ID-1066

ELASTICITY SOLUTION FOR THICK LAMINATED CIRCULAR CYLINDRICAL SHALLOW AND NON-SHALLOW PANELS UNDER DYNAMIC LOAD

M. Shakeri, M. R. Eslami , A. Alibiglu

Department of Mechanical Engineering, Amirkabir Univ. of Tech., Tehran, Iran

SUMMARY: Dynamic response of axisymmetric cross-ply laminated deep and shallow panels subjected to asymmetric load based on three-dimensional elasticity equations are studied. The shell panel is simply supported at four sides and has finite length. The highly coupled partial differential equations (p.d.e.) are reduced to ordinary differential equations (o.d.e.) with constant coefficients for shallow shell panel and variable coefficients for deep panel by means of trigonometric function expansion in circumferential and axial directions. The resulting ordinary differential equation is solved by Galerkin finite element method. Numerical examples are presented for $(0/90)$ and $(0/90/0)$ laminations under dynamic loading ,and the results of deep and shallow panels are compared with each other

KEYWORDS: Elasticity, Panel, Shallow, Dynamic, and Composite

INTRODUCTION

Cylindrical panels composed of advanced homogeneous or laminated composite materials are being increasingly used in the engineering and especially the aerospace industry. The use of high-modulus low-density materials such as fiber reinforced plastics in modern day structures demands an accurate analysis of such structures. It is well known that these materials are anisotropic in nature. However, due to complicated effects, such as a strong influence of transverse shear and transverse normal deformation, bending-extensional coupling due either to nonzero curvature or to lamination, the dynamic behavior of such advanced structural elements is considerably more complicated than that for the corresponding isotropic cases. Hence accurate prediction of their dynamic response often requires analysis, which are based on three-dimensional modeling. There are not many solutions for laminated cylindrical panel based on three-dimensional elasticity, because of the considerable mathematical difficulties in solving governing differential equations for the general boundary and loading conditions.

An exact solution for static response of laminated cylindrical panel with infinite length subjected to asymmetric loading and simply supported boundary conditions has been presented (REN) [1]. In this paper the plane strain assumption is made and the equilibrium equations are solved with introducing an appropriate stress function. Static analysis of simply-

supported and cross-ply laminated cylindrical panel with finite length was also presented by the above author[2]. Displacements and stresses of the solution are expressed in terms of Fourier and power series. Free vibration analysis of doubly curved shallow shells on rectangular platform using three-dimensional elasticity theory was studied (Bhimaraddi) [3]. In this paper the governing partial differential equations is reduced to ordinary differential equations by assuming the solution in the axial and circumferential directions, to be composed of trigonometric function and then solved the resulting equation. An exact three-dimensional thermo elasticity solution for a cross-ply cylindrical panel was obtained (Huang and Tauchert) [4] using the power series method. Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shells was presented (Bhimaraddi and Chandrashekhara) [5]. In this paper solution is obtained by utilizing the assumption that the ratio of the panel thickness to its middle surface radius is negligible as compared to unity. It is shown that the two dimensional shell theories are very inaccurate when the thickness to length ratio of the panel is more than 1/20. Three-dimensional elasticity solution for static response of orthotropic doubly curved shallow shells on rectangular platform was studied (Bhimaraddi) [6]. He obtained the static response such as displacements and stresses in x , y and z directions by assuming the variables in the form of the trigonometric functions expansion. The exact three-dimensional elasticity solution for infinitely long, arbitrarily laminated, anisotropic cylindrical panels with simply supported boundary condition under transverse loading was established (Hung and Kuan) [7] using power series method.
Review of the published literature shows that elasticity solution to the problem of laminated, cross-ply cylindrical panel of finite length under dynamic load has not yet been investigated. Recently the authors have studied the response of closed cylindrical shells and panels with infinite length under dynamic loading , using the elasticity solution (Shakeri) [8,9].

In this paper the dynamic response of axisymmetric cross-ply laminated panels subjected to asymmetric loading based on three-dimensional elasticity equations are studied.

PROBLEM DESCRIPTION

Consider a laminated circular cylindrical panel, as shown in Figure (1), composed of N uniformly thick layers. The layers of the panel are oriented such that the material axes of any layer are aligned with the r,θ and z directions, so that the panel is laminated orthotropic. The panel is simply supported on the edges. The constitutive equations of each layer are stated as;

$$\begin{bmatrix} \sigma_Z \\ \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \\ \tau_{Zr} \\ \tau_{Z\theta} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_Z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{Zr} \\ \gamma_{Z\theta} \end{bmatrix} \qquad (1)$$

where:

$\sigma_i\,(i = r,\theta,z)$, $\varepsilon_i\,(i = r,\theta,z)$ Are the normal stresses and strains

$\tau_{r\theta},\tau_{zr},\tau_{z\theta}$, $\gamma_{r\theta},\gamma_{zr},\gamma_{z\theta}$ are the shear stresses and strians

and

$C_{ij}(i,j = 1,2,3,4,5,6)$ are the elastic constants

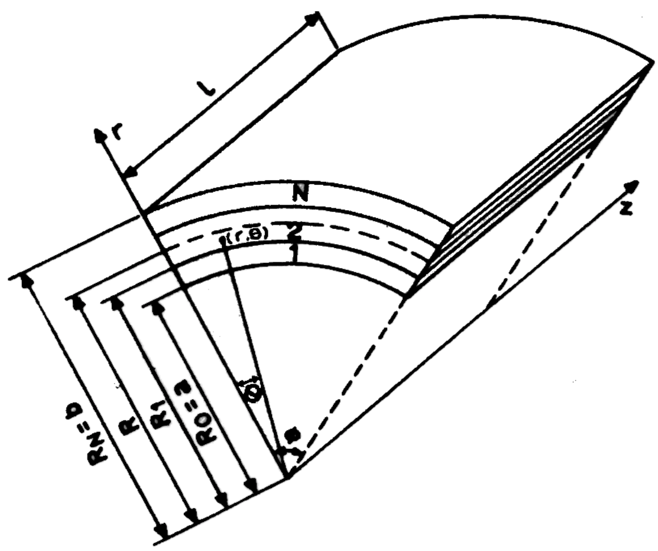

Fig.1 Geometry and the coordinate system of laminated panel

NON-SHALLOW PANEL

The governing equations of three-dimensional boundary value problem in deep cylindrical panels are as;

$$\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{z\theta}}{r\partial\theta}+\frac{\partial\tau_{rz}}{\partial r}+\frac{\tau_{rz}}{r}=\rho\frac{\partial^2 U_z}{\partial t^2}$$

$$\frac{\partial\tau_{\theta z}}{\partial z}+\frac{\partial\sigma_\theta}{r\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}=\rho\frac{\partial^2 U_\theta}{\partial t^2} \qquad (2)$$

$$\frac{\partial\tau_{zr}}{\partial z}+\frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta}+\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}=\rho\frac{\partial^2 U_r}{\partial t^2}$$

where:

U_r,U_θ,U_z are the displacements respectively in the r,θ,z directions.

Strain-displacement relations are expressed as;

$$\varepsilon_r=\frac{\partial U_r}{\partial r}\qquad \varepsilon_\theta=\frac{U_r}{r}+\frac{\partial U_\theta}{r\partial\theta}\qquad \varepsilon_z=\frac{\partial U_z}{\partial z}\qquad \gamma_{zr}=\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z}$$

$$\gamma_{r\theta} = \frac{-U_\theta}{r} + \frac{\partial U_\theta}{\partial r} + \frac{\partial U_r}{r\partial\theta} \qquad \gamma_{z\theta} = \frac{\partial U_\theta}{\partial z} + \frac{\partial U_z}{r\partial\theta} \tag{3}$$

After substitution equations (1) and (3) into equation (2), the governing equations of motion in terms of displacements for each layer of cylinderical panel becomes:

$$C_{11}^{(k)}\frac{\partial^2 U_z}{\partial z^2}+\frac{C_{12}^{(k)}}{r}\left(\frac{\partial U_r}{\partial z}+\frac{\partial^2 U_\theta}{\partial\theta\partial z}\right)+\frac{C_{66}^{(k)}}{r}\left(\frac{\partial^2 U_\theta}{\partial z\partial\theta}+\frac{\partial^2 U_z}{r\partial\theta^2}\right)+$$
$$C_{13}^{(k)}\frac{\partial^2 U_r}{\partial r\partial z}+C_{55}^{(k)}\left(\frac{\partial^2 U_z}{\partial r^2}+\frac{\partial^2 U_r}{\partial r\partial z}\right)+\frac{C_{55}^{(k)}}{r}\left(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z}\right)=\rho\frac{\partial^2 U_z}{\partial t^2}$$

$$C_{66}^{(k)}\left(\frac{\partial^2 U_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial^2 U_z}{\partial\theta\partial z}\right)+\frac{C_{12}^{(k)}}{r}\frac{\partial^2 U_z}{\partial z\partial\theta}+\frac{C_{22}^{(k)}}{r^2}\left(\frac{\partial U_r}{\partial\theta}+\frac{\partial^2 U_\theta}{\partial\theta^2}\right)+$$
$$C_{44}^{k}\left(\frac{-\partial U_\theta}{r\partial r}+\frac{U_\theta}{r^2}+\frac{\partial^2 U_\theta}{\partial r^2}-\frac{\partial U_r}{r^2\partial\theta}+\frac{1}{r}\frac{\partial^2 U_r}{\partial\theta\partial r}\right)+\left(\frac{-U_\theta}{r}+\frac{\partial U_\theta}{\partial r}+\frac{\partial U_r}{r\partial\theta}\right)$$
$$\times\frac{2C_{44}^{k}}{r}+\frac{C_{23}^{k}}{r}\frac{\partial^2 U_r}{\partial r\partial\theta}=\rho\frac{\partial^2 U_\theta}{\partial t^2}$$

(4)

$$\left(C_{55}^{(k)}+C_{13}^{(k)}\right)\frac{\partial^2 U_z}{\partial r\partial z}+C_{55}^{(k)}\frac{\partial^2 U_r}{\partial z^2}+\frac{C_{44}^{(k)}}{r}\left(-\frac{\partial U_\theta}{r\partial\theta}+\frac{\partial^2 U_\theta}{\partial r\partial\theta}+\frac{\partial^2 U_r}{r\partial\theta^2}\right)+$$
$$\left[\left(C_{13}^{(k)}-C_{12}^{(k)}\right)\frac{\partial U_z}{\partial z}+\left(C_{23}^{(k)}-C_{22}^{(k)}\right)\left(\frac{U_r}{r}+\frac{\partial U_\theta}{r\partial\theta}\right)+\left(C_{33}^{(k)}-C_{23}^{(k)}\right)\frac{\partial U_r}{\partial r}\right]$$
$$\times\frac{1}{r}+C_{33}^{(k)}\frac{\partial^2 U_r}{\partial r^2}+C_{23}^{(k)}\left(\frac{-U_r}{r^2}+\frac{\partial U_r}{r\partial r}-\frac{\partial U_\theta}{r^2\partial\theta}+\frac{\partial^2 U_\theta}{r\partial\theta\partial r}\right)=\rho\frac{\partial^2 U_r}{\partial t^2}$$

SHALLOW PANEL

The equations of motion based on three-dimensional elasticity theory for shallow cylindrical panels are:

$$\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{z\theta}}{R\partial\theta}+\frac{\partial\tau_{rz}}{\partial r}+\frac{\tau_{rz}}{R}=\rho\frac{\partial^2 U_z}{\partial t^2}$$

$$\frac{\partial\tau_{\theta z}}{\partial z}+\frac{\partial\sigma_\theta}{R\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{R}=\rho\frac{\partial^2 U_\theta}{\partial t^2} \qquad (5)$$

$$\frac{\partial\tau_{zr}}{\partial z}+\frac{1}{R}\frac{\partial\tau_{\theta r}}{\partial\theta}+\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{R}=\rho\frac{\partial^2 U_r}{\partial t^2}$$

Where;

R is the midradius.

Strain-displacement relations of the 3-D elasticity equations in the cylindrical coordinate system are written as :

$$\varepsilon_r = \frac{\partial U_r}{\partial r} \qquad \varepsilon_\theta = \frac{U_r}{R} + \frac{\partial U_\theta}{R\partial\theta} \qquad \varepsilon_z = \frac{\partial U_z}{\partial z} \qquad \gamma_{zr} = \frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z}$$

$$\gamma_{r\theta} = \frac{-U_\theta}{R} + \frac{\partial U_\theta}{\partial r} + \frac{\partial U_r}{R\partial\theta} \qquad \gamma_{z\theta} = \frac{\partial U_\theta}{\partial z} + \frac{\partial U_z}{R\partial\theta} \tag{6}$$

Substituting the stress-strain relations (1),via strain-displacement relations (6) , the governing equations (5) can be written in terms of three displacements as :

$$C_{11}^{(k)}\frac{\partial^2 U_z}{\partial z^2} + \frac{C_{12}^{(k)}}{R}\left(\frac{\partial U_r}{\partial z} + \frac{\partial^2 U_\theta}{\partial\theta\partial z}\right) + \frac{C_{66}^{(k)}}{R}\left(\frac{\partial^2 U_\theta}{\partial z\partial\theta} + \frac{\partial^2 U_z}{R\partial\theta^2}\right) +$$

$$C_{13}^{(k)}\frac{\partial^2 U_r}{\partial r\partial z} + C_{55}^{(k)}\left(\frac{\partial^2 U_z}{\partial r^2} + \frac{\partial^2 U_r}{\partial r\partial z}\right) + \frac{C_{55}^{(k)}}{R}\left(\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z}\right) = \rho\frac{\partial^2 U_z}{\partial t^2}$$

$$C_{66}^{(k)}\left(\frac{\partial^2 U_\theta}{\partial z^2} + \frac{1}{R}\frac{\partial^2 U_z}{\partial\theta\partial z}\right) + \frac{C_{12}^{(k)}}{R}\frac{\partial^2 U_z}{\partial z\partial\theta} + \frac{C_{22}^{(k)}}{R^2}\left(\frac{\partial U_r}{\partial\theta} + \frac{\partial^2 U_\theta}{\partial\theta^2}\right) +$$

$$C_{44}^{k}\left(\frac{-\partial U_\theta}{R\partial r} + \frac{\partial^2 U_\theta}{\partial r^2} + \frac{1}{R}\frac{\partial^2 U_r}{\partial\theta\partial r}\right) + \left(\frac{-U_\theta}{R} + \frac{\partial U_\theta}{\partial r} + \frac{\partial U_r}{R\partial\theta}\right)$$

$$\times\frac{2C_{44}^{k}}{R} + \frac{C_{23}^{k}}{R}\frac{\partial^2 U_r}{\partial r\partial\theta} = \rho\frac{\partial^2 U_\theta}{\partial t^2}$$

(7)

$$\left(C_{55}^{(k)} + C_{13}^{(k)}\right)\frac{\partial^2 U_z}{\partial r\partial z} + C_{55}^{(k)}\frac{\partial^2 U_r}{\partial z^2} + \frac{C_{44}^{(k)}}{R}\left(-\frac{\partial U_\theta}{R\partial\theta} + \frac{\partial^2 U_\theta}{\partial r\partial\theta} + \frac{\partial^2 U_r}{R\partial\theta^2}\right) +$$

$$\left[\left(C_{13}^{(k)} - C_{12}^{(k)}\right)\frac{\partial U_z}{\partial z} + \left(C_{23}^{(k)} - C_{22}^{(k)}\right)\left(\frac{U_r}{R} + \frac{\partial U_\theta}{R\partial\theta}\right) + \left(C_{33}^{(k)} - C_{23}^{(k)}\right)\frac{\partial U_r}{\partial r}\right]$$

$$\times\frac{1}{R} + C_{33}^{(k)}\frac{\partial^2 U_r}{\partial r^2} + C_{23}^{(k)}\left(\frac{\partial U_r}{R\partial r} + \frac{\partial^2 U_\theta}{R\partial\theta\partial r}\right) = \rho\frac{\partial^2 U_r}{\partial t^2}$$

BOUNDARY CONDITIONS

The simply supported boundary conditions are taken as:

$$U_r = \sigma_\theta = \tau_{\theta z} = 0 \qquad \text{at} \quad \theta = 0 \ , \ \phi$$

(8)

$$U_r = \sigma_z = \tau_{z\theta} = 0 \qquad \text{at}$$

$$Z = 0 \ , \ l$$

For a laminate consisting of N laminae, the continuity conditions to be enforced at any arbitrary interior (k)th interface can be written as:

$$\sigma_r)_k = \sigma_r)_{k+1} \qquad \tau_{r\theta})_k = \tau_{r\theta})_{k+1} \qquad \tau_{rz})_k = \tau_{rz})_{k+1} \tag{9-a}$$

$$U_r)_k = U_r)_{k+1} \qquad U_\theta)_k = U_\theta)_{k+1} \qquad U_z)_k = U_z)_{k+1} \tag{9-b}$$

The boundary conditions on the inner and outer surfaces of the panel are:

$$\sigma_r = p(\theta, t) \quad , \quad \tau_{zr} = \tau_{r\theta} = 0$$ at the outer surface (10-a)

$$\sigma_r = \tau_{zr} = \tau_{r\theta} = 0$$ at the inner surface (10-b)

SOLUTION OF THE GOVERNING EQUATIONS

The solution which satisfies the boundary conditions (8) is:

$$U_r = u_r(r, t) \sin \beta_m \theta . \sin p_n z$$

$$U_\theta = u_\theta(r, t) \cos \beta_m \theta . \sin p_n z \tag{11}$$

$$U_z = u_z(r, t) \sin \beta_m \theta . \cos p_n z$$

where:

$$\beta_m = \frac{m\pi}{\theta m} \quad , \quad p_n = \frac{n\pi}{l}$$

After substituting equation (11) into equations (4) and (7) the p.d.e. reduces to o.d.e. , and applying the formal Galerkin method to the governing o.d.e., results into the following dynamic finite element equilibrium equation for each non boundary element;

$$[M]_k \{\ddot{X}\}_k + [K]_k \{X\}_k = \{F(t)\}_k \tag{12}$$

where:

$[M]$ and $[K]$ are the constant matrices of 6×6

$\{F(t)\}$ is the force matrix of 6×1

Driving equation (9-a) in term of displacements and expressing the derivatives in backward and forward finite differences for (k)th and (k+1)th layer respectively we can obtain U_{rkI}, $U_{\theta kI}$ and U_{zkI} in term of the displacement values of neighbouring nodes as follows;

$$U^k_{rkI} = U^{k+1}_{rkI+1} = A.U^k_{rkI-1} + B.U^{k+1}_{rkI+2} + C.U^k_{\theta kI-1} + D.U^{k+1}_{\theta kI+2} + E.U^k_{zkI-1} + F.U^{k+1}_{zkI+2}$$

$$U^{k}_{\theta kI} = U^{k+1}_{\theta kI+1} = A'.U^{k}_{rkI-1} + B'.U^{k+1}_{rkI+2} + C'.U^{k}_{\theta kI-1} + D'.U^{k+1}_{\theta kI+2} + E'.U^{k}_{zkI-1} + F'.U^{k+1}_{zkI+2}$$

(13)

$$U^{k}_{zkI} = U^{k+1}_{zkI+1} = A''.U^{k}_{rkI-1} + B''.U^{k+1}_{rkI+2} + C''.U^{k}_{\theta kI-1} + D''.U^{k+1}_{\theta kI+2} + E''.U^{k}_{zkI-1} + F''.U^{k+1}_{zkI+2}$$

where:

$U^{k}_{rkI}, U^{k}_{\theta kI}, U^{k}_{zkI}$ Is the displacement at (kI)th node of (k)th element

and

A , B, ... , F'' are constant coefficients.

The dynamic finite element equilibrium equations for two neighbouring elements at interior (k)th and (k+1)th interfaces become;

$$[M]_{k}\{\ddot{X}\}_{k} + [K]_{k}\{X\}_{k} = \{0\} \quad \text{(14-a)}$$

$$[M]_{k+1}\{\ddot{X}\}_{k+1} + [K]_{k+1}\{X\}_{k+1} = \{0\} \quad \text{(14-b)}$$

By applying equation (13) for the first and last nodes, displacement values for these nodes can be obtained, and then from equation (12) the dynamic equations for the first and last element becomes:

$$[M]_{1}.\{\ddot{X}\}_{1} + [K]_{1}.\{X\}_{1} = \{0\} \quad \text{(15-a)}$$

$$[M]_{MI}.\{\ddot{X}\}_{MI} + [K]_{MI}.\{X\}_{MI} = \{0\} \quad \text{(15-b)}$$

By assembling equations (12), (14-a,b), (15-a,b), the general dynamic finite element equilibrium equation is obtained as:

$$[M]\{\ddot{X}\} + [K]\{X\} = \{F(t)\} \quad \text{(16)}$$

where:

$[M]$ and $[K]$ are matrices of $(3MI-6-3N)\times(3MI-6-3N)$ sizes.

MI is the number of elements

N is the number of layers

Once the finite element equilibrium is established, the Newmark direct integration method with suitable time step is used and the equations are solved.

NUMERICAL RESULTS AND DISCUSSION

Two and three-layered cross-ply cylindrical panels which their sequence lay-up are (0 / 90) and (0 / 90 / 0) composed of graphite-epoxy is considered. The forcing function is chosen as:

$$p(\theta, z, t) = P_0(1 - e^{-13100t}) \sin \beta_m \theta \sin p_n z \tag{17}$$

where :

$$p_n = \frac{n\pi}{L}\ ,\ \beta_m = \frac{m\pi}{\phi}$$

The material properties are;

$$E_l = 85\ \text{Mpa} \quad E_T = 2.125\ \text{Mpa} \quad G_{LT} = 1.0625\ \text{Mpa} \quad G_{TT} = 0.425\ \text{Mpa}$$
$$\nu_{LT} = \nu_{TT} = 0.25 \quad \rho = 1408\ (\text{kg}/\text{m}^3)$$

The numerical results are described in the form of maximum nondimensional displacements and stresses as follow;

$$\bar{U}_r = \frac{100 E_T U_r}{P_0 h s^4} \quad , \quad (\bar{U}_x, \bar{U}_\theta) = \frac{100 E_T}{P_0 h s^3}(U_x, U_\theta) \quad , \quad \bar{\sigma}_r = \frac{\sigma_r}{P_0}\ ,\ S = \frac{R_0}{h}$$

$$(\bar{\sigma}_\theta, \bar{\sigma}_x, \bar{\tau}_{x\theta}) = (\sigma_\theta, \sigma_x, \tau_{x\theta})/P_0 S^2 \quad , \quad (\bar{\tau}_{r\theta}, \bar{\tau}_{rx}) = (\tau_{r\theta}, \tau_{rx})/P_0 S$$

The time history of in-plane shear stress ($\bar{\tau}_{r\theta}$) is shown in Figs. 2 The boundary and interlaminar conditions are satisfied in Fig. 2.With increasing the time , these stresses are proportionally increasing until reaching the peak time in loading function.

The variations of radial displacement in midradius ($\bar{U}_r$) with S for (0 / 90) lamination is shown in Fig.3. As it is shown in figure, with increasing S the radial displacement decreases fastly for S up to almost 40 and then flatten-out. From Fig.3 it is also concluded that for smaller S , the error of shallow panel theory is considerable.

The span angle (ϕ) is chosen as a suitable parameter in literature to find the ranges in which the shallow-shell and panel theories are acceptable. To find the suitable span angle, the changes of radial normal stress ($\bar{\sigma}_r$) and in-plane shear stress ($\bar{\tau}_{r\theta}$) through thickness are found for shallow and non-shallow panels and compared. These results for $\phi = 90$, 30 and 20 degs. are shown in Figs. 4 to 11 For $\phi = 20$ deg. the results are exactly the same, for $\phi = 30$ deg. the deviations are negligible, but for 90 deg. the maximum deviations increase up to 50 percent. The results show that for $\phi \le 30$ deg. the simplified shallow-panel formulations give acceptable results and can be used instead of more sophisticated non-shallow formulations.

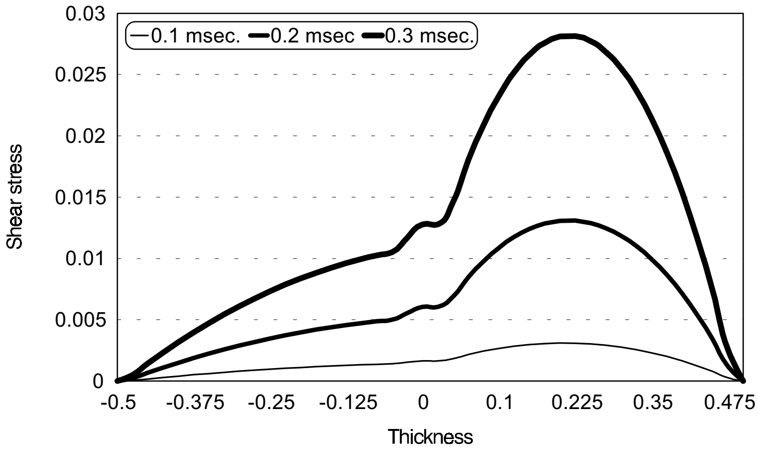

Fig.2 Variation of $\bar{\tau}_{r\theta}$ with Time (0 / 90 , S = 100 , shallow)

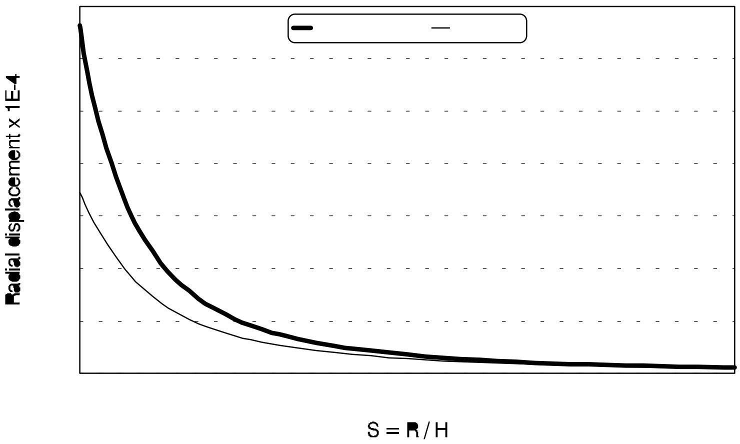

Fig.3 Variation of radial displacement ($\bar{U}_r$) with S (0 / 90)

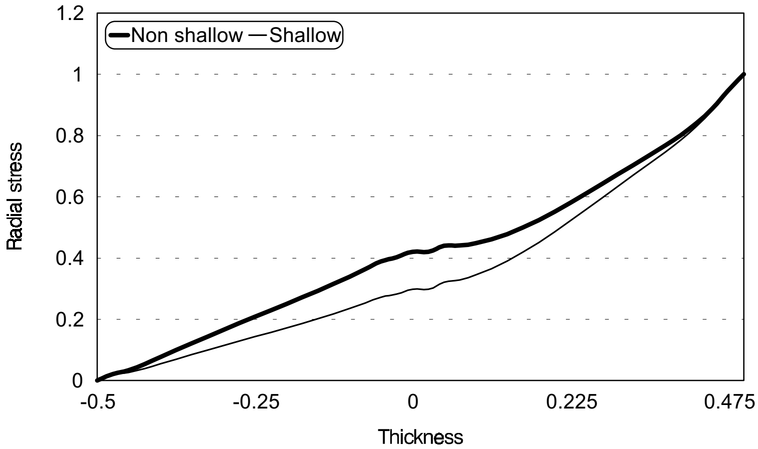

Fig. 4 Distribution of $\bar{\sigma}_r$ across thickness (0 / 90 ,S = 20, ϕ = 90 deg.)

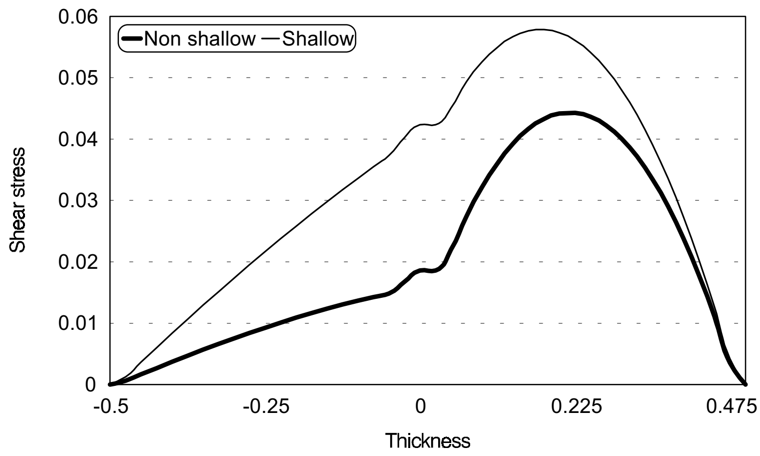

Fig. 5 Distribution of $\bar{\tau}_{r\theta}$ across thickness

(0 / 90 ,S = 20, ϕ = 90 deg.)

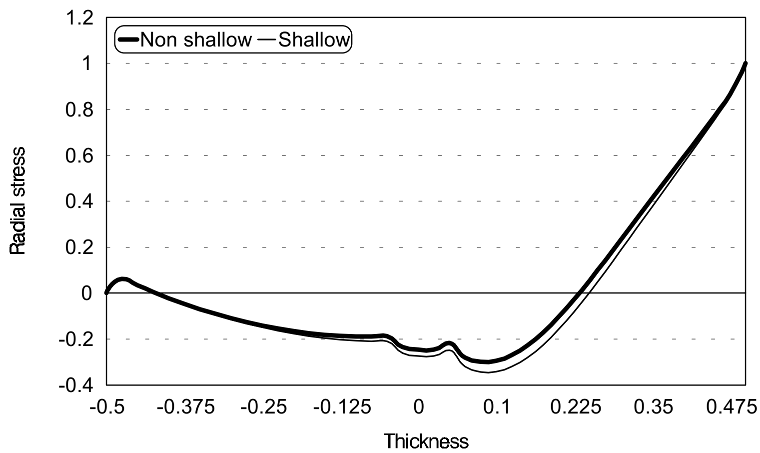

Fig.6 Distribution of $\bar{\sigma}_r$ across thickness

(0 / 90 ,S = 20, ϕ = 30 deg.)

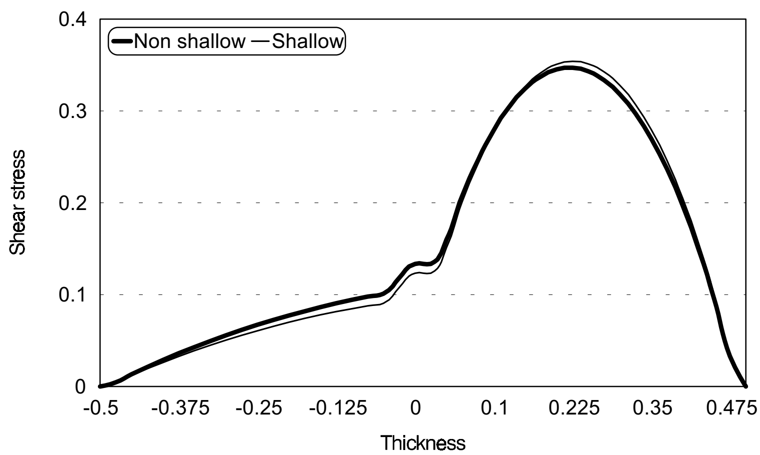

Fig.7 Distribution of $\bar{\tau}_{r\theta}$ across thickness

(0 / 90 ,S = 20, ϕ = 30 deg.)

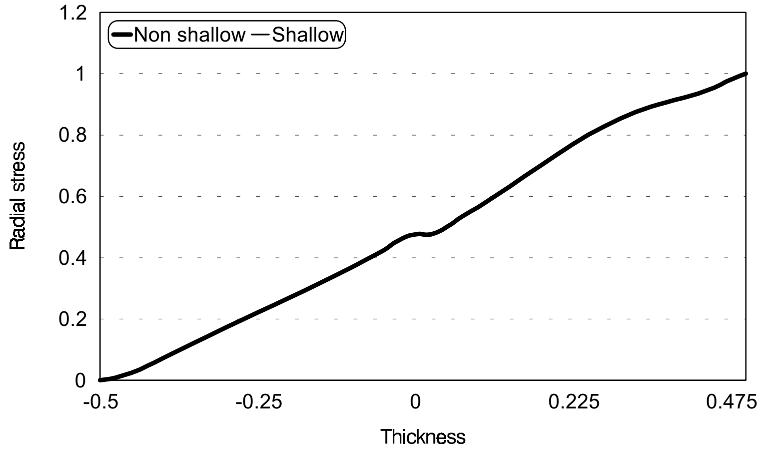

Fig.8 Distribution of $\bar{\sigma}_r$ across thickness

(0 / 90 ,S = 20, ϕ = 20 deg.)

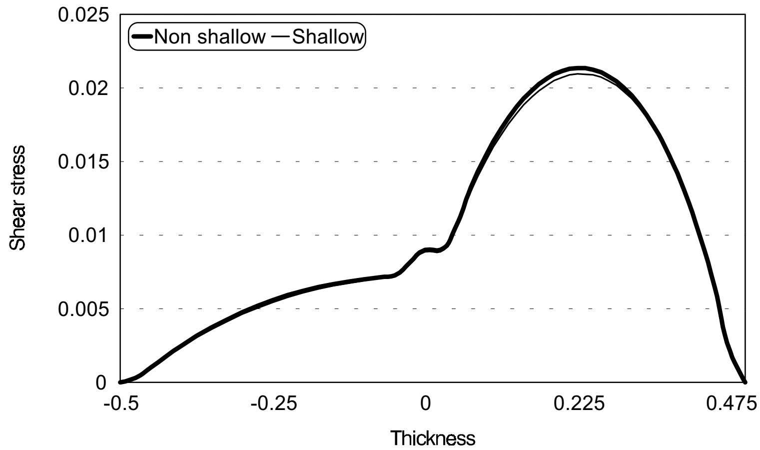

Fig.9 Distribution of $\bar{\tau}_{r\theta}$ across thickness

(0 / 90 ,S = 20, ϕ = 20 deg.)

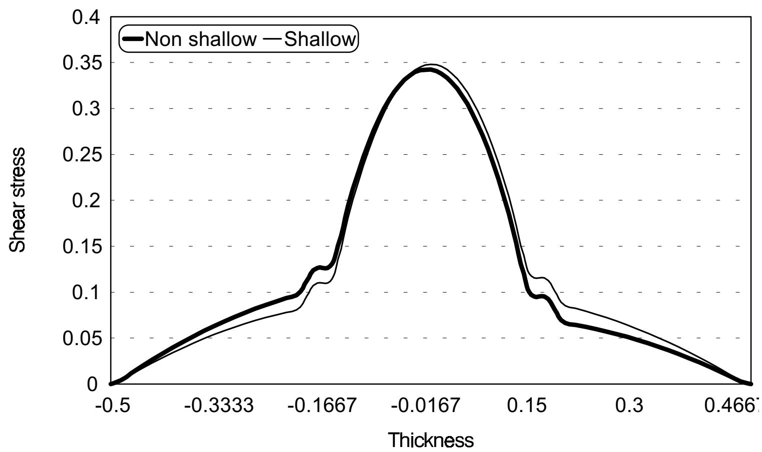

Fig.10 Distribution of $\bar{\tau}_{r\theta}$ across thickness

(0 / 90 / 0 ,S = 20, ϕ = 30 deg.)

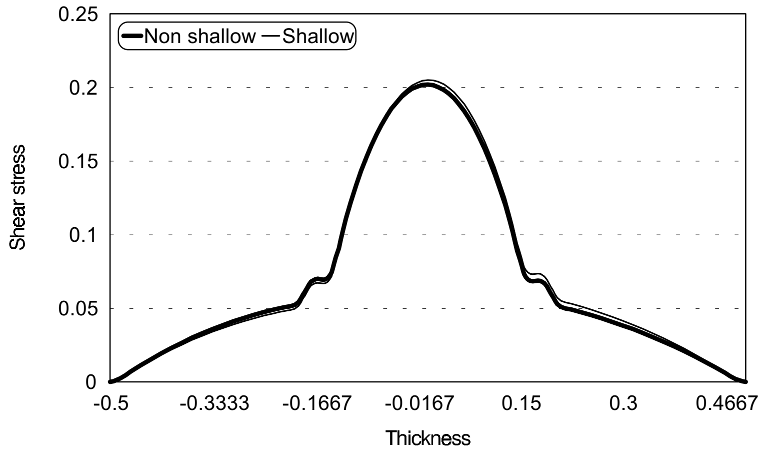

Fig.11 Distribution of $\bar{\tau}_{r\theta}$ across thickness

(0 / 90 / 0 ,S = 20, ϕ = 20 deg.)

REFERENCES

[1] Ren J. G., "Exact solutions for laminated cylindrical shells in cylindrical bending", J. of Composites Science and Technology, Vol.29, PP. 169-187 (1987).

[2] Ren J. G.,"Analysis of simply-supported laminated circular cylindrical shell roofs", Composites Structures, V11, PP. 277-292 (1989).

[3] Bhimaraddi A., "Free vibration analysis of doubly curved shallow shells on rectangular platform using three-dimensional elasticity theory", Int. J. Solids Structures, Vol.23, No.7, PP.897-913(1991) .

[4] Huang, N. N. & Tauchert, T. R., "Thermo elastic solution for cross-ply cylindrical panels", J. of Thermal Stresses, Vol.14, No. , pp. (1991).

[5] Bhimaraddi A. and Chandrashekhara K., "Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shell", J. of composite structures, Vol.20, PP. 227-235 (1992).

[6] Bhimaraddi A., " Three-dimensional elasticity solution for static response of orthotropic doubly curved shallow shells on rectangular planform",J. Composite Structures,Vol.24, PP.67-77(1993)

[7] Hung S. J. and Kuan G. T.,"Elasticity solution for laminated anisotropy cylindrical panels in cylindrical bending" , Composite Structures , Vol.30 , PP. 307-317 ,(1995) .

[8] Shakeri M., Eslami M. R. and Alibiglu A., “Dynamic analysis of thick laminated anisotropic cylindrical panel”,The Sixth East Asia-Pacific Conferences on Structural Engineering & Constraction, Taipei, Taiwan. PP. 714-725, January 14-16, (1998).
[9] Shakeri M., Eslami M. R., Yas M. H. and Alibiglu A. ,”Elasticity solution for laminated anisotropic cylindrical panel under impulse”,First Asian-australasian Conference on Composite Materials(ACCM-1),Osaka ,Japan,PP.746-1,746-2(October7-9,1998).