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GEOMETRICAL NONLINEAR ANALYSIS OF ORTHOTROPIC RECTANGULAR THIN PLATES WITH THREE EDGES CLAMPED AND ONE EDGE SIMPLY SUPPORTED

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ABSTRACT: In the paper, von-Karman type orthotropic rectangular plates are analyzed with three edges clamped and one edge simply supported by Galerkin method. The beam vibration functions that have orthogonality property may accurately satisfy the boundary conditions. Governing nonlinear partial differential equations are transferred to an infinite set of system of nonlinear algebraic equations containing Fourier coefficients. Large scale of sparse matrix linear equations have been solved by Biconjugate Gradients Stabilized Method and nonlinear algebraic equations solved by parameter-regulated iterative procedures. The series of beam vibration functions are rapidly converged. Only a few prior terms of the series may meet the need of accuracy. Numerical results of deflection and stress are obtained for different composite materials.

KEYWORDS: three edges clamped and one edge simply supported, orthotropic, geometrical nonlinear, algebraic equations

INTRODUCTION

In resent years, a lot of research works about geometrical nonlinear analysis of orthotropic plates have been made. Professor C.Y.Chia[1] in Calgary University of Canada made systematic analysis about isotropic and orthotropic plates. At present, the most of study focus on four edges simply supported; four edges clamped; and the elastic rotation constrain boundary conditions of two opposite edges with the same support. But the study of nonlinear problem for three edges clamped and one edge simply supported is seldom reported. In the paper, the vibration functions of beam, which can accurately meet the boundary conditions, are presented. The functions with orthogonality advantage are employed to transfer differential equations to a set of nonlinear algebraic equations that may be directly solved by parameter-regulated iterative procedure. Biconjugate Gradients Stabilized Method may solve the sparse matrix of linear equations. Finally, the numerical results that meet computing accuracy of the deflection and stress of different composite materials are obtained.

FUNDAMENTAL THEORY AND METHOD

Consider an orthotropic rectangular thin plate of length *a*, width *b* and thickness *h*, which is subjected to a uniform lateral pressure q_0。 The reference coordinate system has its origin at the corner of the plate. Let u^0, v^0, w be the mid-plane displacements parallel to a right-hand set of axes (x, y, z), where x is latitudinal and z is perpendicular to the plate. E_1, E_2 and G_{12} are elastic module and shear modulus respectively; γ_{12} and γ_{21} are Poisson's ratios. If plate thickness is very small comparing with transverse size, fundamental Kerchihoff's assumption may be used to get von-Karman governing equations described by displacement form[1]:

$$U,_{\xi\xi}+\lambda^2 C_1 U,_{\eta\eta}+\lambda C_2 V,_{\xi\eta}=-W,_{\xi}(W,_{\xi\xi}+\lambda^2 C_1 W,_{\eta\eta})-\lambda^2 C_2 W,_{\eta} W,_{\xi\eta} \tag{1}$$

$$\lambda C_2 U,_{\xi\eta}+C_1 V,_{\xi\xi}+\lambda^2 C_3 V,_{\eta\eta}=-\lambda W,_{\eta}(C_1 W,_{\xi\xi}+\lambda^2 C_3 W,_{\eta\eta})-\lambda\, C_2 W,_{\xi} W,_{\xi\eta} \tag{2}$$

$$\begin{aligned} W,_{\xi\xi\xi\xi}&+2\lambda^2 C_4 W,_{\xi\xi\eta\eta}+\lambda^4 C_3 W,_{\eta\eta\eta\eta}=Q+W,_{\xi\xi}(U,_{\xi}+\lambda\nu_{21}V,_{\eta})+\\ &+\lambda^2 W,_{\eta\eta}(\nu_{21}U,_{\xi}+\lambda C_3 V,_{\eta})+2\lambda C_1 W,_{\xi\eta}(\lambda U,_{\eta}+V,_{\xi}+\lambda W,_{\xi} W,_{\eta})+\\ &+0.5W,_{\xi\xi}(W,_{\xi}^2+\lambda^2\nu_{21}W,_{\eta}^2)+0.5\lambda^2 W,_{\eta\eta}(\nu_{21}W,_{\xi}^2+\lambda^2 C_3 W,_{\eta}^2)\end{aligned} \tag{3}$$

where

$$\begin{aligned} &\xi=x/a,\quad \eta=y/b,\quad \lambda=a/b,\quad U=\frac{12au^0}{h^2},\quad V=\frac{12av^0}{h^2}\\ &W=2\sqrt{3}\frac{w}{h},\quad Q=\frac{24\sqrt{3}\mu q_0 a^4}{E_1 h^4},\quad C_1=\frac{\mu G_{12}}{E_1},\quad C_2=\nu_{21}+C_1\\ &C_3=E_2/E_1,\quad C_4=\nu_{21}+2C_1,\quad \mu=1-\nu_{12}\nu_{21}\end{aligned} \tag{4}$$

Boundary conditions with three edges clamped and one edge simply supported are as follows:

$$\begin{aligned} &\text{when}\quad \xi=0,\quad U=V=W,_{\xi\xi}=W=0;\\ &\text{when}\quad \xi=1,\quad U=V=W,_{\xi}=W=0;\\ &\text{when}\quad \eta=0,1,\quad U=V=W,_{\eta}=W=0.\end{aligned} \tag{5}$$

Bending and membrane stresses expressed by displacements are:

$$\left.\begin{aligned} \sigma_{\xi}^{b}&=\pm\sqrt{3}(W,_{\xi\xi}+\lambda^2\nu_{21}W,_{\eta\eta})\\ \sigma_{\eta}^{b}&=\pm\sqrt{3}(\nu_{21}W,_{\xi\xi}+\lambda^2 C_3 W,_{\eta\eta})\\ \sigma_{\xi\eta}^{b}&=\pm 2\sqrt{3}\lambda C_1 W,_{\xi\eta}\end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} \sigma_{\xi}^{m}&=U,_{\xi}+\lambda\nu_{21}V,_{\eta}+0.5(W,_{\xi}^2+\lambda^2\nu_{21}W,_{\eta}^2)\\ \sigma_{\eta}^{m}&=\nu_{21}U,_{\xi}+\lambda C_3 V,_{\eta}+0.5(\nu_{21}W,_{\xi}^2+\lambda^2 C_3 W,_{\eta}^2)\\ \sigma_{\xi\eta}^{m}&=C_1(\lambda U,_{\eta}+V,_{\xi}+\lambda W,_{\xi} W,_{\eta})\end{aligned}\right\} \tag{7}$$

ANALYSIS

Displacement functions which can accurately meet the boundary conditions (5) are expressed by double Fourier series ：

$$U=\sum_{m,n=1}^{\infty}u_{mn}I_m(\xi)J_n(\eta)\ ;\ \ V=\sum_{m,n=1}^{\infty}v_{mn}I_m(\xi)J_n(\eta)\ ;\ \ W=\sum_{m,n=1}^{\infty}w_{mn}X_m(\xi)Y_n(\eta) \qquad (8)$$

where u_{mn}, v_{mn}, w_{mn} are undetermined coefficients of series; $X_m(\xi)$ and $Y_n(\eta)$ are vibration functions of beam:

$$I_m(\xi)=\sin m\pi\xi\ ;\ \ J_n(\eta)=\sin n\pi\eta \qquad (9)$$

$$X_m(\xi)=\sin\lambda_m\xi-\alpha_m\sinh\lambda_m\xi \qquad (10)$$

$$Y_n(\eta)=\sin\lambda_n^{'}\eta-\sinh\lambda_n^{'}\eta-\alpha_n^{'}\cos\lambda_n^{'}\eta+\alpha_n^{'}\cosh\lambda_n^{'}\eta \qquad (11)$$

The $\lambda_i, \alpha_i \lambda_i^{'}, \alpha_i^{'}$ are determined by following formulas:

$$\tan\lambda_i-\tanh\lambda_i=0\ \ ;\ \ \alpha_i=\sin\lambda_i/\sinh\lambda_i \qquad (12)$$

$$1-\cos\lambda_i^{'}\cosh\lambda_i^{'}=0 \qquad \alpha_i^{'}=\frac{\sin\lambda_i^{'}-\sinh\lambda_i^{'}}{\cos\lambda_i^{'}-\cosh\lambda_i^{'}} \qquad (13)$$

The beam vibration functions $X_m(\xi)$ and $Y_n(\eta)$ possess orthogonality properties:

$$\int_0^1 X_i(\xi)X_j(\xi)d\xi=\begin{cases}0 & i\neq j\\ H_i & i=j\end{cases} \qquad \int_0^1 Y_i(\eta)Y_j(\eta)d\eta=\begin{cases}0 & i\neq j\\ H_i^{'} & i=j\end{cases} \qquad (14)$$

The uniform transverse pressure is also transferred to double Fourier series:

$$Q=\sum_{m,n=1}^{\infty}Q_{mn}X_m(\xi)Y_n(\eta) \qquad (15)$$

which coefficient Q_{mn} is easily determined by equation (14):

$$Q_{mn}=\frac{Q}{H_m H_n^{'}}\int_0^1\int_0^1 X_m(\xi)Y_n(\eta)d\xi d\eta \qquad (16)$$

Using Galerkin method, both sides of the prior two governing equations are multiplied by $I_m(\xi)J_n(\eta)$ and the last governing equation multiplied by $X_m(\xi)Y_n(\eta)$， making integration of ξ and η from 0 to 1:

$$u_{mn}+H_{4mn}\sum_{k,l=1}^{\infty}v_{kl}P_{13}^{mk}P_{13}^{nl}=H_{5mn}\sum_{k,l,r,s=1}^{\infty}w_{kl}w_{rs}(P_{14}^{mkr}Q_{17}^{nls}+C_1\lambda^2P_{15}^{mkr}Q_{18}^{nls}+C_2\lambda^2P_{15}^{mrk}Q_{19}^{nls}) \qquad (17)$$

$$v_{mn}+H_{6mn}\sum_{k,l=1}^{\infty}u_{kl}P_{13}^{mk}P_{13}^{nl}=H_{7mn}\sum_{k,l,r,s=1}^{\infty}w_{kl}w_{rs}(\lambda^2C_3P_{17}^{mkr}Q_{14}^{nls}+C_1P_{18}^{mkr}Q_{15}^{nls}+C_2P_{19}^{mkr}Q_{15}^{nsl}) \qquad (18)$$

$$\begin{aligned}
w_{mn}&+H_{1mn}\sum_{k,l=1}^{\infty}w_{kl}P_1^{mk}Q_1^{nl}=H_{2mn}Q_{mn}+\\
&+H_{3mn}\sum_{k,l,r,s=1}^{\infty}u_{kl}w_{rs}(P_2^{mkr}Q_4^{nls}+\lambda^2\nu_{21}P_5^{mkr}Q_3^{nls}+2\lambda^2C_1P_6^{mkr}Q_7^{nls})+\\
&+H_{3mn}\sum_{k,l,r,s=1}^{\infty}v_{kl}w_{rs}(\lambda\nu_{21}P_3^{mkr}Q_5^{nls}+\lambda^3C_3P_4^{mkr}Q_2^{nls}+2\lambda\,C_1P_7^{mkr}Q_6^{nls})+\\
&+0.5H_{3mn}\sum_{i,j,k,l,r,s=1}^{\infty}w_{ij}w_{kl}w_{rs}(P_8^{mikr}Q_{10}^{njls}+\lambda^2\nu_{21}P_9^{mikr}Q_{11}^{njls}+\\
&+\lambda^2\nu_{21}P_{12}^{mrki}Q_9^{njls}+\lambda^4C_3P_{10}^{mikr}Q_8^{njls}+4\lambda^2C_1P_{12}^{mikr}Q_{11}^{nljs})
\end{aligned} \qquad (19)$$

H_{imn} and P_i are constants related to equations (4,9-14). The parameter-regulated iterative procedure is used to solve nonlinear algebraic equations. Following formula may effectively solve the difficult problems that result becomes divergent when load becomes larger.

$$\begin{aligned} u_{mn}^{i} &= (u_{mn}^{i-1} + u_{mn}^{i-2} + \ldots + u_{mn}^{i-N_1}) / N_1 \\ v_{mn}^{i} &= (v_{mn}^{i-1} + v_{mn}^{i-2} + \ldots + v_{mn}^{i-N_1}) / N_1 \\ w_{mn}^{i} &= (w_{mn}^{i-1} + w_{mn}^{i-2} + \ldots + w_{mn}^{i-N_1}) / N_1 \end{aligned} \tag{20}$$

where N_1 is a modified parameter which takes smaller value when load is smaller and takes larger value when load is larger.

NUMERICAL RESULTS AND CONCLUSIONS

The deflection and stresses of two kinds of different composite materials(shown in Tab.1) and an isotropic material (γ= 0.3) are computed in the paper. Fig.1 shows typical relationship between load and deflection. In use of Material 2, Fig.2 illustrates varying curves of bending stress and membrane stress with deflection. Following conclusions are finally obtained by above analysis and computation:

Table1: Elastic constants of materials

Material	E_1/E_2	G_{12}/E_2	γ_{12}
Material 1	10	1/3	0.22
Material 2	3	0.5	0.25

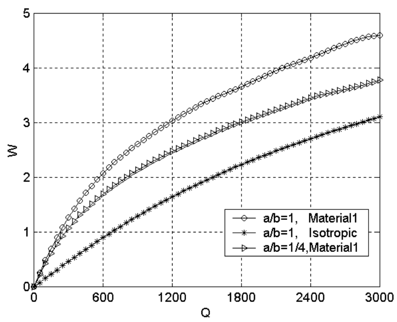

Fig.1 Relationship between central deflection and uniformly distributed loads

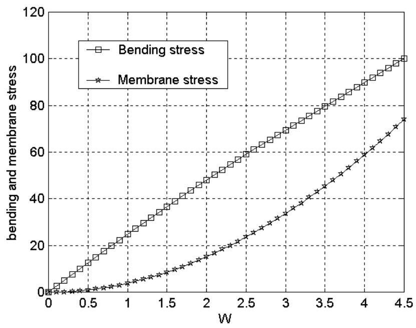

Fig.2 Relationship of deflection *vs.* bending and membrane stress

(1) Beam vibration functions presented in the paper may accurately meet boundary conditions and possess orthogonality property and have faster convergent speed;

(2) The method presented may solve both linear problems and nonlinear problems and make comparison between them;

(3) Combining Biconjugate Gradients Stabilized Method with parameter-regulated iterative procedure is one of the most effective plans to solve geometrical nonlinear problems;

(4) It only needs first term of the displacement series to compute linear deflection but needs prior 9 terms of the series to compute nonlinear deflection. Prior 25 terms of series is needed to compute the stress.

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