{11} & B_{12} & D_{12} & D_{16}+B_{16}/\kappa & -C_{11}/\kappa & -(C_{16}+B_{16})/\kappa \\ B_{21} & B_{22} & D_{22} & D_{26}+B_{26}/\kappa & -C_{21}/\kappa & -(C_{26}+B_{26})/\kappa \\ B_{61} & B_{62} & D_{62} & D_{66}+B_{66}/\kappa & -C_{61}/\kappa & -(C_{66}+B_{66})/\kappa \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \\ \kappa_x^1 \\ \kappa_{xy}^1 \end{Bmatrix} \tag{36}$$

This relationship relates the stress resultants and the moment resultants to the mid-plane strains $\{\varepsilon^0\}$ and the deformed curvatures $\{\kappa^0\}$ and $\{\kappa^1\}$ through the 6x6 ABCD matrix. This new curved laminate theory will provide the fundamental basis to the analyses of curved laminate structures (e.g. curved laminate beams and plates etc.). It is noted that the ABCD matrix of the curved laminate theory is an unsymmetric matrix, which is dissimilar to the ABD matrix of the classic laminate theory.

Stresses within the Layers

Due to different ply orientation and the presence of geometrical curvature, the stresses within the individual layers can be highly non-uniform and nonlinear, even for very simple loadings. However, these stresses can be determined from the equations given in the above sections. The basic scheme is that the strains in the curved laminate can be determined as part of solution process for the particular problem (e.g. 1D beam problem or 2D plate problem). For example, consider a simple statically determined problem in which the stress resultants and moment resultants are known, the mid-plane strains of ε_x^0 and ε_y^0 and deformed curvatures can be obtained by inverting Eq. (36). The mid-plane shear strain of γ_{xy}^0 can be obtained by Eq. (35). Then, the strain distributions throughout the thickness of the curved laminate can be computed from Eq. (22). Furthermore, the through-thickness stresses in the x,y coordinates can be found from Eq. (24). Finally, the stresses in the fiber directions are easily obtained from a simple coordinate transformation procedure. Generally, the determination of individual ply stresses is the basis for strength design in laminated composites.

DISCUSSIONS AND EXAMPLE

Consider a curved laminate of thickness h as depicted in Figure 1a. Symmetric laminates, each ply above the mid-plane will be paired with a same lay-up orientation below the mid-plane, are of significant interest. They are discussed in what follows.

Symmetric Laminates

If the lay-up of laminate is symmetric with respective to the mid-plane, the B matrix will vanish. The 6x6 ABCD matrix constitutive relation of the curved laminate becomes

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{16}/\kappa & C_{11} & C_{16} - A_{16}/\kappa \\ A_{21} & A_{22} & 0 & A_{26}/\kappa & C_{21} & C_{26} - A_{26}/\kappa \\ A_{61} & A_{62} & 0 & A_{66}/\kappa & C_{61} & C_{66} - A_{66}/\kappa \\ 0 & 0 & D_{12} & D_{16} & -C_{11}/\kappa & -C_{16}/\kappa \\ 0 & 0 & D_{22} & D_{26} & -C_{21}/\kappa & -C_{26}/\kappa \\ 0 & 0 & D_{62} & D_{66} & -C_{61}/\kappa & -C_{66}/\kappa \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \\ \kappa_x^1 \\ \kappa_{xy}^1 \end{Bmatrix} \quad (37)$$

The stress resultants and the moment resultants are coupled to mid-plane strains and deformation curvatures through the matrix described in Eq. (37). This response is different to the classic ABD constitutive relation of the flat laminate, in which the vanishing of B matrix gives an uncoupling of the extensional and bending responses. This coupling effect will significantly complicate the analyses for curved symmetric laminates.

Example

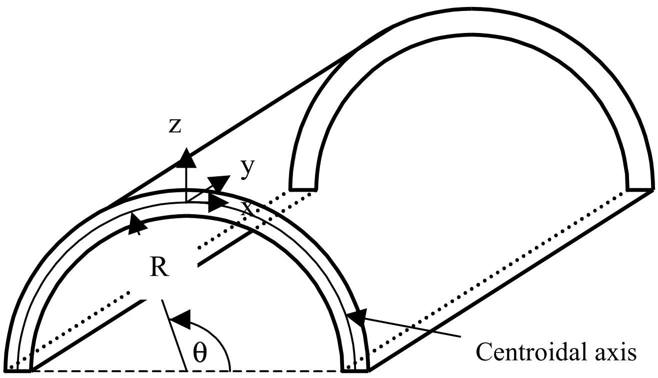
The AS4/3501-6 carbon/epoxy laminated composite is used for case study. Now consider a statically determined problem in which a curved laminate beam with the radius R=0.5 in and the width b=0.1 in is under a loading of P=5 lb applied in the x direction. The composite systems of $[0_5/90_5]_s$ and $[0_{10}/90_{10}]_s$ laminates are chosen for evaluating the effect of h/R ratio on the stress predictions. For this simple loading, only the stress resultants and the moment resultants in the x direction (i.e., N_x and M_x) exist. The bending stress, σ_x, distributions through the thickness (i.e., z axis) at the location of $\theta=\pi/2$ are, respectively, illustrated in Figures 2 and 3, where the predictions by the classic laminate theory are plotted for comparison. For the $[0_5/90_5]_s$ laminate, the difference of the predictions by the curved and classic laminate theories is small. As the h/R ratio increases to the situation of $[0_{10}/90_{10}]_s$ laminate, the difference of predictions by two theories becomes more significant. As seen in Figures (2-3), the maximum bending stress occurs at the inside radius due to the presence of geometrical curvature κ and the tensile stress resultant N_x. Depending on the loading and the dimensions of the cross section in relation to the location of the center of curvature, the ratio of stresses at the inside and outside surfaces can be large.

CONCLUSION

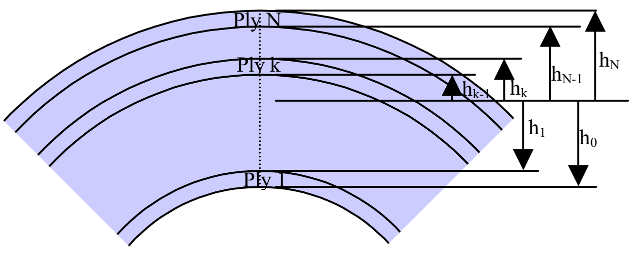
1. A general theory that describes the linear elastic response of a curved laminate subjected to in-plane loads and bending moments is developed in this paper. Similar to the classic 6x6

ABD matrix constitutive relation of a flat laminate, a new 6x6 ABCD matrix constitutive relation between stress resultants, moment resultants, mid-plane strains and deformed curvatures for a curved laminate has been formulated. This new curved laminate theory will provide the fundamental basis for the analyses of curved laminate structures. The thermal behavior of a curved laminate has also been investigated in the present analysis.

2. Unlike the classic ABD constitutive relation of the flat symmetric laminate, in which the vanishing of B matrix gives an uncoupling of the extensional and bending responses, the stress resultants and the moment resultants in the ABCD constitutive relation are still coupled to mid-plane strains and deformed curvatures for a curved symmetric laminate. The coupling effect is caused from the presence of geometrical curvature of the curved laminate.
3. The application of the curved laminate theory is demonstrated by the stress calculations of the curved laminate beam under the statically determined loading. In situations where the h/R ratio is small, the classic laminate theory continues to give acceptable accuracy. However, as the h/R ratio is getting larger, the classic laminate theory can no longer predicts the stresses accurately and the curved laminate theory is essentially required for proper stress analyses.



(a)



(b)

Figure 1 a. Geometry for a curved laminate; b. Notation for location of ply interfaces.

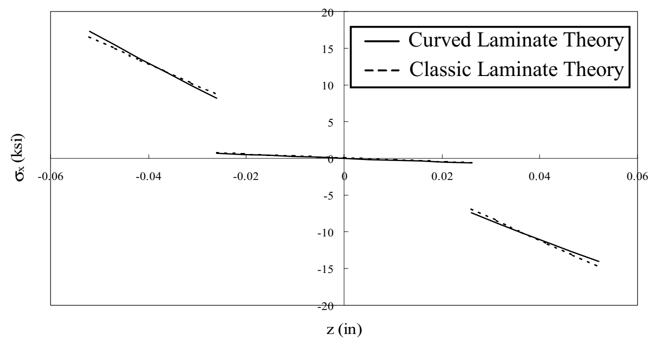


Figure 2 Bending Stress σ_x vs. beam thickness for $[0_5/90_5]_s$ curved laminate.

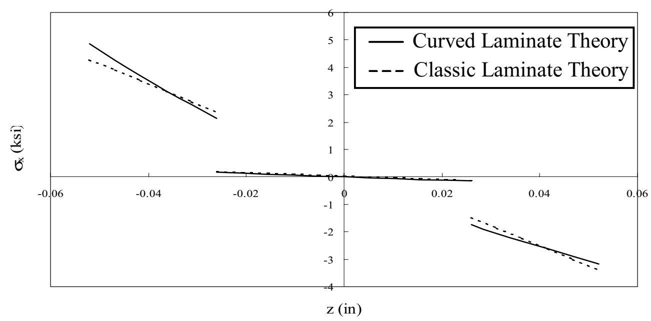


Figure 3 Bending Stress σ_x vs. beam thickness for $[0_{10}/90_{10}]_s$ curved laminate.

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