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INTERFACE STRESSES EVALUATION SOFTWARE FOR INELASTIC LAMINATES WITH FREE EDGES

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SUMMARY: We propose a software (DEILAM) able to evaluate the free edge effects in multi-layered inelastic structures, bearing essentially tensile loads. We take into account inelastic behaviors of layers and interfaces. Therefore, the program involves inelastic strain fields in the layers and fields of displacement discontinuities at the interfaces. These inelastic fields such as thermal strains or interface sliding are supposed to be known by the user. The program uses a simplified model for evaluating the 3D interfacial stresses. Calculations are validated by comparing our results with finite element calculations for a tensile loading and for a thermal loading. The authors show then how the software can help to determine a delamination onset criterion taking into account the ply thickness effect.

KEYWORDS : Inelastic Laminates, Software, Approximate Model, Variational formulation, Interfacial Stresses, Delamination

INTRODUCTION

Behavior and failure prediction is basic for structures design. This task is particularly difficult for multi-layered composite materials due to the edge effects and the complexity of failure modes and inelastic phenomena (Dvorak)[6]. Among all the failure modes, perhaps the most critical one is delamination. Consequently, the evaluation of stresses at the interfaces is important for designing a multi-layered structure. However, very few practical softwares taking into account edge effects have been developed. The softwares can be classified by the method they use to evaluate stresses. Several techniques considering each ply an homogeneous material enable us to evaluate these interface stresses; we can use for example finite elements (FE) or simplified models (Pagano)[8]. The results of the first technique with a very fine meshing are close to the 3D exact solution. Nevertheless, at the free edges, the exact interfacial stresses are often singular (Wang and Crossman)[11][12]. Hence, it isn't possible to create a failure criterion using maximum values of stresses. Moreover, the fact that stresses have infinite values has no physical sense. In this way, it is better to use simplified models (a less local study of stresses) which results are finite and more realistic than FE (Pagano)[8].
Besides, up to now, there is no software that takes into account inelasticity in multi-layers with free edges. In fact, investigators are still working for modelling inelastic phenomena such as plasticity or visco-elasticity for unidirectional (UD) laminates (Nigam etal.)[7]. The modelling of inelastic behavior in any other laminates is more complex, because it must take into account not only inelastic strains in the layer but also inelastic phenomena at the interfaces that could be modelled by fields of displacement discontinuities at the interfaces.
Meanwhile, waiting for a practical modelling (yield conditions, flow rule...) of the inelastic

behavior of laminates, we propose a software called DEILAM able to evaluate edge effects in multi-layered inelastic structures for which inelastic strain fields in the layers and fields of displacement discontinuities at the interfaces are known (by preprocessing or experimental measures for example). We assume that these structures bear essentially tensile loads. The radii of the structure are supposed to be much greater than the laminate thickness. Thus, the calculations for a long flat specimen under a tensile load help us to obtain the values of interfacial stresses at the structure free edges (Raju and Crews)[9] (see fig. 1). DEILAM uses an approximate model called ***M4-5N M***ulti-particle ***M***odel of ***M***ulti-layered ***M***aterials with ***5*** kinematic fields per layer for a ***N*** layer laminate (Chabot)[3] which equations are obtained from an adaptation of the Hellinger-Reissner variational formulation. The model is inspired of (Pagano)[8] and has already been developed (Chabot)[3] and validated for elastic problems (Carreira)[2]. The multi-layer (3-D object) becomes a superposition of *2n* Reissner plates (2D object with *2n* particles in each geometrical point) coupled by interlaminar stresses.

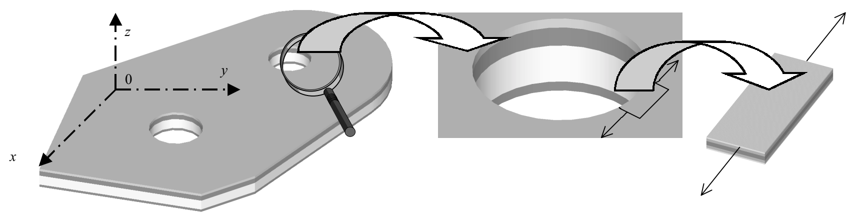


Fig. 1. Multi-layered structure with free edges.

Problem description and notations

Let us consider the $(\theta_1,..,\theta_n)_s$ multi-layer made up of *2n* layers of orthotropic materials (see fig. 2). The *z*-direction is an orthotropy direction for each layer.
For all the sections below,

- the coefficients $S_{opqr}(x,y,z)$ denote the compliances and are constant in each layer,
- subscripts *o, p, q,* and *r* indicate the components in the (*x,y,z*) space and they take the values *1*, *2* and *3*,
- subscripts α and β indicate the components in the (*x,y*) plane and they take the values *1*, and *2*,
- subscripts " *,1* " , " *,2* " and " *,3* " denote respectively partial derivatives with respect to x, y and z,
- the interfaces between layers *j* and *j+1* are represented by $\Gamma_{j,j+1}$,
- superscripts *i* and *j,j+1* indicate respectively the layer *i* and the interface $\Gamma_{j,j+1}$,

The specimen (*2b* width, *2t* thickness) is subjected to a $\pm\Delta$ displacement on the *x*-direction on its $\pm L$ ends. An inelastic strain field ε_{op}^{in} in the multi-layer and fields of displacement discontinuities $\gamma_{op}^{j,j+1}$ at the interfaces $\Gamma_{j,j+1}$ exist. These inelastic fields are supposed to be known and are x-independent.

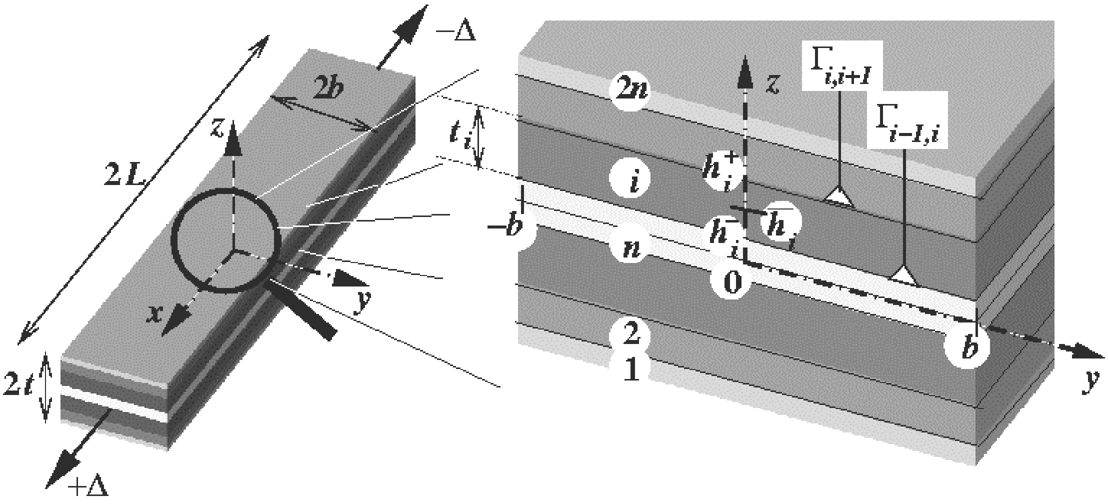


Fig. 2. Tensile loading.

MODEL DESCRIPTION AND RESOLUTION

We suppose that the specimen length L is much greater than the other specimen dimensions. Thus, we can suppose that strain and stress fields in a subvolume $\Omega = \{-l \leq x \leq l, -b \leq y \leq b, -t \leq z \leq t\}$ far from the $\pm L$ ends are x-independent (Pagano)[8], (Wang and Crossman)[11][12]. The 3D displacement field in Ω is

$$\mathbf{U}(\mathbf{x}) = \begin{pmatrix} U_1(x,y,z) = \frac{\Delta}{L} x + u_1(y,z) \\ U_2(x,y,z) = u_2(y,z) \\ U_3(x,y,z) = u_3(y,z) \end{pmatrix} \quad \textbf{(1)}$$

where $(\mathbf{x}) = (x, y, z) \in \Omega$. The equations of the mechanical problem are

- $\sigma_{op,p}(\mathbf{x}) = 0$: equilibrium where $\boldsymbol{\sigma}$ is the 3D stress field **(2)**
- $\varepsilon_{op}(\mathbf{x}) - \varepsilon_{op}^{in}(\mathbf{x}) = S_{opqr}(\mathbf{x})\sigma_{qr}(\mathbf{x})$: behavior where $\varepsilon_{op} = \frac{1}{2}\left(U_{o,p} + U_{p,o}\right)$ **(3)**
- $\sigma_{p2}(x, \pm b, z) = \sigma_{2p}(x, \pm b, z) = 0$ for $p = 1, 2, 3$: free edge boundary conditions **(4)**
- $\lim\limits_{z \xrightarrow{>} h_j^+} U_p(x,y,z) - \lim\limits_{z \xrightarrow{<} h_j^+} U_p(x,y,z) = \gamma_p^{j,j+1}(x,y)$ for $1 \leq j \leq n-1$: displacement discontinuity conditions at the interfaces $\Gamma_{j,j+1}$ **(5)**

Now, we're going to approximate the solution ($\mathbf{U}$,$\boldsymbol{\sigma}$) of the problem, using an adaptation of the Hellinger-Reissner variational formulation of 3D elastic problems. In the formulation, the elastic strain is $\varepsilon - \varepsilon^{in}$ and the displacement discontinuities $\gamma^{j,j+1}$ at the interfaces appear naturally. Let us consider a couple $(\mathbf{U}^*, \boldsymbol{\sigma}^*)$ where $\mathbf{U}^*$ is a 3D first order tensor verifying equation (1) and $\boldsymbol{\sigma}^*$ is a second order 3D symmetric tensor, both are piecewise C^1 fields on Ω . For such a couple, the Hellinger-Reissner functional is:

$$H.R.(\mathbf{U}^*,\boldsymbol{\sigma}^*) = -\int_{\Omega} \left[\sigma^*_{op,p}(\mathbf{x}) U^*_o(\mathbf{x}) + \sigma^*_{op}(\mathbf{x}) \varepsilon^{in}_{op}(\mathbf{x}) + \frac{1}{2} \sigma^*_{op}(\mathbf{x}) S_{opqr}(\mathbf{x}) \sigma^*_{qr}(\mathbf{x}) \right] d\Omega$$

$$- \sum_{j=1}^{2n-1} \left(\int_{\Gamma_{j,j+1}} \sigma^*_{p3}(\mathbf{x}) \gamma^{j,j+1}_p(\mathbf{x}) \right) dS \qquad \textbf{(6)}$$

$$- \int_{y=-b} \sigma^*_{p2}(\mathbf{x}) U^*_p(\mathbf{x})\, dS + \int_{y=b} \sigma^*_{p2}(\mathbf{x}) U^*_p(\mathbf{x})\, dS$$

Reissner's theorem (Reissner) [10]: *the solution of the problem is the couple ($\mathbf{U}, \boldsymbol{\sigma}$) which makes the H.R. functional stationary*.

We can show easily that the variational property $\delta H.R. = 0$ for any variation of $\mathbf{U}^*$ and $\boldsymbol{\sigma}^*$ yields equations (2) to (5).

Choice of the approximate 3D stresses

In each layer, we choose in-plane approximate stresses $\sigma^*_{\alpha\beta}$ ($\alpha, \beta = 1,2$) linearly dependent on z. The polynomial degrees of the approximate out-of-plane stresses $\sigma^*_{\alpha 3}$ and σ^*_{33} are deduced from the 3-D equilibrium equations and are respectively 2 and 3. The coefficients of these polynomials are y-fields and are linearly related to the following generalized resultants and interfacial stresses:

- in-plane stress resultants of the layer i $\qquad N^i_{\alpha\beta}(y) = \int_{h_i^-}^{h_i^+} \sigma^*_{\alpha\beta}(y,z) dz$
- in-plane moment resultants of the layer i $\qquad M^i_{\alpha\beta}(y) = \int_{h_i^-}^{h_i^+} (z - \bar{h}_i) \sigma^*_{\alpha\beta}(y,z) dz$
- out-of-plane shear stress resultants of layer i $\qquad Q^i_{\alpha}(y) = \int_{h_i^-}^{h_i^+} \sigma^*_{\alpha 3}(y,z) dz$
- shear stresses at the interface $\Gamma_{i,i+1}$ $\qquad \tau^{i,i+1}_{\alpha}(y) = \sigma^*_{\alpha 3}(y, h_i^+)$

  and at the interface $\Gamma_{i-1,i}$ $\qquad \tau^{i-1,i}_{\alpha}(y) = \sigma^*_{\alpha 3}(y, h_i^-)$
- normal stresses at the interface $\Gamma_{i,i+1}$ $\qquad \nu^{i,i+1}(y) = \sigma^*_{33}(y, h_i^+)$

  and at the interface $\Gamma_{i-1,i}$ $\qquad \nu^{i-1,i}(y) = \sigma^*_{33}(y, h_i^-)$

These interfacial stresses have a physical meaning and represent the exact value of the out-of-plane 3-D approximate stresses at the interface between two layers. By introducing the 3-D approximate stress fields into the Hellinger-Reissner's functional, we highlight the following $5N$ generalized displacement fields ($N = 2 \times n$, number of layers in the whole laminate):

$$U_1^i(x,y)=\frac{\Delta}{L}x+u^i(y)=\frac{\Delta}{L}x+\int_{h_i^-}^{h_i^+}\frac{1}{t^i}u_1^*(y,z)dz\,,\qquad U_2^i(x,y)=v^i(y)=\int_{h_i^-}^{h_i^+}\frac{1}{t^i}u_2^*(y,z)dz$$

$$U_3^i(x,y)=w^i(y)=\int_{h_i^-}^{h_i^+}\frac{1}{t^i}u_3^*(y,z)dz\,,$$

$$\Phi_1^i(x,y)=\varphi_1^i(y)=\int_{h_i^-}^{h_i^+}\frac{12}{t^{i2}}\frac{z-\overline{h}_i}{t_i}u_1^*(y,z)dz \text{ and } \Phi_2^i(x,y)=\varphi_2^i(y)=\int_{h_i^-}^{h_i^+}\frac{12}{t^{i2}}\frac{z-\overline{h}_i}{t_i}u_2^*(y,z)dz\,.$$

In the same way, we highlight the generalized total strains and inelastic strains. The generalized strains are linear algebraic functions of $\frac{\Delta}{L}$, u^i, v^i, w^i, φ_1^i, φ_2^i and their derivatives. The generalized inelastic strains are linear algebraic functions of $\gamma^{j,j+1}$ and of some integrals of ε^{in} through the thickness of each layer.

Finally, the variational properties of the Hellinger-Reissner functional give the generalized constitutive equilibrium equations (Reissner)[10].

Resolution of the equations

Using the problem equations and owing to the problem symmetries, we can write all the unknown fields as linear functions of the components of the following vector

$$\boldsymbol{\kappa}(y)=\left(Q_2^1,\dots,Q_2^n,u^2-u^1,\dots,u^n-u^{n-1},v^2-v^1,\dots,v^n-v^{n-1},\varphi_x^1,\dots,\varphi_x^n,\varphi_y^1,\dots,\varphi_y^n\right).$$

We distinguish in the vector components some model fields defined above. Calculations show that this *5n-2* dimensional vector is a solution of the following second order differential equation set:

$$\boldsymbol{\kappa}''(y)=\boldsymbol{F}\cdot\boldsymbol{\kappa}(y)+\boldsymbol{\psi}^{in}(y)\qquad y\in[0,b] \tag{7}$$

where $\boldsymbol{F}$ is a $5n\text{-}2\times 5n\text{-}2$ constant matrix and $\boldsymbol{\psi}^{in}$ is a *5n-2* known vector which components are linear functions of the generalized inelastic strains. We don't show in this paper the boundary conditions of the problem (symmetries with respect to the point $y=0$ and boundary conditions at the free edge $y=b$). We can find this conditions in (Chabot)[3].

The numerical resolution of equation (7) is done by a variational formulation (Díaz Díaz and Caron)[5]. The segment $[0,b]$ is divided into p segments $[a_r,a_{r+1}]$. The variational problem is reduced to the following linear equation set:

$$\boldsymbol{R}\cdot\boldsymbol{\xi}=\boldsymbol{c} \tag{8}$$

where $\boldsymbol{R}$ is a $(p\text{-}1)(5n\text{-}2)\times(p\text{-}1)(5n\text{-}2)$ constant matrix, $\boldsymbol{c}$ is a $(p\text{-}1)(5n\text{-}2)$ constant vector and $\boldsymbol{\xi}$ is a $(p\text{-}1)(5n\text{-}2)$ unknown vector. The components of $\boldsymbol{\xi}$ are the nodal values of $\boldsymbol{\kappa}$ components. The equilibrium and constitutive equations of the model give the nodal values of the remaining unknown fields of the model.

The system stiffness matrix $\boldsymbol{R}$ is a sparcely populated matrix. This matrix is stored by means of the sky-line storage scheme (Dhatt and Touzot)[4]. In this way, a considerable saving in storage is achieved.

DEILAM DESCRIPTION

The variational formulation and its resolution is made by a C++ program. The program needs some input data to carry out the calculations. Several modules help the user to input the data. First of all, the "laminate creation module" is run and the multi-layer is defined. Then, the user defines in the "loading and inelastic fields module" the values of the tensile loading. Notice the possibility to input hygrothermal loads δH and δT for calculating residual stresses. In this module, one gives also the inelastic fields $\varepsilon_{pq}^{in}(y,z)$ and $\gamma_p^{i,i+1}(y)$. Finally, the user selects or defines a meshing of the [*0,b*] segment. The calculations are then carried out by the software. The output data are the approximate stresses and displacements expressed as *z*-polynomials at each point of the meshing. This data can be plotted by the "plotting module".

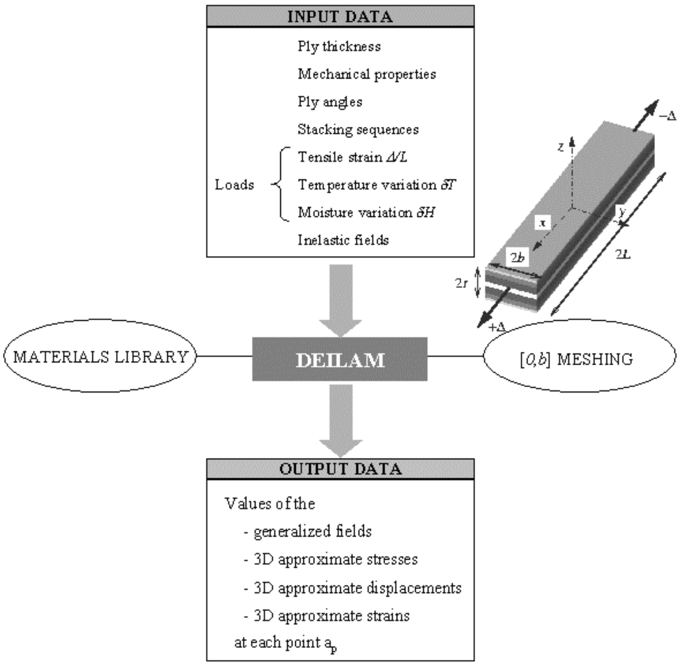


Fig. 3. Software description (input/output data).

The mechanical model ***M4-5N*** used in the program gives finite values. The [*0,b*] default meshing uses 50 points set appropriately in order to have convergent results and a good description of fields. The user can also mesh the [*0,b*] segment using a module that creates geometrical sequences.

Using a personal computer equipped with a 500MHz micro-processor, DEILAM takes less than 4 minutes to evaluate the edge effect in a symmetric 50-layer laminate for the default meshing.

CALCULATION VALIDATION

To validate our calculations, we have chosen the same ply properties as (Wang and Crossman)[11][12], (Carreira)[2], (Pagano)[8] (typical high modulus graphite-epoxy unidirectional composite):

$$E_L = 137.90\ GPa,\ E_T = E_N = 14.48\ GPa$$
$$G_{LT} = G_{LN} = G_{TN} = 5.86\ GPa$$
$$\nu_{LT} = \nu_{LN} = \nu_{TN} = 0.21$$
$$\alpha_{LL} = 0.36\times 10^{-6}\ °C^{-1},\ \alpha_{TT} = \alpha_{NN} = 28.8\times 10^{-6}\ °C^{-1}$$

where the subscripts L, T, N refer to the longitudinal, transverse and thickness directions of the individual ply.

For the laminate we have chosen a thickness-to-width ratio $\frac{2t}{2b} = \frac{1}{4}$. For all the calculations shown below, we used the 50-point default meshing in order to have quickly convergent results and a good description of the fields all along the width. In fact, all the approximate fields of the ***M4-5N*** model are finite as in (Pagano)[8] and the convergence problem of the 3D finite elements method doesn't take place in our calculations. In all this section, we show interfacial stresses against y/b curves.

Results for the free edge problem.

In this subsection, we study the influence of the **single mechanical loading** $\frac{\Delta}{L} = 1$. We compare our results with finite element (FE) calculations (Wang and Crossman)[11], (Carreira)[2], with the local model in (Pagano)[8] and with an analytical resolution of our model equations for 4-layer symmetric laminates (Chabot)[3].

$(-45,45)_s$ *laminate (figure 4).*

Far enough from the free edge, figure 4 shows that finite elements (Carreira)[2] and our software give almost the same shear stress τ_1 shape at the interface 45/-45. In fact, τ_1 is singular at the free edge for finite elements calculations while τ_1 is finite for the approximate models. The analytical (Chabot)[3] and the numerical resolutions give the same results for all the generalized fields. DEILAM and Pagano's local model calculations are almost the same.

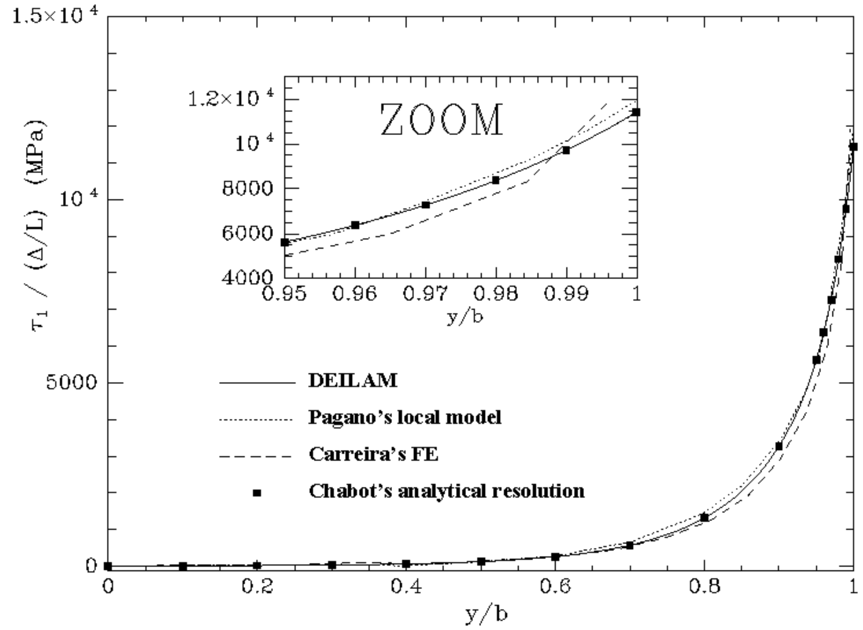


Fig. 4. Shear stress τ_1 at the 45/-45 interface of a $(\pm 45)_s$ laminate.

$(90,0,45,-45)_s$ laminate (figure 5).

We've chosen this laminate to compare our results with FE calculations in (Wang and Crossman)[11]. Figures 5 shows that our calculations give very accurate results similar to the FE for the interfacial stresses. The FE calculations aren't as accurate as in the case of 4-layer symmetric laminates. In fact, the number of elements was the same but the mesh is less refined because the number of layers increases. The storage limits of the finite elements methods were reached.

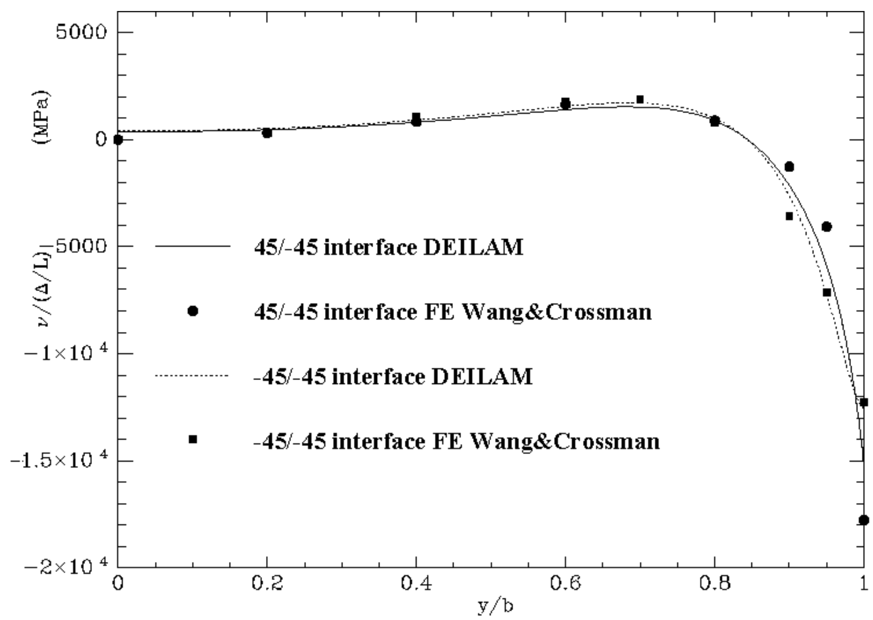


Fig. 5. Normal stress v at the 45/-45 and -45/-45 interfaces of a $(90,0,\pm 45)_s$ laminate.

Results for the edge effect on thermal loading.

For the case of a single thermal loading, in the "loading and inelastic fields" module, the user leaves a blank in the $\frac{\Delta}{L}$ space and gives the δT value. The program must determine then the value of $\frac{\Delta}{L}$ which is the thermal expansion ε_x^t in the *x*-direction. Since the specimen is in equilibrium and no mechanical load exists, the exact value of $\frac{\Delta}{L}$ is the one that yields an overall axial force

$$F_x = \sum_{i=1}^{i=2n} \int_{-b}^{b} N_{11}^i(y)dy = 0 .$$

In this way, the program is able to attribute the correct value to the overall axial strain $\frac{\Delta}{L} = \varepsilon_x^t$ and to calculate the generalized resultants and displacements. The program uses the same method for evaluating residual stresses due to the single moisture variation δH . In this section we compare our results with some FE results (Wang and Crossman)[12], (Carreira)[2].

$(90,0)_s$ *laminate (figure 6).*

For this laminate and for a $1°C$ temperature variation, the 90/0 interface is compressed and the 0/0 interface is stretched (see figure 6). There's a good agreement between all the calculation methods for the interfacial normal stresses. Besides, (Wang and Crossman)[12] gives $\varepsilon_x^t = 3.49 \times 10^{-6}$ while the DEILAM gives $\varepsilon_x^t = 3.43 \times 10^{-6}$. Thus, the two overall axial strain predictions are almost the same.

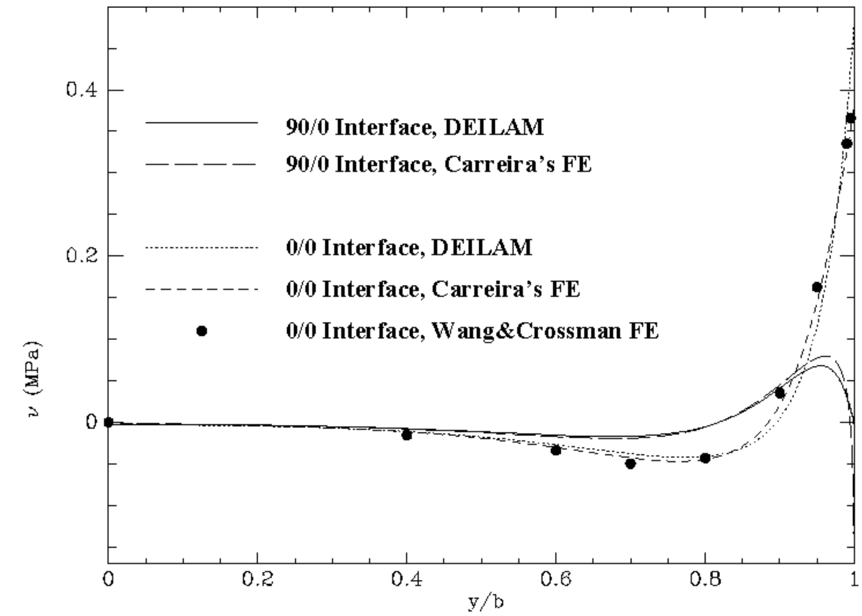


Fig. 6. Normal stress ν at the interfaces of a $(90,0)_s$ laminate for a $\delta T = 1°C$.

DELAMINATION ONSET PREDICTION

Now that we have proved that DEILAM gives realistic finite stresses for a thermo-mechanical loading, we're going to show how it can help to determine a relevant delamination criterion onset. In fact, a relevant delamination criterion must take into account the well known thickness effect (Brewer)[1], (Díaz Díaz and Caron)[5].

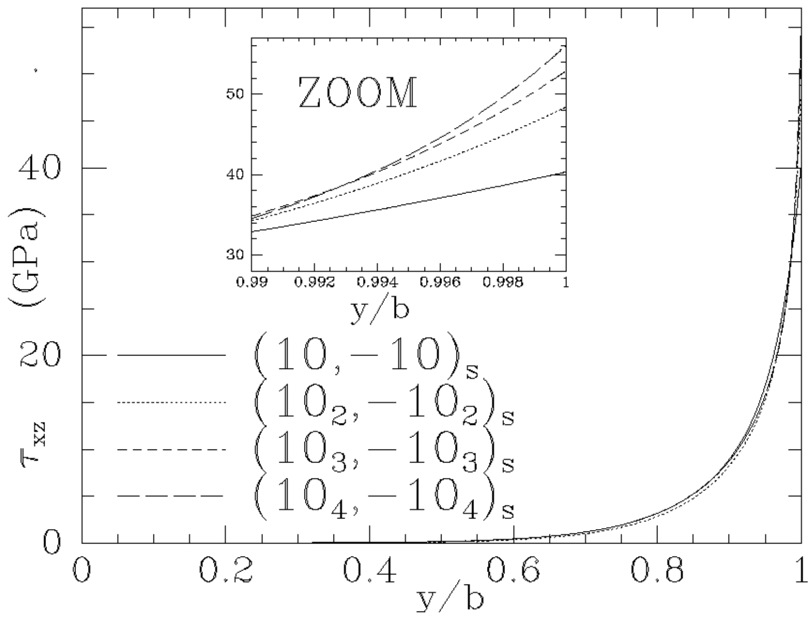
Thickness effect modelling.

Let us consider two homothetic structures $(\theta_1,\theta_2,...,\theta_n)_s$ and $((\theta_1)_m,(\theta_2)_m,...,(\theta_n)_m)_s$ with free edges and under a same tensile loading $\frac{\Delta}{L}$. We assume that the constitutive ply (its thickness is t_l) is made up of an elastic brittle material.

This laminates are homothetic because we can pass from a laminate to another by a similarity through the thickness. If we determine the exact 3D stress state in the elastic structures, some calculated stresses are singular at the free edges and at the interfaces. The analysis of fields with 3-D FE using homothetic meshes gives obviously the same result for the approximate stresses in the elements near the free edge or near the interfaces. Nevertheless, when physical stress fields are singular, approximate fields calculated by FE don't converge using a better meshing refinement. Moreover, for brittle materials, the strength value must be associated with a material volume. The strength is a function of this volume. Thus, if we make a 3-D FE calculation to study the failure in various homothetic structures using a stress criterion, we need to analyse the fields with the same mesh refinement, in order to associate the same material volume to the elements. Therefore, the calculated stresses in elements near the edge will be different for homothetic structures. Unfortunately, this scale effect is very different from the experimental one (Brewer)[1].

Besides, if we model the homothetic structures by means of our ***M4-5N*** model using $2n$ "model plies" which thickness is $t_l \times m$ (t_l is the physical ply thickness), we obtain the same strain and stress results. This is a general property of homothetic modelling of homothetic structures. Nevertheless, if we want to propose a stress criterion for brittle materials, according to what we've said above for FE meshing and material volume, the modelling must have the same thickness refinement (all the "model plies" have the same thickness t_l) for all the structures. In this way, the number of "model plies" increases with the structure size and it's equal to the number of physical plies (Díaz Díaz and Caron)[5]. If we use this modelling and if we carry out the calculations with DEILAM, we see that the edge values of stresses aren't the same for the homothetic structures. In that way, a thickness effect appears in the modelling.

An example is shown in figure 7 for the case of $\left(10_n,-10_n\right)_s$ carbon-epoxy specimens under a tensile loading. In the calculations, we have chosen a thickness-to-width ratio $\frac{2t}{2b}=\frac{1}{4}$. The material properties are the same as in (Díaz Díaz and Caron)[5].



7. Shear stresses at the *10/-10* interfaces in $\left(10_n,-10_n\right)_s$ laminates for $\frac{\Delta}{L}=1$

At this step, we just have to verify that this theoretical thickness effect can predict accurately the thickness effect on delamination onset.

An example of delamination criterion.

DEILAM has already been used for determining a delamination onset criterion for carbon-epoxy laminates showing essentially mode III delaminations (Díaz Díaz and Caron)[5].

The criterion involves the maximum values (the edge values) of τ_1 :

$$\left|\tau_1^{j,j+1}\right| \leq \tau^c = 248MPa\,.$$

This criterion is based on experimental data of $\left(10_n,-10_n\right)_s$ and $\left(20_n,-20_n\right)_s$ ($n=1$, 2, 3 and 4) specimens under a tensile loading. Using this criterion and DEILAM we can determine the theoretical overall critical tensile load $\bar{\sigma}_{xx}$ that will lead to delamination. If we calculate the theoretical / experimental ratios we can say that the errors made by our theoretical predictions lie between 0.4 and 6.0% (see Table 1).

Delamination initiation stress *MPa*	$(10_n,-10_n)_s$ laminate		$(20_n,-20_n)_s$ laminate	
	Calculated values	Experimental values	Calculated values	Experimental values
n = 1	*908*	*941*	*626*	*601*
n = 2	*762*	*765*	*528*	*503*
n = 3	*701*	*731*	*486*	*459*
n = 4	*664*	*679*	*461*	*479*

Table 1. Critical stress $\bar{\sigma}_{xx}$: calculated and measured values (Díaz Díaz and Caron)[5].

CONCLUSION

We have presented in this paper a software able to calculate interfacial stresses in inelastic symmetric laminates bearing essentially tensile loads. Inelastic strains in the layers and displacement discontinuities at the interfaces are taken into account and are supposed to be known by the user. The software DEILAM uses the approximate model ***M4-5N*** for evaluating stresses. The model equations are resolved by means of a variational formulation. Calculations have been validated by a comparison with FE results for the elastic and thermo-elastic free edge problems. The software gives finite realistic results and enables us to carry out calculations for inelastic laminates having a great number of layers. An example of its utility is the evaluation of residual hygro-thermal stresses in any laminate. Inelastic symmetric fields which values depend on the position through the width and through the thickness can be input directly as well in order to calculate the approximate stresses in the laminate.

In this paper we have also shown that the software can help to determine delamination onset criteria involving the maximum values of the interfacial stresses. The criteria take into account the ply thickness effect.

For other inelastic problems such as interface sliding or plasticity, the knowledge of the inelastic fields isn't evident. Investigators are still trying to model in a practical way these complex inelastic phenomena in unidirectional laminates. When the inelastic equations (flow rule and yield conditions) will be found, we'll be able to write the generalized inelastic equations. Then, iterating appropriately the computations of DEILAM and the generalized inelastic equations, we will determine the generalized inelastic fields and the generalized stresses. To conclude, the software might be modified later in order to calculate by itself more complex inelastic fields than the constant hygro-thermal strains. However, the software could be already an useful tool for multi-layered structures design.

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