ID 1065

Vibration Response of Thin Piezoelectric Laminated Plates

Jin H. Huang
Department of Mechanical Engineering, Feng Chia University, Taichung 40745,
Taiwan, China

SUMMARY: In this article, theoretical and experimental studies on the vibration response of a thin piezoelectric plate are presented. The theoretical study is based on a coupled first-order shear deformation theory for piezoelectric plates, in which the fundamental unknowns such as the displacements and the electric potential are assumed to be expandable through the plate thickness coordinate. In terms of analyzing a single vibrating PIC-151 layer plate, theoretical results for the vibration mode shapes and natural frequencies are presented. The results correlate well with the experimental data conducting by the electronic speckle pattern interferometry (ESPI), which is a full-field, noncontact, and real-time optical method for measuring dynamic response of a structure subjected to electromechanical loading. Finally, the proposed formulation is further applied to study a piezoelectric laminated plate with fiber-reinforced layers. The influences of fiber orientation and length-to-width ratio on the natural frequency and the electroelastic response of the laminated plate are also presented.

KEYWORDS: AF-ESPI, piezoelectric laminated plate, mode shapes, natural frequency.

INTRODUCTION

Piezoelectric materials, especially of lead zirconate titanate (PZT) are widely used as materials of ultrasonic devices such as transducer, filters, resonators, and motors. This is due to their accuracy as transducers, large electro-mechanical coupling constant that in turn allows the actuators output larger power. In the past few years, the application of piezoelectric ceramics material used as a sensor for active vibration control or actuator for precise motion control are carried out in many research areas. The study of the vibration behavior for piezoelectric materials is a problem of great practical interest. It appears, however, that relatively little work has been done regarding the non-contact measurement of real-time vibration response for piezoelectric materials. The most significant measurement in this area is that Butters and Leendertz [1] that was directed toward the study of the out-of-plane displacement of a vibrating disk by employing electronic speckle pattern interferometry (ESPI). ESPI is also known as TV-Holography or electro-optic holography. It combined the object and reference beams and directed them collinearly towards the detector array of frame-transfer CCD which interfaced to a computer for video electronics processing of speckle patterns, then displayed on a TV-monitor in real-time (about 30 ms). With its full-field, non-contact and high accuracy characteristics of ESPI, it has contributed much for deformation analysis [2,3], especially for vibration measurement [4-8].

In this paper, ESPI will be employed to measure the vibration characteristics of piezoelectric beams with different boundary conditions. In light of the results presented in the work, theoretical predictions based on the variational approach are also made. The acceptable correspondence of the experimental data with numerical results reveals that the presented methodologies on vibration response of piezoelectric materials own an available exactitude.

BASIC EQUATIONS

In the first part of the present work, an attempt is made to introduce the governing equations for a piezoelectric laminated plate with a dimension of L_1xL_2. The total thickness of the plate made up of N thin layers. Some of these layers are made of piezoelectric materials as sensors or actuators while others are made of fiber-reinforced composite materials. Both sensors and actuators are poled in the thickness direction and their major surfaces are fully electroded with negligible thickness. Suppose the displacement vector u_i and the electric potential E_i are chosen as independent variables, while the dependent variables are the elastic stress tensor σ_{ij} and the electric displacement vector D_i. The system of equations of motion are given by [9]

$$\mathbf{L}_d\mathbf{d} - \mathbf{L}_c\boldsymbol{\phi} + \mathbf{p} = \mathbf{L}_\rho\ddot{\mathbf{d}}, \tag{1}$$

$$\mathbf{L}_c^*\mathbf{d} + \mathbf{L}_\phi\boldsymbol{\phi} + \mathbf{q} = \mathbf{0}, \tag{2}$$

and the characteristic equation is

$$\left(\begin{bmatrix}\mathbf{K}_d & \mathbf{K}_\phi \\ \mathbf{K}_\phi^{\mathrm{T}} & \mathbf{K}\end{bmatrix} - \omega^2\begin{bmatrix}\mathbf{L}_\rho & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{z}_d \\ \mathbf{z}_\phi\end{Bmatrix} = \mathbf{f}, \tag{3}$$

where the force vector **f**, the amplitude vectors $\mathbf{z}_d$ and $\mathbf{z}_\phi$, **q**, $\mathbf{L}_c$, $\mathbf{L}_d$, $\mathbf{L}_\phi$, $\mathbf{K}_d$, $\mathbf{K}_\phi$, and **K** can be found in the paper of Huang [9]. Thus, in the static analysis, set ω^2 to zero in the 8×8 matrix Eq. (3) and solve the equation for the amplitude vectors $\mathbf{z}_d$ and $\mathbf{z}_\phi$ and, then, the solution is given by Eqs. (1) and (2). In the mode-frequency analysis, set the generalized force vector **f** to zero and solve the resulting eigenvalue problem for natural frequencies.

PRINCIPLE OF EXPERIMENTAL TECHNIQUES

The most familiar way of ESPI used for vibration analysis is the time-averaging method with an image sensor (mostly CCD array is used) integrating the speckle interferogram field pixel by pixel during camera frame period. Two different optical design setups are commonly used (out-of-plane sensitivity and in-plane sensitivity) for the vibration measurement. Fundamentals with regards to AF-ESPI method for in-plane and out-of-plane measurement can be found in references 7 and 8. Since the piezoelectric material adopted in this work is thin, it will display large transverse (out-of-plane) motion compared to any possible in-plane components, especially in the low natural frequencies. Hence only the out-of-plane sensitivity optical setup is used in this study. As shown in Fig.1, a He-Ne Laser (Uniphase 1135P) with 30 mw and wavelength $\lambda = 632.8$ nm is used as the coherent light source. The emitting laser beam is split into two parts by a variable beamsplitter. One beam is directed toward the tested piezoelectric specimen and then reflects to the CCD camera acting as the object beam. The second one served as a reference beam is illuminated on the surface of a reference plate and reflects into CCD camera via the beamsplitter. The object and reference beams are combined into the CCD sensor array through a zoon lens (Nikon Micro-Nikkor 55mm). It is important to note that the optical path and the light intensity of these two beams should remain identical in the experimental setup. A CCD camera (Pulnix TM-7CN) and a frame grabber (Dipix P360F) with a digital signal processor on board are used to record and process the images obtaining from interferogram of the object and reference beams. Once the object vibrating, the interferogram recorded by the CCD camera is stored in an image buffer as a reference image. Then the next frame is grabbed and is subtracted by the image processing system. The CCD camera converts the intensity distribution of the interference pattern of the object into a corresponding video signal at 30 frames per second. The signal is electronically processed and finally converted into an image on the video monitor. The interpretation of the fringe image is similar to reading of a displacement contour. To achieve a sinusoidal output, a function generator (Hewlett Packard, HP-33120A) connected to a power amplifier (NF Electronic Instruments 4005 type) is employed as an input source, which generates periodical exciting force to the specimen.

The detailed experimental procedure of the AF-ESPI technique is performed as follows. First, a reference image is taken after the specimen vibrates, then the second image is taken, and the reference image is subtracted by the image processing system. If the vibrating frequency is not at the natural frequency, only randomly distributed speckles are displayed and no fringe patterns will be shown. However, if the vibrating frequency is in the neighborhood of the natural frequency, stationary distinct fringe patterns will be observed. Then the function generator is carefully and slowly turned, the number of fringes will increase and the fringe pattern will become clearer but the position of nodal lines will not change, as the natural frequency is approached. From the aforementioned experimental procedure, the natural frequencies and the corresponding mode shapes can be determined at the same time using the AF-ESPI optical system.

EXPERIMENTAL AND NUMERICAL RESULTS

In this section, three studies on the vibration and electroelastic response of a piezoelectric thin plate are presented. The first focused on the numerical results of the vibrating response of a single PIC-151 layer plate by using the proposed approach. To validate the present analysis, the second study is on the comparisons with experimental results by employing AF-ESPI measurement method. Figure 1 and Table 1 show the experimental and theoretical results for the vibration mode shapes and resonant frequencies of a simply supported thin PIC-151 plate. The results indicate that numerical calculations of the proposed approach correlate well with the experimental data.

In the last study, numerical results are presented for a piezoelectric five-layered laminated plate with four Graphite/Epoxy fiber reinforced layers. The stacking sequences of the laminated plate are $(\theta/0/\text{PIC-151})_{\text{sym}}$ and $(\theta/-\theta/\text{PIC-151})_{\text{sym}}$, implying that the plate is symmetric in geometric and material properties about PIC-151 layer. Figure 3 summarizes the effect of fiber orientation on the natural frequency of the simply supported laminated plate. For each stacking sequence, Fig.3 displays the dependence of the natural frequency for three-square plates as a function of the fiber orientation θ. As θ increases, the natural frequencies follow wave-like curves symmetric with respect to $\theta = 90^\circ$, where a minimum is reached. Maximum values of natural frequency are located at $\theta = 45^\circ$ and 135°. At a given θ, our results indicate that a smaller plate leads to a higher natural frequency, and the stacking sequence $(\theta/0/\text{PIC-151})_{\text{sym}}$ has higher frequency than $(\theta/-\theta/\text{PIC-151})_{\text{sym}}$.

CONCLUSIONS

This work has examined the natural frequencies and mode shapes of a vibrating piezoelectric laminated plate by using theoretical and AF-ESPI approaches. It has shown that AF-ESPI has the advantages of non-contact, real-time and high-resolution measurement for investigating the vibration problem. According to our results, the natural frequencies measured by AF-ESPI correlate well with theoretical. It has been shown that the fiber orientation and length-to-width ratio have significant influences on the natural frequency and the electroelastic response of the laminated plate. The good correspondence of the experimental data with numerical results by the present method verifies that the presented methodologies on measuring vibration response of piezoelectric materials have an available accuracy. These results demonstrate that the proposed analysis is applicable to many situations in engineering vibration analysis for piezoelectric materials with different boundary conditions. Moreover, the proposed methodologies for measuring out-of-plane vibration response of a piezoelectric beam can be equally applied not only to measure in-plane response, but also to gain simultaneously both out-of-plane and in-plane responses for any vibrating materials with/without piezoelectric coupling effect.

ACKNOWLEDGMENTS

This research was supported by the National Science Council of the Republic of China under contracts Nos. NSC 89-2212-E-035-003 and NSC 89-2745-P-035-004. Partial support from the Research Foundation of the Feng Chia University under Grant No. FCU-RD-87-A01 is also appreciated. The author is grateful toProfessor Chien-Ching Ma and Mr. Hsien-Yang Lin of Department of Mechanical Engineering, National Taiwan University, for their many contributions to this work.

REFERENCES

1. Butters, J.N. and Leendertz, J.A., “Speckle Patterns and Holographic Techniques in Engineering Metrology,” *Optics and Laser Technology*, 1971, Vol. 3, No. 1

2. Jones, R. and Wykes, C., “Holographic and Speckle interferometry,” Cambridge University Press, Cambridge, England, UK, 1989.

3. Wykes, C., “Use of Electronic Speckle Pattern Interferometry (ESPI) in the Measurement of Static and Dynamic Surface Displacements,” *Optics and Laser Tech.*, 1982, Vol. 21, No.3

4. Creath, K. and Gudmunn, A., “Vibration-Observation Techniques for Digital Speckle-Pattern Interferometry,” *Journal of the optical society of America A*, Vol. 2, No.10

5. Nakadate, S., “Vibration Measurement using Phase-Shifting Speckle Pattern Interferometry,” *Applied Optics*, 1986, Vol. 25, No.22

6. Wang, W.C., Hwang, C.H. and Lin, S.Y, “Vibration Measurement by the Time-Averaging Electronic Speckle Pattern Interferometry Method,” *Applied Optics*, 1996, Vol. 35, No. 22

7. Huang, C.H. and Ma, C.C., “Vibration Characteristics for Piezoelectric Cylinders using Amplitude-Fluctuation Electronic Speckle Pattern Interferometry ,” *AIAA Journal*, 1998, Vol. 36, No. 12

8. Ma, C.C. and Huang, C.H., “The Investigation of Three-Dimensional Vibration for Piezoelectric Rectangular Parallelepipeds Using the AF-ESPI Method,” *IEEE Transactions on Ultrasonics, Ferroelectrics*, *and Frequency Control,* 2001, Vol. 48, No.1

9. Huang, J.H. and Yu, H.I., "Dynamic Response of Piezoelectric Plates as Sensors or Actuators," *Materials Letters*, 2000, Vol. 46, No. 11

Table 1. The fist twelve resonant frequencies of a fixed-fixed PIC-151 thin plate.

	AF-ESPI (Hz)	Theoretcal method (Hz)
Mode 1	2223	2453
Mode 2	2589	2886
Mode 3	3342	3678
Mode 4	4455	4849
Mode 5	5851	6396
Mode 6	6078	6547
Mode 7	6240	6997
Mode 8	7058	7770
Mode 9	7688	8306
Mode 10	8148	8893
Mode 11	9528	10380
Mode 12	9768	10576

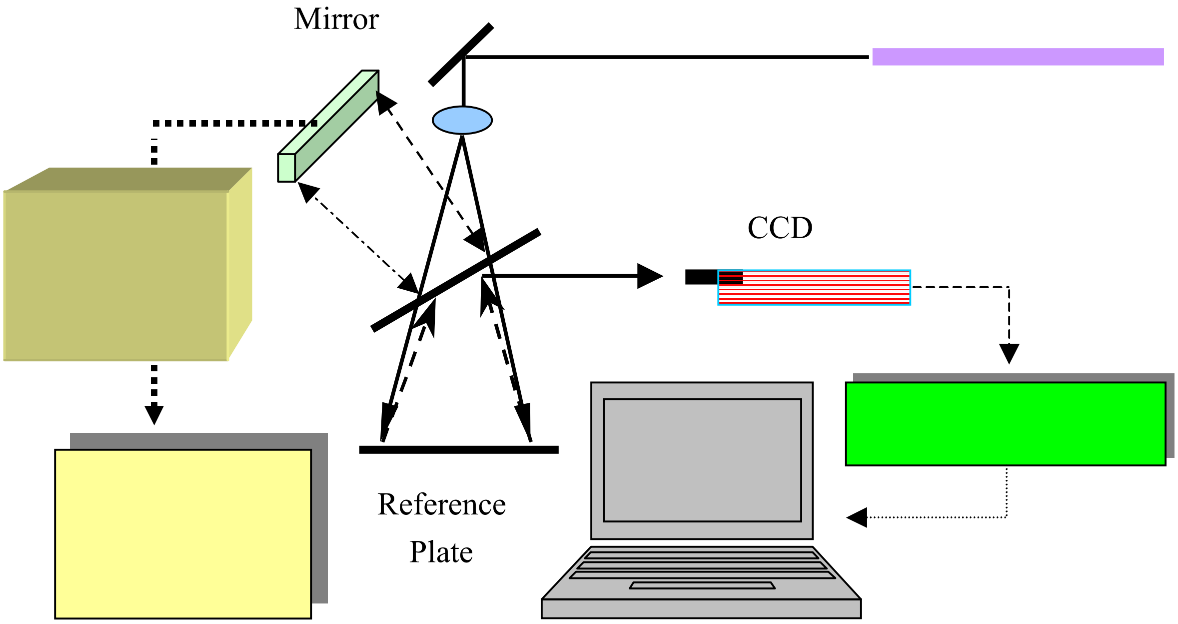
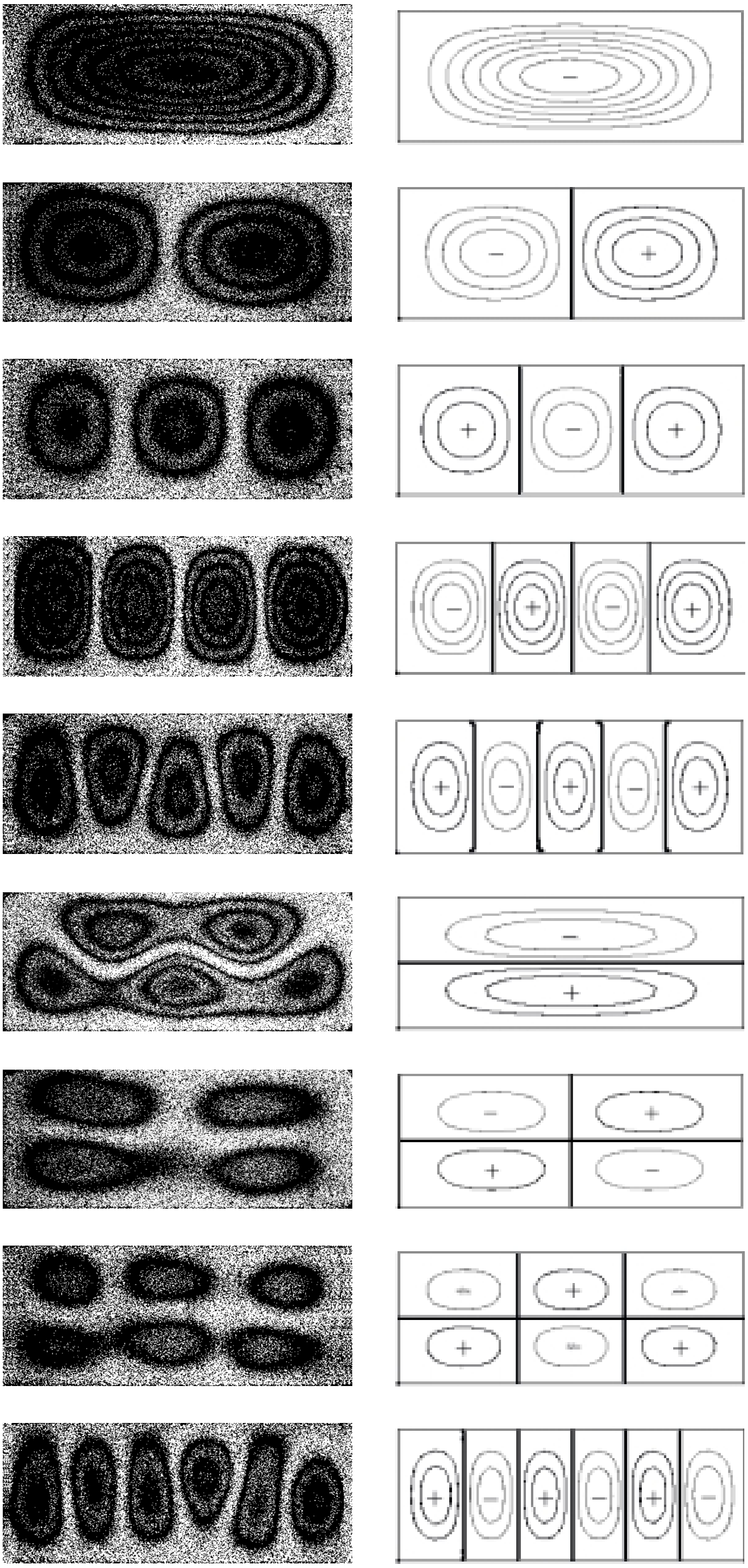


Fig. 1 Optical setup with out-of-plane sensitivity for ESPI.



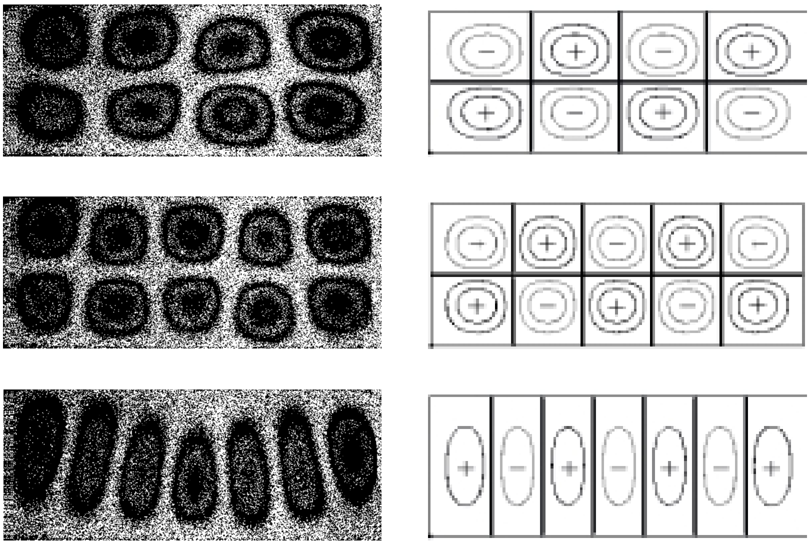


Fig.2 The mode shapes of a fixed-fixed PIC-151 thin plate.

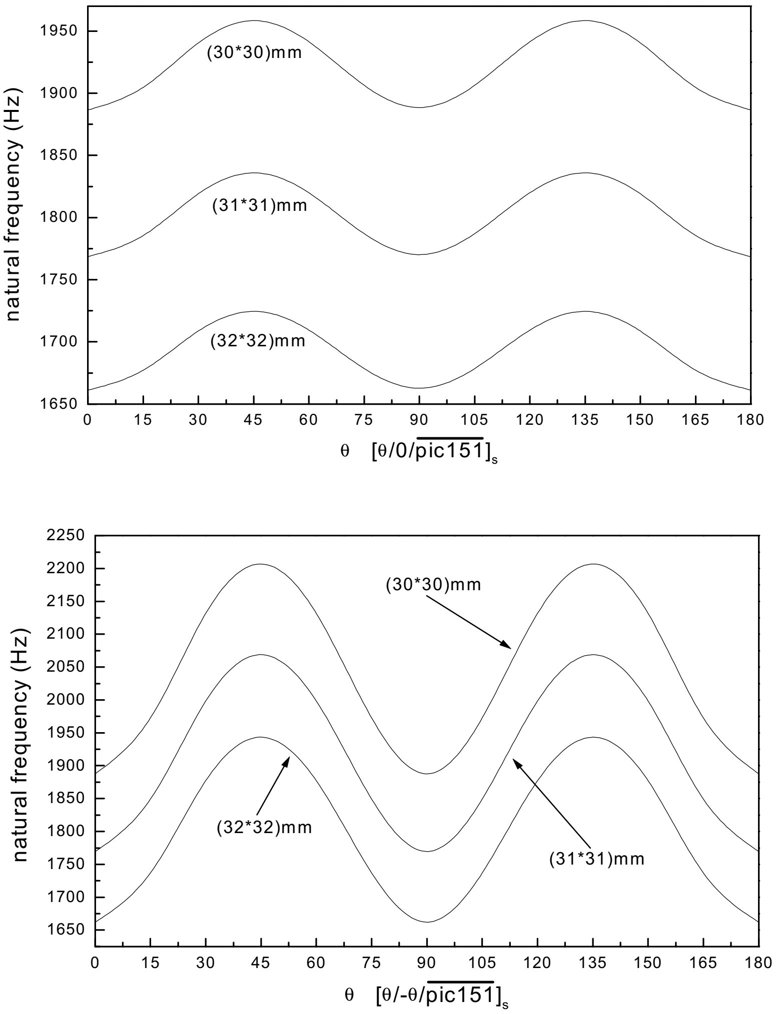


Fig.3 Effect of the fiber orientation θ on natural frequency for a five-layered laminated plate.