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ANALYSIS OF SPATIALLY REINFORCED COMPOSITE SHELLS OF REVOLUTION

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SUMMARY: The objective of this paper is to demonstrate an approach to the stress-strain analysis of thin-walled composite shells of revolution manufactured by the 3D helical lay-up method. The analysis is based on the applied theory of spatially reinforced composite shells. The hypothesis of a free thermal expansion in the direction orthogonal to the reference surface of the shell was implemented in order to obtain a basic set of governing equations. Geometrical modeling of the shells of revolution under consideration reflects major features of the spatial orientation of reinforcement and manufacturing technology. Structural parameters of the wall material are determined in terms of the three angles of reinforcement orientation and coefficients representing geometrical, physical and mechanical properties of the elementary layers. An example of a symmetrically loaded shell is considered and the results of the stress-strain analysis are presented.

KEYWORDS: Composite Shell, Shell Theory, Helical Lay-up Method, Spatial Reinforcement, Involute Structure.

INTRODUCTION

Conventional ways for manufacturing composite shells of revolution require the use of one of the standard filament winding techniques. Usually, such shells consist of a number of orthotropic elementary layers produced by helical winding and, correspondingly, they have their typical shortcomings – the transverse (normal to the layer plane) stiffness and strength are substantially lower than the corresponding in-plane characteristics. To improve the material properties under tension or compression and in shear in the transverse direction, composites should be additionally reinforced with fibers directed along the axis orthogonal to the layer or making an angle ($<90^\circ$) with this axis. The latter way of spatial reinforcement has been realized for the shells of revolution produced by the 3D helical lay-up technology as shown in Fig. 1. In this case, the spatially reinforced structure is formed by a unidirectional composite material whose principal axes x_1', x_2', x_3' make three angles $\phi(y)$, $\psi(y)$, $\eta(y)$ with the global shell

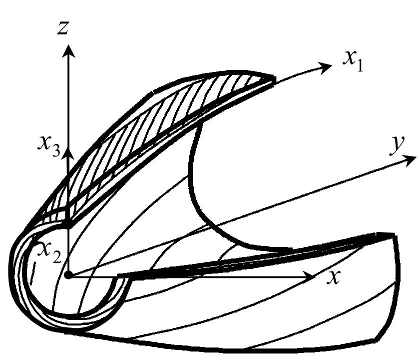

Fig. 1 Shell of revolution

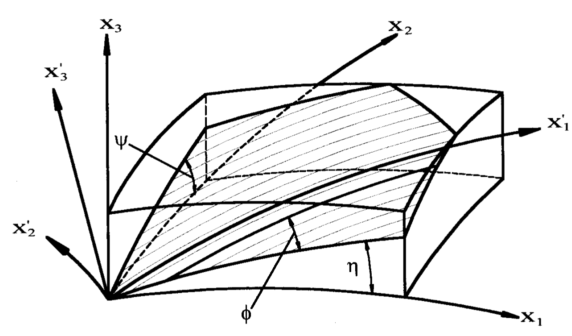

Fig. 2 Orientation of the elementary layer

axes x_1, x_2, x_3 as shown in Fig. 2. The specific way of spatial fiber orientation makes the material and structure rather special in terms of mechanical characterization [1, 2]. The approach presented in this paper allows for the major structural features of the shells under consideration.

Due to complex character of the composite structure the stress-strain analysis needs to be performed on the basis of more complicated mechanical models. In this work, the governing equations have been derived on the basis of specific averaging procedure described in [3, 4]. The basic system of equations modeling the thin-walled spatially reinforced composite structures includes the sets of geometric equations, constitutive equations, and equations of equilibrium. Using a geometrical modeling the stiffness coefficients of the shell were determined in terms of the angles of reinforcement orientation, structural parameters and mechanical properties of elementary composite layer.

MATERIAL STRUCTURE AND MODEL ANALYSIS

Consider an element of the shell referred to the system of curvilinear coordinates x_1, x_2, x_3 in such a way that axes x_1 and x_2 coincide with the meridian coordinate and circumferential coordinate respectively (see Fig.1). Axis x_3 coincides with the outward normal vector to a certain initial surface $x_3 = 0$ - the so-called reference surface. The elementary reinforcing layer is referred to the system of principal material axes in the direction of fibers x_1', normal to the fibers in the plane of the layer x_2', and normal to the plane of the layer x_3' as shown in Fig. 2. As it can be seen there are three angles of the reinforcement orientation $\phi(y)$, $\psi(y)$, $\eta(y)$ in this case, while the filament wound shells are characterized with the only one helical angle $\pm\phi(y)$, ($\eta = \psi = 0$). The angle $\psi(y)$ can be found from the geometry considerations [4]

$$\psi = \sin^{-1}\left[\delta\left(\delta^2 + \frac{4\pi^2 r^2}{N^2}\right)^{-1/2}\right]$$

provided that the number of fabric flags N and the thickness of elementary layer δ are known. The other two angles $\phi(y)$ and $\eta(y)$ are determined by the specific shape and geometry of the elementary fabric pattern and the design of the shell itself as a structural component.

Analysis of the spatially reinforced shells of revolution is based upon an applied theory of shells discussed in [3]. Implementation of that kind of theory allows for the major structural features of the shell and general character of the material anisotropy. It is assumed that the considered shells of revolution operate under axisymmetric mechanical and thermal loading, and the fabric flags are made of one and the same material. Hence, the middle surface of the shell can be taken as the reference surface. Under these circumstances, the constitutive equations given in [3] have the following form

$$\begin{aligned}
\{N\} &= h\left[\overline{B}_{mn}\right]\begin{Bmatrix}\varepsilon\\ \psi\end{Bmatrix} - \{T\}_0, && m = 1,\ldots,3;\ n = 1,\ldots,5 \\
\{M\} &= \frac{h^3}{12}\left[\overline{B}_{mn}\right]\{\kappa\} - \{T\}_1, && m,n = 1,\ldots,3 \\
\{Q\} &= \left[I_{pq}\right]\begin{Bmatrix}\varepsilon\\ \psi\end{Bmatrix} - \{I\}_t, && p = 1,\ldots,2;\ q = 1,\ldots,5
\end{aligned} \tag{1}$$

where $\{N\}=\{N_1,N_2,N_{12}\}^T$, $\{M\}=\{M_1,M_2,M_{12}\}^T$, and $\{Q\}=\{Q_1,Q_2\}^T$ are the vectors of stress resultants, bending moments, and transverse shear stress resultants respectively; $\begin{Bmatrix}\varepsilon\\ \psi\end{Bmatrix}=\{\varepsilon_1,\varepsilon_2,\varepsilon_{12},\psi_1,\psi_2\}^T$ is the vector of strains $\varepsilon_1,\varepsilon_2,\varepsilon_{12}$ and transverse shear strains ψ_1,ψ_2; $\{\kappa\}=\{\kappa_1,\kappa_2,\kappa_{12}\}^T$ is the vector of variables induced by bending and torsion of the shell element $\{T\}_0=\{T_1^{(0)},T_2^{(0)},T_{12}^{(0)}\}^T$, $\{T\}_1=\{T_1^{(1)},T_2^{(1)},T_{12}^{(1)}\}^T$, $\{I\}_t=\{I_{t1},I_{t2}\}^T$ are the vectors of thermal coefficients; *h* is the total thickness of the shell.

For the case under consideration, the constitutive equations reflecting general character of anisotropy of the material structure and three-dimensional orientation of the reinforcement are given by

$$\sigma_{ij}=B_{ijkl}e_{kl}-S_{ij}t \tag{2}$$

Stiffness, B_{ijlk}, and thermal, S_{ij}, coefficients are calculated in terms of the mechanical and physical properties of the elementary layers as follows

$$B_{ijlk}=B'_{i'j'k'l'}l_{ij'}l_{ji'}l_{kl'}l_{lk'}\,; \qquad S_{ij}=B'_{i'j'k'l'}l_{ij'}l_{ji'}\alpha'_{k'l'}$$

where, according to Fig.2

$$\begin{gathered}
l_{11}=\cos\chi\cos\zeta\,;\quad l_{12}=-\sin\chi\cos\zeta\,;\quad l_{13}=-\sin\zeta\,;\\
l_{21}=\sin\chi\cos\psi-\sin\psi\sin\zeta\cos\chi\,;\\
l_{22}=\cos\chi\cos\psi+\sin\psi\sin\chi\sin\zeta\,;\quad l_{23}=-\sin\psi\cos\zeta\,;\\
l_{31}=\sin\psi\sin\chi+\cos\psi\cos\chi\sin\zeta\,;\\
l_{32}=\sin\psi\cos\chi-\cos\psi\sin\chi\sin\zeta\,;\quad l_{33}=\cos\psi\cos\zeta\,;\\
\zeta=\tan^{-1}(\tan\eta\cos\psi)\,;\quad \chi=\phi+\tan^{-1}(\sin\zeta\tan\psi)\,;
\end{gathered} \tag{3}$$

The tensor components of the elastic constants $B'_{i'j'k'l'}$ and the coefficients of linear thermal expansion $\alpha'_{k'l'}$ for the elementary orthotropic layer referred to the system of coordinates x'_1,x'_2,x'_3 are expressed in terms of engineering constants as presented below

$$B'_{1111}=\bar{E}'_1\,;\; B'_{1122}=B'_{2211}=\bar{E}'_1\bar{\mu}'_{12}\,;\; B'_{1133}=B'_{3311}=\bar{E}'_1\bar{\mu}'_{13}\,;\; B'_{2222}=\bar{E}'_2\,;\; B'_{2233}=B'_{3322}=\bar{E}'_2\bar{\mu}'_{23}\,;$$

$$B'_{3333}=\bar{E}'_3\,;\quad B'_{1212}=G'_{12}\,;\quad B'_{1313}=G'_{13}\,;\quad B'_{2323}=G'_{23}\,; \tag{4}$$

$$\alpha'_{11}=\alpha'_1\,;\quad \alpha'_{22}=\alpha'_2\,;\; \alpha'_{33}=\alpha'_3$$

where

$$\bar{E}'_{1,2,3}=\Omega'_{1,2,3}\,/\,(1-\mu'_{23}\mu'_{12}\mu'_{31}-\mu'_{32}\mu'_{21}\mu'_{13}-\mu'_{13}\mu'_{31}-\mu'_{21}\mu'_{12}-\mu'_{32}\mu'_{23})\,;$$

$$\Omega'_1=E'_1(1-\mu'_{32}\mu'_{23})\,;\quad \Omega'_2=E'_2(1-\mu'_{31}\mu'_{13})\,;\quad \Omega'_3=E'_3(1-\mu'_{12}\mu'_{21})\,;$$

$$\bar{\mu}'_{12}=\frac{\mu'_{12}+\mu'_{32}\mu'_{13}}{1-\mu'_{32}\mu'_{23}}\,;\quad \bar{\mu}'_{21}=\frac{\mu'_{21}+\mu'_{31}\mu'_{23}}{1-\mu'_{31}\mu'_{13}}\,;\quad \bar{\mu}'_{13}=\frac{\mu'_{13}+\mu'_{23}\mu'_{12}}{1-\mu'_{32}\mu'_{23}}\,;$$

$$\overline{\mu}'_{23} = \frac{\mu'_{23} + \mu'_{13}\mu'_{21}}{1-\mu'_{31}\mu'_{13}}\,; \quad \overline{\mu}'_{31} = \frac{\mu'_{31} + \mu'_{32}\mu'_{21}}{1-\mu'_{12}\mu'_{21}}\,; \quad \overline{\mu}'_{32} = \frac{\mu'_{32} + \mu'_{31}\mu'_{12}}{1-\mu'_{21}\mu'_{12}}\,;$$

Here E'_1, E'_2, are the elastic moduli along and across the reinforcement orientation (warp orientation), E'_3 is the elastic modulus in the direction orthogonal to the elementary layer, μ'_{12}, μ'_{21}, μ'_{13}, μ'_{31}, μ'_{23}, μ'_{32} are the Poisson's ratios, G'_{12}, G'_{13}, G'_{23} are the shear moduli, α'_1, α'_2, α'_3 are the coefficients of linear thermal expansion. These characteristics can be experimentally determined for the specific composite material.
The generalized stiffness coefficients, $\overline{B}_{mn}$ entering Eqs.(1) are linked with the stiffness characteristics B_{ijlk} in the following way

$$\overline{B}_{m1} = B_{mm11}\,; \quad \overline{B}_{m2} = B_{mm22}\,; \quad \overline{B}_{m3} = B_{mm12}\,; \quad \overline{B}_{m4} = B_{mm13}\,; \quad \overline{B}_{m5} = B_{mm23} \qquad (m = 1,\ldots,3)$$

The components of the vectors of thermal coefficients are determined as:

$$T_1^{(k)} = \int_{-h/2}^{h/2} (S_{11}t - B_{1111}e_1^t - B_{1122}e_2^t - B_{1112}e_{12}^t)x_3^k\,dx_3$$

$$T_2^{(k)} = \int_{-h/2}^{h/2} (S_{22}t - B_{2222}e_2^t - B_{2211}e_1^t - B_{2221}e_{21}^t)x_3^k\,dx_3$$

$$T_{12}^{(k)} = \int_{-h/2}^{h/2} [S_{12}t - 2(B_{1211}e_1^t + B_{1222}e_2^t + 2B_{1212}e_{12}^t)x_3^k\,dx_3 \qquad (k = 0,\,1)$$

The coefficients I_{pq}, I_{tq} ($p = 1,\ldots,2$; $q = 1,\ldots,5$) are presented by the following relationships

$$I_{1m} = K_1(\nu_{13,23}U_m - V_m)\,, \qquad m = 1,\ldots,3$$

$$I_{14} = K_1(1+\nu_{13,23}U_4 - V_4)\,, \qquad I_{15} = K_1[\nu_{13,23}(U_5 - 1) - V_5]$$

$$I_{1n} = K_2(\nu_{23,13}V_n - U_n)\,, \qquad n = 1,\ldots,3$$

$$I_{24} = K_2[\nu_{23,13}(V_4 - 1) - U_4]\,, \qquad I_{25} = K_2(1 - U_5 + \nu_{23,13}V_5) \tag{5}$$

$$I_{t1} = \frac{K_1}{h}\int_{-h/2}^{h/2}\left\{\nu_{13,23}[(U_t + \alpha_{23})t - U_1e_1^t - U_2e_2^t] - [(V_t + \alpha_{13})t - V_1e_1^t - V_2e_2^t]\right\}dx_3$$

$$I_{t2} = \frac{K_2}{h}\int_{-h/2}^{h/2}\left\{\nu_{23,13}[(V_t + \alpha_{23})t - V_1e_1^t - V_2e_2^t] - [(U_t + \alpha_{23})t - U_1e_1^t - U_2e_2^t]\right\}dx_3$$

where t is the temperature change, and

$$U_m = \frac{\eta_{23,1}}{E_1}\overline{B}_{1m} + \frac{\eta_{23,2}}{E_2}\overline{B}_{2m} + \frac{\nu_{23,12}}{G_{12}}\overline{B}_{3m}\,, \qquad m = 1,\ldots,5,t$$

$$V_n = \frac{\eta_{13,1}}{E_1}\overline{B}_{1n} + \frac{\eta_{13,2}}{E_2}\overline{B}_{2n} + \frac{\nu_{13,12}}{G_{12}}\overline{B}_{3n}\ , \qquad n = 1,\ldots,5,t$$

$$K_1 = hG_{13}\,/\,(1-\nu_{23,31}\nu_{13,23})\,, \qquad K_2 = hG_{23}\,/\,(1-\nu_{13,23}\nu_{23,13})$$

$$(\ \overline{B}_{1t} = S_{11}\,,\ \overline{B}_{1t} = S_{22}\,,\ \overline{B}_{3t} = S_{12}\ )$$

Elastic constants E_1, E_2, G_{12}, $\nu_{ij,kl}$, $\eta_{ij,k}$, and coefficients of thermal deformation α_3, α_{13}, α_{23} are calculated through the same set of constants of the elementary reinforcing layers and the angles of layer orientation as the coefficients B_{ijlk} [3, 5].

Thermal strain components for the shell of revolution with the axis of rotation *y* are:

$$e_1^t = \frac{u_3^t}{R_1} - \frac{1}{A_1}\frac{du_1^t}{dy}\,, \quad e_2^t = \frac{u_3^t}{R_2} - \frac{u_1^t}{A_1 A_2}\frac{dr}{dy}\,, \quad e_{12}^t = 0$$

where $A_1 = \sqrt{1+\left(\frac{dr}{dy}\right)^2}$, $A_2 = r(y)$.

Here *r(y)* is the function specifying the shape of the shell meridian, and R_1, R_2 are the two principal curvature radii

$$R_1 = -\frac{1}{(d^2 r / d^2 y)}\left[1+\left(\frac{dr}{dy}\right)^2\right]^{3/2}, \qquad R_2 = r\sqrt{1+\left(\frac{dr}{dy}\right)^2}$$

Thermal components of the displacements have the form [4]

$$u_1^t = \frac{1}{A_1}\int_0^{x_3}\frac{du_3^t}{dy}dx_3\,, \qquad u_2^t = 0\,, \qquad u_3^t = \int_0^{x_3}\alpha_3 t dx_3 \qquad (6)$$

The components e_{ij} of the strain tensor are expressed in terms of the thin-walled shell theory strain characteristics as follows [3]

$$e_{11} = \varepsilon_1 + x_3\kappa_1 + e_1^t\,; \qquad e_{22} = \varepsilon_2 + x_3\kappa_2 + e_2^t\,; \qquad e_{33} = \alpha_3 t$$

$$e_{12} = (\overline{\varepsilon}_{12} + \overline{\varepsilon}_{21}) + x_3(\overline{\kappa}_{12} + \overline{\kappa}_{21}) + e_{12}^t\,; \quad e_{13} = \psi_1\,; \quad e_{23} = \psi_2 \qquad (7)$$

where, for the type of the composite shell considered in this work

$$\varepsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\partial x_1} + \frac{w}{R_1}\,; \qquad \varepsilon_2 = \frac{1}{A_2}\left(\frac{\partial u_2}{\partial x_2} + \frac{u_1}{A_1}\frac{\partial A_2}{\partial x_1}\right) + \frac{w}{R_2}\,;$$

$$\overline{\varepsilon}_{12} = \frac{1}{A_1}\frac{\partial u_2}{\partial x_1}\,; \qquad \overline{\varepsilon}_{21} = \frac{1}{A_2}\left(\frac{\partial u_1}{\partial x_2} - \frac{u_2}{A_1}\frac{\partial A_2}{\partial x_1}\right);$$

$$\kappa_1 = \frac{1}{A_1}\frac{\partial \theta_1}{\partial x_1}\ ; \qquad \kappa_2 = \frac{1}{A_2}\left(\frac{\partial \theta_2}{\partial x_2} + \frac{\theta_1}{A_1}\frac{\partial A_2}{\partial x_1}\right); \tag{8}$$

$$\tilde{\kappa}_{12} = \frac{1}{A_1}\frac{\partial \theta_2}{\partial x_1}; \qquad \tilde{\kappa}_{21} = \frac{1}{A_2}\left(\frac{\partial \theta_1}{\partial x_2} - \frac{\theta_2}{A_1}\frac{\partial A_2}{\partial x_1}\right);$$

$$\theta_1 = \psi_1 + \frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial w}{\partial x_1}; \qquad \theta_2 = \psi_2 + \frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial w}{\partial x_2}\ ;$$

$$(\varepsilon_{12} = \tilde{\varepsilon}_{12} + \tilde{\varepsilon}_{21}; \quad \kappa_{12} = \tilde{\kappa}_{12} + \tilde{\kappa}_{21})$$

Here u_1 , u_2 , w are the displacements of the reference surface along the axes x_1 , x_2 , x_3 respectively, θ_1, θ_2 are the rotation angles of the element initially orthogonal to the reference surface.

To complete the model analysis the equations of equilibrium should be obtained. For the shell of revolution under consideration the following relationships can be obtained [4]

$$\frac{d(rN_1)}{dy} - \frac{dr}{dy}N_2 + \frac{A_1 A_2}{R_1}Q_1 = 0\,, \qquad \frac{d(rM_1)}{dy} - \frac{dr}{dy}M_2 + A_1 A_2 Q_1 = 0$$

$$\frac{d(rQ_1)}{dy} - A_1 A_2\left(\frac{N_1}{R_1} + \frac{N_2}{R_2}\right) + A_1 A_2 p(y) = 0 \tag{9}$$

Function *p(y)* characterizes the pressure distribution. For the case under consideration, the theoretical model of the spatially reinforced composite shell of revolution is determined by the constitutive equations (1), strain-displacement relationships (8), and equilibrium equations (9). These sets of equations make up a closed system with respect to the displacements u_1, u_2, w; rotation angles θ_1, θ_2; generalized strains ε_1, ε_2, ε_{12}, κ_1, κ_2, κ_{12}; stress resultants N_1, N_2, N_{12}; bending moments M_1, M_2, M_{12}, and transverse shear stress resultants Q_1, Q_2.

EXAMPLE

Consider a spatially reinforced conical shell made from carbon-carbon composite material. Physical and mechanical properties of the composite are: $E_1' = 1.9 \cdot 10^4$ MPa, $E_2' = 8000$ MPa, $E_3' = 5000$ MPa, $G_{12}' = 4000$ MPa, $G_{13}' = 5000$ MPa, $G_{23}' = 5000$ MPa, $\mu_{21}' = 0.15$, $\mu_{13}' = \mu_{23} = 0.2$, $\alpha_1' = 2 \cdot 10^{-6}$ 1/K, $\alpha_2' = 3.6 \cdot 10^{-6}$ 1/K, $\alpha_3' = 5 \cdot 10^{-6}$ 1/K. The reinforcing elementary layers are made from the carbon fabric. The analysis and calculations are performed for the case of thermal loading. The shell is supposed to be uniformly heated up to the temperature of t =1000 K. There is no any mechanical load applied and the shell is fixed at one end. The geometry and the angles of the reinforcement orientation are shown in Fig. 3.

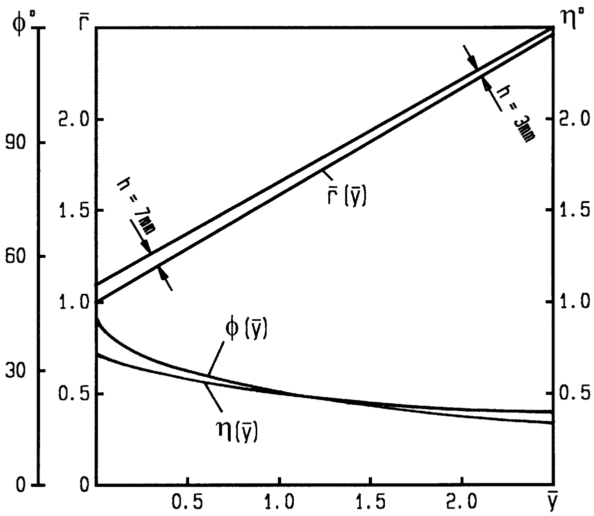

Fig. 3 Shape of the shell and the angles of the reinforcement orientation
$(\bar{y} = y / y_0,\ \bar{r} = r / r_0)$

The number of the fabric flags (3D-oriented layers) N is equal to 180, and the thickness of the elementary layer $\delta = 0.25 \cdot 10^{-3}$ m. The orientation angle ψ has been calculated according to the corresponding equation given in the previous Section.

Taking into account the uniform distribution of the temperature over the thickness the simplified approach based on the membrane shell theory can be employed for the analysis. This implies that $M_1 = M_2 = M_{12} = Q_1 = Q_2 = 0$. According to the considered loading conditions the membrane stress resultants N_1, N_2, N_3 are equal to zero. The coefficients $T_1^{(1)}$, $T_2^{(1)}$, $T_{12}^{(1)}$ are equal to zero too, due to the uniform character of the temperature distribution. As a result the second group of the equations (1) yields $\kappa_1 = \kappa_2 = \kappa_{12} = 0$. The rest of the relations (1) effectively represent the linear system of five equations from which the strain distributions ε_1, ε_2, ε_{12}, ψ_1, ψ_2 can be found. The strain in the transverse direction is calculated as $\varepsilon_3 = \alpha_3 t$. For the considered in this work shell these strain distributions are presented in Fig. 4.

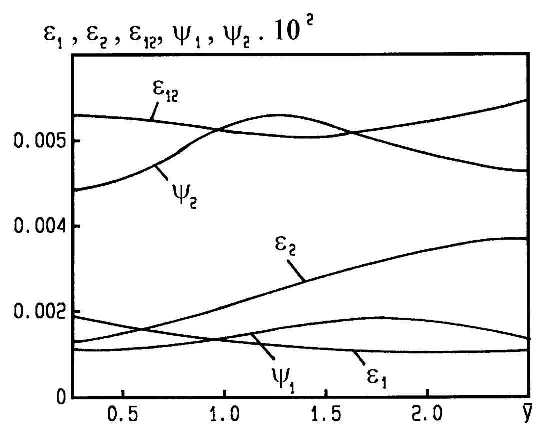

Fig. 4 The strains of the shell

The strains of the elementary composite layer ε_1', ε_2', ε_{12}' can be calculated in terms of the shell strains ε_1, ε_2, ε_{12}, ψ_1, ψ_2 making use of the transformation relations linking the coordinates ($x_1, x_{2,} x_3$) with the coordinates (x_1', x_2', x_3')

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = [T] \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix}, \qquad [T] = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \tag{10}$$

The resulting relationships are:

$$\varepsilon_1' = \varepsilon_1 l_1^2 + \varepsilon_2 m_1^2 + \varepsilon_3 n_1^2 + \varepsilon_{12} m_1 l_1 + \psi_1 n_1 l_1 + \psi_2 m_1 n_1$$

$$\varepsilon_2' = \varepsilon_1 l_2^2 + \varepsilon_2 m_2^2 + \varepsilon_3 n_2^2 + \varepsilon_{12} m_2 l_2 + \psi_1 n_2 l_2 + \psi_2 m_2 n_2 \tag{11}$$

$$\varepsilon_{21}' = \varepsilon_1 2 l_1 l_2 + \varepsilon_2 2 m_1 m_2 + \varepsilon_3 2 n_1 n_2 + \varepsilon_{12}(l_1 m_2 + m_1 l_2) + \psi_1 (n_1 l_2 + n_2 l_1) + \psi_2 (m_1 n_2 + m_2 n_1)$$

Stresses σ_1', σ_2', τ_{12}' are calculated using the equations of the generalized Hooke's law considered for the elementary reinforcing layer. The transverse stress σ_3 and the transverse shear stresses τ_{13}, τ_{23} are to be determined integrating the three-dimensional equilibrium equations of the theory of elasticity written for the orthogonal curvilinear system of coordinates [6], and taking into account the static conditions at the boundary surfaces. For the shell under consideration such relationships can be obtained in the following form [4]

$$\tau_{13} = -\frac{1}{A_1 A_2} \int_{-h/2}^{x_3} \left(\frac{d(r\sigma_1)}{dy} - \sigma_2 \frac{dr}{dy} \right) dx_3, \quad \tau_{23} = -\frac{1}{A_1 A_2^2} \int_{-h/2}^{x_3} \frac{d(A_2^2 \tau_{12})}{dy} dx_3$$

$$\sigma_3 = -\frac{1}{A_1 A_2} \int_{-h/2}^{x_3} \left[\frac{d(A_2 \tau_{13})}{dy} + \frac{r(d^2 r / dy^2)}{1 + (dr/dy)^2} \sigma_1 - \sigma_2 \right] dx_3 - A_1 A_2 p \tag{12}$$

Here the stresses σ_1, σ_2, τ_{12} are calculated in terms of the shell strains as follows

$$\sigma_1 = \overline{B}_{11}(\varepsilon_1 + x_3 \kappa_1) + \overline{B}_{12}(\varepsilon_2 + x_3 \kappa_2) + \overline{B}_{13}(\varepsilon_{12} + x_3 \kappa_{12}) + \overline{B}_{14}\psi_1 + \overline{B}_{15}\psi_2 -$$
$$-(\overline{B}_{1t} t - \overline{B}_{11} e_{1t} - \overline{B}_{12} e_{2t} - \overline{B}_{13} e_{12t})$$

$$\sigma_2 = \overline{B}_{21}(\varepsilon_1 + x_3 \kappa_1) + \overline{B}_{22}(\varepsilon_2 + x_3 \kappa_2) + \overline{B}_{23}(\varepsilon_{12} + x_3 \kappa_{12}) + \overline{B}_{24}\psi_1 + \overline{B}_{25}\psi_2 -$$
$$-(\overline{B}_{2t} t - \overline{B}_{21} e_{1t} - \overline{B}_{22} e_{2t} - \overline{B}_{23} e_{12t}) \tag{13}$$

$$\tau_{12} = \overline{B}_{31}(\varepsilon_1 + x_3 \kappa_1) + \overline{B}_{32}(\varepsilon_2 + x_3 \kappa_2) + \overline{B}_{33}(\varepsilon_{12} + x_3 \kappa_{12}) + \overline{B}_{34}\psi_1 + \overline{B}_{35}\psi_2 -$$
$$-(\overline{B}_{3t} t - \overline{B}_{31} e_{1t} - \overline{B}_{32} e_{2t} - \overline{B}_{33} e_{12t})$$

Finally, the stresses σ'_3, τ'_{13}, τ'_{23} acting in the elementary reinforcing layers can be calculated using the transformation relationships

$$\sigma'_3 = \sigma_1 l_3^2 + \sigma_2 m_3^2 + \sigma_3 n_3^2 + 2m_3 l_3 \tau_{12} + 2n_3 l_3 \tau_{13} + 2m_3 n_3 \tau_{23}$$

$$\tau'_{13} = \sigma_1 l_1 l_3 + \sigma_2 m_1 m_3 + \sigma_3 n_1 n_3 + (l_3 m_1 + l_1 m_3)\tau_{12} + +(n_1 l_3 + n_3 l_1)\tau_{13} + (m_1 n_3 + m_3 n_1)\tau_{23} \quad (14)$$

$$\tau'_{23} = \sigma_1 l_2 l_3 + \sigma_2 m_2 m_3 + \sigma_3 n_2 n_3 + (l_3 m_2 + l_2 m_3)\tau_{12} + +(n_2 l_3 + n_3 l_2)\tau_{13} + (m_2 n_3 + m_3 n_2)\tau_{23}$$

The corresponding stress calculations are presented in Fig. 5. Numerical results obtained here are valid for the major part of the shell excluding the narrow area adjoined to the fixed cross-section. The analysis of this specific portion of the shell should be performed on the basis of the full model described by the equations (1), (8), and (9).

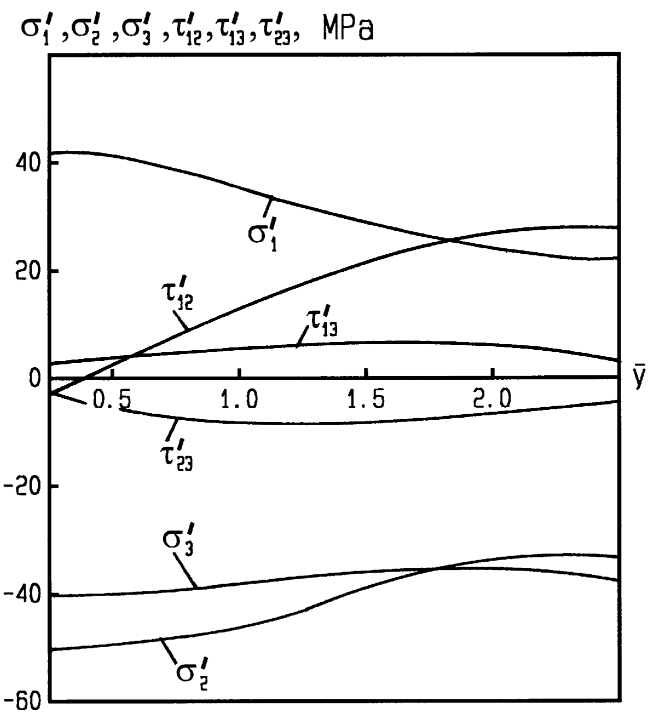

Fig. 5 Stress distributions in the elementary reinforcing layers

CONCLUSIONS

Using the models and equations discussed in this paper the joint thermal and mechanical analysis of the stress and strain state can be performed for the symmetrically loaded composite shells of revolution. The full range of stresses and strains can be calculated including transverse stress and transverse shear stresses. Numerical results demonstrate the fact that due to a spatial nature of the composite reinforcement orientation the axisymmetric twisting deformation of the shell plays an important role in the stress and strain distributions together with the tensile and compressive strains in the reference surface. The stresses and strains induced by this deformation are essential for the analysis of the structures under consideration and should be taken into account at the design stage. Due to its relative simplicity the approach presented in the paper can be applied to the design and optimization of thin-walled structural components with spatial type of the reinforcement orientation.

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